non-compact Cauchy surface, then $x \in I^+(y)$ if $\mathrm{Link}(X,Y)>0$, $x \in I^-(y)$ if $\mathrm{Link}(X,T)<0$, and $x \in M \setminus J(y)$ if $\mathrm{Link}(X,Y)=0$.

4) In curved space-times, the above results still hold locally in the sense that given any point $x \in M$ x has a causally convex globally hyperbolic neighbourhood with Cauchy slice $\mathbb{R}^3$, and for y lying in this set we obtain exactly the same relationship between linking of skies in the corresponding space of null geodesics as for Minkowski space.

Thanks to R. Baston and R. Penrose.

Robert Low.

References.

[1] Greenberg and Harper, Algebraic topology: a first course.

[2] Rolfsen, Knots and Links.

Abstract

A twistor conformal field theory for four space-time dimensions

A. P. Hodges R. Penrose M. A. Singer

A definition is proposed for four-dimensional conformal field theory in which a class of complex 3-manifolds and holomorphic sheaf cohomology replace the Riemann surfaces and holomorphic functions of two-dimensional theory. Suggestions are made about the interpretation of such a theory, in terms of the interaction of fundamental particles, in relation to the theory of twistor diagrams, and the possibility of extending it to incorporate gravity.

to appear in Physics Letters

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Complex paraconformal manifolds—their differential geometry and twistor theory

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October 7, 1988

Abstract

A complex paraconformal manifold is a pq-dimensional complex manifold ($p, q \geq 2$) whose tangent bundle factors as a tensor product of two bundles of ranks p and q. We also assume that we are given a fixed isomorphism of the highest exterior powers of the two bundles. Examples of such manifolds include 4 dimensional conformal manifolds (with spin structure) and complexified quaternionic, quaternionic Kähler and hyperKähler manifolds.

We develop the differential geometry of these structures, which is formally very similar to that of the special case of four dimensional conformal structures [30].

The examples have the property that they have a rich twistor theory, which we discuss in a unified way in the paraconformal category. In particular, we consider the 'non-linear graviton' construction [29], and discuss the structure on the twistor space corresponding to quaternionic Kähler and hyperKähler metrics.

We also define a family of special curves for these structures which in the 4-dimensional conformal case coincide with the conformal circles [34,2]. These curves have an intrinsic, naturally defined projective structure. In the particular case of complexified $4k$-dimensional quaternionic structures, we obtain a distinguished $8k+1$ parameter family of special curves satisfying a third order ODE in local coordinates.

*This work was carried out with support from the Australian Research Council. T.N.B. would also like to thank the University of Adelaide for hospitality.

Preprints Available.

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Conformal Circles and Parametrizations of Curves in Conformal Manifolds

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Abstract

We give a simple O.D.E. for the conformal circles on a conformal manifold, which gives the curves together with a family of preferred parametrizations. These parametrizations endow each conformal circle with a projective structure. The equation splits into two pieces, one of which gives the conformal circles independent of any parametrization, and another which can be applied to *any* curve to generate explicitly the projective structure which it inherits from the ambient conformal structure [1].

We discuss briefly the use of conformal circles to give preferred co-ordinates and metrics in the neighbourhood of a point, and sketch the relationship with twistor theory in the case of dimension four.

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*This work was carried out with support from the Australian Research Council. T.N.B. would like to thank the University of Adelaide for hospitality during this time.

AMS subject classifications. Primary 53A30; Secondary 58G30, 58G35.

Preprints Available.

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On the Twistor Description of Sourced Fields

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Abstract

Massless fields with source on an analytic world-line are double-valued, and it was shown by Bailey [1985] that a large family of such fields have a twistor description in terms of relative cohomology groups. In this paper it is proved that *all* right-handed massless fields are obtained in this way, and that if the sheaves $\mathcal{O}(n-2)$ are quotiented by the polynomials, then the relative cohomology of the resulting sheaves describes all left-handed sourced massless fields. The proof for right-handed fields uses techniques developed by Singer [1987,1988] for applying the Penrose transform to situations in which the 'pull-back mechanism' is non-trivial. For the left-handed fields it is necessary to use some additional arguments involving the conserved quantities (e. g. momentum and angular momentum for spin 2) of these fields; it is shown that the conserved quantities are the obstructions to a twistor description of left-handed fields in terms of the cohomology of $\mathcal{O}(n-2)$.

Relative cohomology and projective twistor diagrams

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10 October 1988

Abstract

The use of relative cohomology in the investigation of functionals on tensor products of twistor cohomology groups is considered and yields a significant reduction in the problem of looking for contours for the evaluation of (projective) twistor diagrams. The method is applied to some simple twistor diagrams and is used to show that the standard twistor kernel for the first order massless scalar ϕ^4 vertex admits a (cohomological) contour for only one of the physical channels. A new kernel is constructed for the ϕ^4 vertex which admits contours for all channels.

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A Hamiltonian Interpretation of Penrose's Quasi-Local Mass

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Abstract

A connection is established between Penrose's definitition of quasi-local mass and the more conventional notions of mass and momentum etc. arising from the canonical formalism of general relativity (which exist at least asymptotically). It is shown that the each component of the 'angular momentum' twistor can be thought of as the value of a Hamiltonian which generate motions of regions of the space-time which tend towards one of a collection of 'quasi-Killing vectors' on the bounding 2-surface on which the computations take place. The quasi-Killing vectors are obtained from solutions of the twistor equation, and essential use is made of the spinorial version of the gravitational Hamiltonian first employed in Witten's simplified proof of positive energy in general relativity.

These ideas are then used to suggest a variation on Penrose's quasi-local mass definition using 'quasi conformal Killing vectors' rather than quasi-Killing vectors. This has the advantage that there are only 16 *real* quantities rather than the 20 real (10 complex) ones from Penrose's original definition.

†Esmée Fairbairn Junior Research Fellow and Andrew Mellon Postdoctoral Fellow supported also in part by NSF grant no. PHY 80023. + FULBRIGHT SCHOLARSHIP.

To appear in Classical and Quantum Gravity

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Insights from Twistor Theory

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Abstract

This article discusses how twistor methods may be applied to problems arising from the canonical quantization of gravity. First of all the hypersurface twistor space construction is briefly reviewed, and a correspondence between (complex) initial data sets and a complex 3-manifold together with two cohomology classes is described.

Three possible applications of the methods are discussed. Firstly, a polarization condition analagous to that of positive frequency for initial data sets is presented. Secondly, it is argued that a canonical quantization procedure based on the use of the twistorial data would realize Penrose's suggestion that one should quantize gravity in such a way as to 'fuzz' out space-time points, leaving null directions well defined; the usual procedure smears the metric and therefore the null directions but leaves the space-time events fixed. Thirdly, it is pointed out that the gauge group for the twistor data is unrelated to the space-time diffeomorphism group so that the technical difficulties associated with factoring out the diffeomorphism group can be avoided.

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To appear in "Conceptual Problems in Quantum Gravity", Osgood Hill Conference Proceedings, Boston.

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Bäcklund transformations for the anti-self-dual

Yang-Mills equations

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Beginning from any given (local) solution of the GL(n,$\mathbb{C}$) anti-self-dual Yang-Mills (ASDYM) equations on Minkowski space, a simple technique for the generation of large classes of solutions (perhaps in some sense all) is given. The origin of this technique is described in terms of two versions of the Ward construction. The resulting description of Bäcklund transformations is sufficiently simple that it is then possible to identify the group generated by the collection of all such Bäcklund transformations and the space on which it acts in terms of concrete functions.

in J. Math. Phys. 29 (4) April 1988

Abstracts: -

Poon's Self-Dual Metrics
and Kähler Geometry.

Claude LeBrun

It is shown that the self-dual conformal metrics on connected sums of $\mathbb{CP}_2$'s recently produced by Y. S. Poon arise from zero scalar curvature Kähler metrics on blow-ups of $\mathbb{C}^2$ by adding a point at infinity and reversing the orientation.

[In J Differential Geometry 28 (1988) 341-343]

The Integral Constraint Vectors of Traschen
and Three-Surface Twistors.

K. P. Tod.

I relate the integral constraint vectors (ICV's) of Traschen to covariant constant sections of a connection at a hypersurface. I find the conditions for ten linearly independent ICV's to exist as the vanishing of the curvature of this connection. I interpret these conditions in terms of embeddability of the hypersurface in a space of constant curvature and I relate the ICV's to 3-surface twistors.

[In Gen Rel and Grav, 20 № 12, 1988]