

has to do with the leading term in the asymptotic expansion (8). The fact that this term diverges is an indication that there is a mis-match at  $i^0$  between the following two limits: sliding down a generator of  $\mathcal{I}^+$ , and, approaching  $i^0$  along a space-like direction and then making an infinite boost. The mis-match is a measure of the ADM 4-momentum. There is a similar mis-match in the way  $\mathcal{I}^-$  is attached to  $i^0$ . This mis-match should show up in the metric coefficients rather than the curvature. If one allows  $E$  to acquire a non-symmetric part under reflection, one can, by a logarithmic translation, remove the mis-match between  $i^0$  and  $\mathcal{I}^+$ . However, then the mis-match with  $\mathcal{I}^-$  is twice as big. There is something "cohomological" here: with one coordinate choice near  $i^0$ , it appears that  $\mathcal{I}^+$  matches "smoothly" on to  $i^0$ , whereas for another coordinate choice near  $i^0$  it would be  $\mathcal{I}^-$  that matches smoothly to  $i^0$ . The 4-momentum represents the mismatch between these two attempts at a smooth structure at  $i^0$ . It would seem that these two choices correspond to whether we use the intersections with  $\mathcal{I}^+$  or with  $\mathcal{I}^-$  to represent light rays in the space-time. There appears to be a relation to twistor theory here. A clearer treatment of this issue is needed to make further progress in a proper understanding of asymptotically flat space-times, e.g. along the lines initiated by Friedrich.

We have presented here only the overall picture. Some details are yet to be worked out fully. Also, the results can probably be generalized in a number of ways. A more complete account will appear elsewhere.

Abhay Ashtekar & Roger Penrose

#### References

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2. Abhay Ashtekar, In: General Relativity and Gravitation, Vol 2, edited by A. Held (Plenum, New York, 1980).
3. Abhay Ashtekar, Found. Phys. **15**, 419 (1985).

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*Bull. LMS to appear:*

## Conformally Invariant Operators: Singular cases.

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#### Abstract

All invariant linear differential operators between bundles of singular weight on flat conformal manifolds are determined and shown to have analogues on general conformal manifolds, obtained by adding suitable curvature correction terms.

obtained in this manner from the set of all invariants on any other given weight.

For invariants of densities this procedure and these results also work in other dimensions and for other structures (for example projective geometries) and also for the corresponding curved cases. On the other hand it is difficult to imagine that *this* scheme for producing the families of invariants can be generalised to deal with quantities other than densities (i.e. weighted tensors and spinors). However, at least in the flat case, these families can be generated in other ways [1] that work equally well for tensors. It is likely that similar results hold for this more general case – i.e. that all invariants occur in families. If this is true then the problem of producing a complete theory of invariants is considerably reduced.

## References

- [1] A.R.G. A theory of invariants for flat conformal and projective structures. *preprint*.

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## Quaternionic complexes.

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### Abstract

Each regular or semi-regular integral affine orbit of the Weyl group of  $\mathfrak{gl}(2n+2, \mathbb{C})$  invariantly determines a locally exact differential complex on a  $4n$  dimensional quaternionic manifold.

Duke Math J.:

## Almost Hermitian Symmetric Manifolds I Local Twistor Theory

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### Abstract

Conformal and projective structures are examples of structures on a manifold which are modelled on the structure groups of Hermitian symmetric spaces. We show that each such structure has associated a distinguished vector bundle (or *local twistor bundle*) equipped with a connection (*local twistor transport*). For projective and conformal manifolds, this is Cartan's connection. The curvature of the connection provides an tensor invariant which vanishes if and only if the manifold is locally isomorphic to a Hermitian symmetric space.

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## Almost Hermitian Symmetric Manifolds II Differential Invariants

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### Abstract

We use local twistor connections and Lie algebra cohomology to construct linear differential operators depending invariantly on almost Hermitian symmetric structures. All *standard* homogeneous (flat) operators admit curved analogues whilst most *nonstandard* ones appear obstructed; these obstructions yield further invariants of the AHS structure. The methods of the paper, applied in the flat case to irreducible quotients of Verma modules, construct nonstandard homomorphisms of Verma modules given (relative) Kazhdan-Lustzig polynomials.

# Zuckerman functors, the Penrose transform and homomorphisms of Verma modules.

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## A multiplicity one theorem for Zuckerman's functors

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### Abstract

Let  $\mathfrak{g}$  be a complex semisimple Lie algebra and  $\mathfrak{p}, \mathfrak{r}$  standard parabolic subalgebras with  $\mathfrak{k}$  a reductive Levi factor of  $\mathfrak{p}$ . We prove, via the *Penrose transform* and results of Enright-Shelton on semi-regular categories, that irreducibles occur with multiplicity at most one in the cohomologically induced modules  $\oplus_i \Gamma_{\mathfrak{k}}^i N$ . Here  $N$  is a (dual) Verma module induced from  $\mathfrak{r}$ , and  $\mathfrak{p}, \mathfrak{r}$  are of a restricted class which includes all Hermitian symmetric cases. This, with Vogan's  $U_{\alpha}$  calculus, gives an effective method for computing the irreducible subquotients of the  $\Gamma_{\mathfrak{k}}^i N$ .

The methods used generalize to a geometric setting, in which  $\Gamma_{\mathfrak{k}}^i N$  is replaced by holomorphic sheaf cohomology over an open subvariety of  $G/R$  and may be viewed as an extension of the Bott-Borel-Weil theorem to such domains.

Invariants of *CR* Densities

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## Invariants of Conformal Densities

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**Abstract**

This article is concerned with problems in parabolic invariant theory arising from flat conformal geometry. We show that such problems may be formulated in terms of the variational complex, taken from the formal theory of the calculus of variations. From known properties of this complex we are able to write down the general scalar differential invariant of functions in odd dimensions under conformal motions.