Abstracts

On Bell non-locality without probabilities: some curious geometry

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Abstract. In 1966, John Bell showed how Gleason's 1957 theorem can be used to demonstrate the incompatibility of the predictions of quantum theory with "non-contextual" hidden variable models. Later, Kochen and Specker independently found a set of 117 (unoriented) spatial directions that exhibited this incompatibility in a finite explicit way. Such configurations have been used (Heywood and Redhead 1983, Stairs 1983, Brown and Svetlichny 1990) as part of an EPR system, to show that the non-contextual assumption can be replaced by one of locality. This, like results obtained recently by Greenberger, Horne, Zeilinger (GHZ) and others illustrates a conflict between quantum mechanics and locality that shows up in yes/no constraints on the results of certain idealized experiments, no probabilities being involved. Kochen and Specker's original set of 117 directions, for a 3-state (spin 1) system , has recently been reduced to 33 by Peres (1990a) (and to 31 by Conway and Kochen). Peres has also exhibited a set of 24 Hilbert-space directions, with similar properties, for a 4-state system, these being the common eigenstates of sets of commuting operators among a set of 9 found by Peres (1990b) (and Mermin). In this article, I show how Peres's set of 33 directions can be directly visualized in terms of a geometrical configuration (three interpenetrating cubes) that appears in the Escher print "Waterfall". Using the Majorana description of general spin states, I also exhibit a quite different set of 33 idealized measurements that can be performed on a spin 1 system. These measurements are specified in terms of an explicit set of 18 oriented directions in space. The configuration involved in Peres's set of 24 Hilbert-space directions can be understood in terms of a 4-dimensional regular polytope known as the "24-cell", and they are, in principle, ideally suited to providing an EPR-type of GHZ non-locality without probabilities. Unfortunately, if each 4-state system is taken to be a spin 3/2 particle, no simple spatial geometrical description of the needed measurements seems to emerge. Instead, I provide an alternative configuration for spin 3/2, based on a regular dodecahedron, in which only 20 oriented directions are explicitly used.

Existence and Deformation Theory for Scalar-Flat Kähler Metrics on Compact Complex Surfaces

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Abstract

Let M^4 be a compact complex 2-manifold which admits a Kähler metric whose scalar curvature has integral zero. Suppose, moreover, that $\pi_1(M)$ does not contain an Abelian subgroup of finite index. Then if M is blown up at sufficiently many points, the resulting complex manifold $\tilde{M}$ admits Kähler metrics with scalar curvature identically zero. The proof, which proceeds by deforming the explicit metrics constructed in [27], hinges on a remarkable relationship between Kodaira-Spencer theory and the Futaki invariant that arises via the Penrose transform. In the process, we point out a relationship between the existence problem for scalar-flat Kähler metrics and the parabolic stability of vector bundles in the sense of Seshadri [38].

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POSITIVE EINSTEIN METRICS WITH SMALL $L^{n/2}$-NORM OF THE WEYL TENSOR

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Abstract: A gravitational analogue is given of Min-Oo's gap theorem for Yang-Mills fields.

Keywords: Riemannian manifold, Einstein metric, Weyl tensor, L^p-norm, Sobolev constant, Euler characteristic.

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INTRODUCTION

In this note we prove

Theorem 1. *Let M be a compact oriented n-manifold ($n = 2m \geq 4$) with non-vanishing Euler characteristic $\chi(M)$ and let g be a (Riemannian) positive Einstein metric on M with Weyl curvature W. Then there is a constant $\varepsilon > 0$, depending only upon n and $\chi(M)$, such that if $||W||_{L^{n/2}} < \varepsilon$, then $W = 0$ (and so M is isometric to a quotient of S^n with the standard metric).*

The Fröhlicher Spectral Sequence on a Twistor Space

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1 Introduction

Associated to any compact self-dual four-manifold M is a compact complex three-dimensional manifold Z known as its twistor space [1,18]. Twistor spaces provide a source of interesting complex three-manifolds (cf. [6]). The purpose of this article is to investigate the Fröhlicher spectral sequence [9]

$$E_1^{p,q} = H^q(Z, \Omega^p) \Longrightarrow H^{p+q}(Z, \mathbf{C})$$

where Ω^p denotes the sheaf of holomorphic p-forms on Z. The Penrose transform [2,3,4,7,12] interprets the Dolbeault cohomology $H^q(Z, \Omega^p)$ in terms of differential equations on M. In this way, the Fröhlicher spectral sequence has differential-geometric consequences on M and vice versa.

We shall explain this interpretation and its consequences. For, example we shall show that $E_1 = E_\infty$ if and only if a certain conformally invariant system of linear differential equations has only constant solutions. The classical case in which $E_1 = E_\infty$ is when Z admits a Kähler metric. Hitchin [13] has shown that there are only two such twistor spaces, namely $\mathbf{CP}_3$ and the space of flags in $\mathbf{C}^3$. However, we shall construct other twistor spaces with $E_1 = E_\infty$. We shall show that if $E_1 \neq E_\infty$, then $E_2 = E_\infty$ and that this possibility does occur.