Gravitational Collapse of the Wave Function: New Thoughts

A desire for a non-local basis for physical reality, as a means of making sense of the puzzling non-locality that occurs with Einstein-Podolsky-Rosen-type quantum measurements, was one powerful original motivation underlying twistor theory. According to this "twistor" view, it will not be possible to provide an adequate understanding of the state-vector reduction that quantum measurement induces, in ordinary space-time terms. In TN 26, p.25, I indicated a (somewhat fanciful) picture of how a twistor-type picture might conceivably provide some kind of resolution of the non-local puzzles that state-vector reduction confronts us with, but a properly coherent viewpoint must await appropriate fundamental changes in our present theory.

Despite the fact that we are a long way from a coherent theory, I believe that it has been possible to make some new progress which is consistent with the motivational ideas that I have described elsewhere (cf. TN 22, p.1-3, and "The Emperor's New Mind" Chapter 8) — that state-vector reduction should be a gravitational phenomenon. But the new suggestions that I am presenting here differ quite significantly from the specific proposals that I made earlier.

The starting point for this development is an expression that had some relevance to my earlier ideas, namely the integral that describes the symplectic form $\{K_1, K_2\}$ on the space of spin 2 massless fields in Minkowski space, whose linearized curvatures are given by

$$K_{abcd} = \psi_{ABCD}\,\varepsilon_{A'B'}\varepsilon_{C'D'} + \varepsilon_{AB}\,\varepsilon_{CD}\,\overline{\psi}_{A'B'C'D'}\,.$$

This is given (up to a factor) by

$$\{K_1, K_2\} = \int_\Sigma \left(\psi_{1\,ABCD}\,\overline{\eta}_{2\,A'}{}^{BCD} + \overline{\psi}_{1\,A'B'C'D'}\,\eta_{2\,A}{}^{B'C'D'} \right) d^3x^{AA'}$$

taken over a (normally spacelike) 3-surface Σ. Integrating by parts we find, schematically

$$\int \psi_1\,\overline{\eta}_2 = -\int \gamma_1\,\overline{\chi}_2 = \int \chi_1\,\overline{\gamma}_2 = -\int \eta_1\,\overline{\psi}_2 \quad \text{(whence } \{K_1, K_2\} = -\{K\ K\}\text{)}$$

where we have a (Dirac) chain of potentials

$$\nabla_{BB'}\eta_{A}{}^{B'C'D'} = \chi_{AB}{}^{C'D'}, \quad \nabla^{AA'}\eta_{A}{}^{B'C'D'} = 0,$$

$$\nabla_{CC'}\chi_{AB}{}^{C'D'} = \gamma_{ABC}{}^{D'}, \quad \nabla^{AA'}\chi_{AB}{}^{C'D'} = 0$$

$$\nabla_{DD'}\gamma_{ABC}{}^{D'} = \psi_{ABCD}, \quad \nabla^{AA'}\gamma_{ABC}{}^{D'} = 0$$

$$\nabla^{AA'}\psi_{ABCD} = 0$$

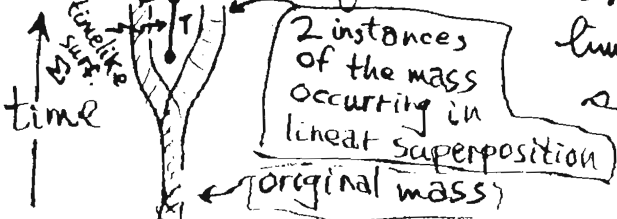
, all spinors η, χ, γ, ψ being symmetric.

N.B. $\gamma_{\dots}$ are linearized spin-coefficients and $\Gamma \sim \gamma + \bar{\gamma}$ Christoffel
Also $h \sim \chi + \bar{\chi}$ lin. metric

The symplectic form $\{,\}$ is closely related to the scalar product $\langle\,|\,\rangle$, but for that a positive/negative frequency split must be made. In fact $\langle K|K\rangle = i\{K, JK\}$ where J multiplies the positive-frequency part of K by i and the negative-frequency part by $-i$.

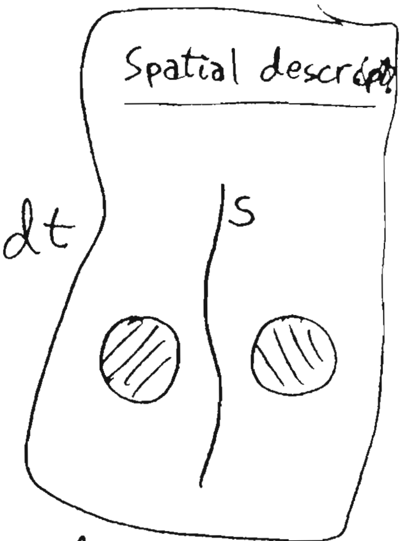
The idea underlying my earlier proposal was that for a quantum state consisting of a superposition of two significantly different $\underset{1}{\psi}$ and $\underset{2}{\psi}$ fields, state-vector reduction would take place when the difference between these states reached roughly the (longitudinal) "one-graviton level".
This might be measured in terms of the "graviton number" in the difference field $\underset{1}{\psi} - \underset{2}{\psi}$, which might be obtained as an appropriate integral over a spacelike hypersurface.
The view had been that some measure of <u>longitudinal</u> graviton number would accordingly be required, but this leads to profound problems, owing to the fact that the concept is not gauge invariant. The present idea is to use something closer to the symplectic form and to try to examine the Newtonian limit — since most situations of wave-function collapse occur when gravitational fields are weak and velocities small.

Consider a simple "Schrodinger's cat" type of situation in which a quantum measurement is to be achieved by moving a mass from one location to another. We thus have a superposition of the mass in one place with the mass in another. The question, according to the present point of view is: at what stage, as the masses are moved apart, should collapse occur? It appears that in the Newtonian limit, it makes more sense to do something like using a <u>timelike</u> hypersurface



between the two masses and to perform the symplectic form integral for the two fields on that hypersurface. We take collapse to occur roughly when the region of integration is large enough that the integral reaches order unity. The integral can be written

$$\int_\Sigma \gamma_1 \bar{\chi}_2 + \int_\Sigma \bar{\gamma}_1 \chi_2 \sim \int_\Sigma (\Gamma_1 h_2 - \Gamma_2 h_1)$$



which in Newtonian terms becomes

$$\int_t^{t+T}\left(\int_S (\phi_2 \vec{\nabla}\phi_1 - \phi_1 \vec{\nabla}\phi_2)\cdot d\vec{x}\right)dt$$

$$= T\left(\int_S (\quad " \quad)\cdot d\vec{x}\right).$$

Here T is the ~~total~~ time that the surface S persists for, so for the integral to be order unity we require T (= ~ collapse time) $= \dfrac{1}{\int_S(\phi_2\vec{\nabla}\phi_1 - \phi_1\vec{\nabla}\phi_2)}$.
Note that $\phi_2\vec{\nabla}\phi_1 - \phi_1\vec{\nabla}\phi_2$ is divergence-free in vacuum, so the integral is unchanged as S is moved continuously without crossing matter. When matter density ρ is present we see $\vec{\nabla}\cdot(\phi_2\vec{\nabla}\phi_1 - \phi_1\vec{\nabla}\phi_2) = -4\pi\phi_2\rho_1 + 4\pi\rho_2\phi_1$, so moving S across one mass, we find that our expression is just the gravitational energy of one mass in the gravitational field of the other ($\times 4\pi$). the collapse time this suggests is the reciprocal of this energy.

As it stands, this cannot be quite right, but it suggests a much more plausible expression, namely the energy that it would take to move the two masses apart from their initial positions of coincidence (gravitational energy only). It is worthwhile to consider the Newtonian expression in the case of a uniform sphere, mass m, radius a, distance between centres b. We get, with $\lambda = b/2a$

$$\text{Energy to separate} = \begin{cases} \dfrac{m^2G}{a}\left(6/5 - 2\lambda^2 + \tfrac{3}{2}\lambda^3 - \tfrac{1}{5}\lambda^5\right) & (0 \le \lambda \le 1) \\ \dfrac{m^2G}{2a\lambda} & (1 \le \lambda). \end{cases}$$

This is $\sim \frac{m^2G}{a}$. In natural units ($G=c=1=\hbar$; gram $=10^5$, sec $=10^{43}$, cm $=10^{33}$) we get: for a neutron, $T \sim 10^{60}$ = Hubble time
" " droplet of water of $a = 10^{-5}$ cm, $T = 10^{48}$ = day ; $a = 10^{-4}$ cm, $T = 10^{43}$ = sec;
$a = 10^{-3}$ cm, $T = 10^{38} = 10^{-5}$ sec. All this seems fairly plausible.
Thanks especially to Ted Newman and Abhay Ashtekar. ~ R Penrose