Puzzle Page (as in TN 34)

Who wrote this? When was it published?

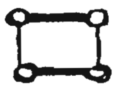
What does it mean?

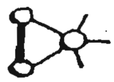
$f =$ 

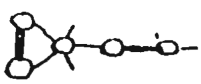
$g =$ 

~~izzotor.~~

$i =$ 

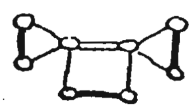
$A =$ 

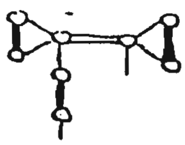
$j =$ 

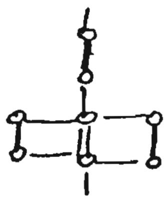
$k =$ 

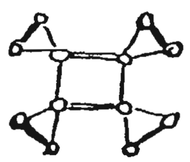
$\tau =$ 

$$\Delta_k = A\tau - Bi = \upsilon$$

$B =$ 

$\vartheta =$ 

$\upsilon =$ 

$C =$ 

$$D_{A\tau - Bi} = A^2C - AB^2 + AB^2$$
$$= A^2C$$

$$\Theta = \kappa^2 + \frac{A}{2}\lambda^2$$

$$J = \kappa j + \lambda k$$

$$f_F = F = \kappa f + \lambda g$$

$$g_F = \frac{1}{2}\Theta . F\Theta$$

Answers on the next page

Solution to Puzzle in TN 34

The "twistor diagrams" were not drawn by Andrew Hodges (or by me) but by William Kingdon Clifford, the great 19th century twistor theorist (1845–1879), discoverer of Clifford algebras (on which the theory of n-dimensional spinors – and twistors – is based) and of Clifford parallels on S^3 (whose stereographic projection to Euclidean 3-space provides the twistor picture that now adorns all TN covers), and who also anticipated Einstein in suggesting that the presence of matter is to be identified with the curving of space (a profound insight that twistor theory still strives doggedly to come to terms with). After he died, at an early age (34), several of Clifford's unpublished papers were collected together posthumously and published in 1881 under the title "Mathematical Fragments, being Facsimilies of his Unfinished Papers Relating to the Theory of Graphs" — which suggests to me that those who collected the papers together did not really understand what they were about!

What are they about then? It should be borne in mind that one of the prevailing interests among algebraists of the late 19th century was the theory of invariants and, in particular, the construction of "complete sets of invariants" for binary forms. A binary form is a homogeneous polynomial (normally with complex coefficients) in two variables ξ, η. (Three or more variables was considered to be too hard, at that time.) An invariant of a binary form (or collection of such forms) would be a polynomial in the coefficients of the form(s) which is invariant under (complex) unimodular linear transformations of ξ and η. A covariant would be a corresponding thing, but which is a homogeneous polynomial in ξ, η. Nowadays, we would use tensor algebra to construct such things, but in the late 19th and early 20th century, the "symbolic calculus" was widely used. This calculus employed a strange notation that can be described roughly as follows, using a comparison with the spinor notation that is more familiar to us now.

Spinor notn.	Symbolic notn
Take $x^0 = \xi$, $x^1 = \eta$	All factors commute,
All spinors symmetric	But $(\alpha\beta) = -(\beta\alpha)$

Examples	Spinor notn.	Symbolic notn.
forms	$\alpha_A x^A$	$\alpha_x = \beta_x = \dots$
	$\alpha_{AB} x^A x^B$	α_x^2 or $\alpha_x \alpha_x$ or β_x^2 etc.
	$\alpha_{\underbrace{AB\dots L}_{n}} x^A x^B \dots x^L$	$\alpha_x^n = \beta_x^n = \gamma_x^n = \dots$
invariants	$\alpha_{AB}\alpha^{AB} = \alpha_{AB}\alpha_{CD}\varepsilon^{AC}\varepsilon^{BD}$	$(\alpha\beta)^2 = (\alpha\beta)(\alpha\beta)$
	$\alpha_{ABC}\alpha^{ABD}\alpha_{EFD}\alpha^{EFC}$	$(\alpha\beta)^2(\alpha\gamma)(\beta\delta)(\gamma\delta)^2$
covariants	$\alpha_{AB}\alpha_{CD}x^A x^B x^C x^D$	$\alpha_x^2 \beta_x^2$
	$\alpha_{AB}\alpha_{CD}\alpha_{EF}x^A x^F \varepsilon^{BC}\varepsilon^{DE}$	$\alpha_x(\alpha\beta)(\beta\gamma)\gamma_x$

A complete set of invariants [covariants] is a set from which all others can be constructed as polynomials in members of the set.

Now Clifford, with characteristic insight, not only effectively employs tensor (or spinor) ideas, but also uses a <u>diagrammatic</u> notation (similar to that described in the appendix to vol. I of Spinors and Space-Time). On the Puzzle Page, TN 34, we have (all expressions up to sign — I'm not sure how Clifford dealt with signs)

f standing for, say α_{ABCDE} (i.e. for $\alpha_{\dots\dots}x^{\cdot}x^{\cdot}x^{\cdot}x^{\cdot}$, i.e. α_x^5)

i " " $\alpha_{ABCD(E}\alpha_{F)}{}^{ABCD}$ (i.e. $\alpha_{ABCDE}\alpha_F{}^{ABCD}x^E x^F$, i.e. $(\alpha\beta)^4\alpha_x\beta_x$)

(I'm not <u>sure</u> abt. symmetry bracket.)

g " " $\alpha_{ABCD(E}\alpha^{ABCDF}\alpha_{GHJK)F}$ (i.e. $(\alpha\beta)^4(\gamma\beta)\alpha_x\gamma_x^4$)

etc. The thick lines seem to be 4-fold lines.

At the bottom of p. xxii Clifford seems to be working out a (complete?) set of covariants for a binary cubic (black spots) together with a binary quartic (white spots). Note the times at the left-hand side, evidently indicating how long it took him to work things out, using his notation.

All credit to K.P.T. for spotting this publication among the books being cleared out of St Johns library.

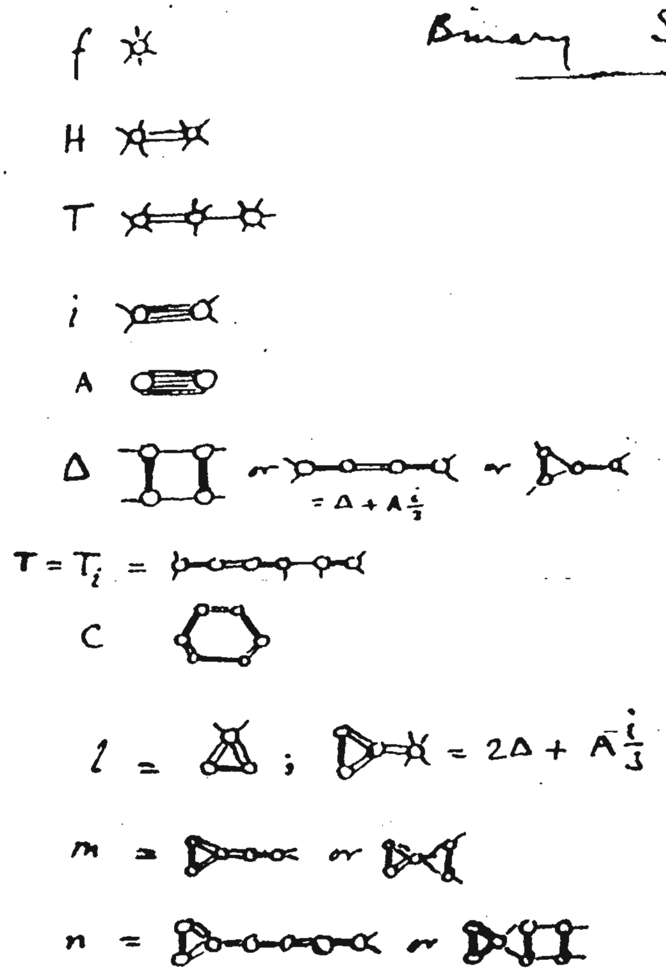
Roger Penrose

Reference

Grace, J.H. & Young, A. (1903) The Algebra of Invariants (Camb. Univ. Press)

39

Binary Sextic



these are really equivalent because i is symmetrical

Table of invariants of ~~cubic &~~ quadratic & cubic, f_2, φ_3

, 0 $D = (ab)^2$

, 4 $R = (\Delta\Delta')^2$

, 2 $E = (a\Delta)^2$

, 2 $F = (ap)^2 =$

$M = (ap)(ar)$

here is a series of invariants of a quadratic and any other binary form analogous to the resultant: viz. $(x\alpha . x\beta . y\alpha . y\beta)^k \left(\overline{x\alpha' y\beta}^{\,n-k} + \overline{x\beta . y\alpha}^{\,n-k} \right)$ where x, y are factors of the quadratic and $\alpha_2^n \cdot \beta_1^n$ is the form of the n^{th} order. Their evanescence is the condition that one element of the quadric should be the k^{th} polar of the other.

...ne in the case of a conic C_2 and a curve K_n ~~we ms~~ there is a curve of

MATHEMATICAL FRAGMENTS

BEING

FACSIMILES OF HIS UNFINISHED PAPERS

RELATING TO THE

THEORY OF GRAPHS

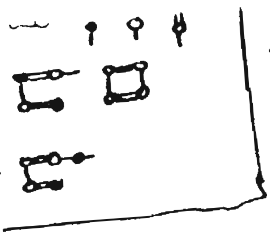
BY THE LATE

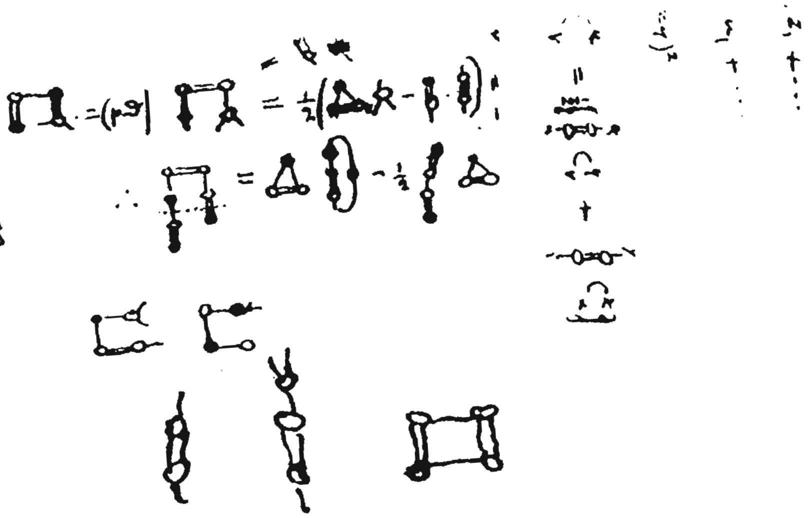
W. K. CLIFFORD

London

MACMILLAN AND CO.

1881



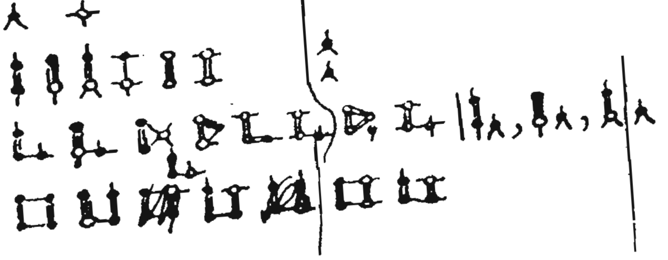


$$a(xu).b(yu) + a(yu).b(xu) = a(yu).b(yu)..|xy|$$

$$a(xu).b(yu).c(yz) + a(yu).b(xu).c(yz) = -a(yu).b(yu)..c(xz)$$

$$a(yu).b(zu).c(yx) + a(yu).b(xu).c(yz) = a(yu).b(zu)\,c(yx)..|xz|$$

$$\therefore a(xu).b(yu).c(yz) + c(xy).a(uy).b(uz)$$



1

$\frac{1}{2}$ hr
10 min