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Twistors and the Einstein Equations

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Abstract It has been proposed that the appropriate global definition of a twistor, applicable to general curved vacuum space-times, would be as a charge for a massless field of helicity $^3/_2$. In flat space-time, using the Dirac form of these potentials, these twistor charges arise as the "gauge freedom of the second kind" in a long exact sequence involving the first and second potentials for the field.

A construction due to Ward is recalled, in which potentials for massless fields can act as partial connections on non-linear bundles, integrable on β-planes. This is generalized, in the case of helicity $^3/_2$, to provide a full connection on a vector bundle of rank 3, leading to an expression whereby the usual Rarita-Schwinger potential is supplemented by a second potential.

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NEW MASSLESS FREE FIELDS IN OLD SPACETIMES

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Abstract

A kind of massless free field, a "symmetric recurrent" spinor field is defined. The principal spinors of such a field define shear-free ray congruences. A vacuum solution of Einstein's equations is type {2,2},{4} or conformally flat iff its Weyl spinor is symmetric recurrent. The massless free fields of the Robinson-Sommers theorem are symmetric recurrent. A spacetime with a certain kind of symmetric recurrent spinor admits a Killing spinor. A massless free field associated with a Killing spinor is symmetric recurrent. Symmetric recurrent fields are constructed for spacetimes with certain types of Killing spinor, essentially one per Killing spinor.

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ABSTRACT OF D.Phil THESIS by F.Müller

The ladder representations $\mathcal{H}_k$, $\overline{\mathcal{H}}_k$ ($k \in \mathbb{Z}$) of $SU(2,2)$ which via the Penrose transform correspond to free massless fields of helicity $\mp k\hbar/2$ have analogues for all $SU(p,q)$. We extend the formalism of twistor diagrams accordingly and construct projection operators from tensor products of such representations into irreducible subspaces by means of these generalised twistor diagrams. We first consider the case $p = q = 1$ where $\mathcal{H}_k$ ($\overline{\mathcal{H}}_k$) exhaust all discrete series representations with lowest (highest) weight. We formulate projection operators on $\mathcal{H}_k \otimes \mathcal{H}_l$ in algebraic, diagrammatic and analytic ways and obtain formulae to translate between these various guises. For realisations of $\mathcal{H}_k$ on spaces of sections we then establish an equivalence of diagram composition with the algebraic composition of operators which is expected to carry over to realisations on higher cohomology groups. We achieve this by explicit construction of a contour of integration on which a power series expansion of the linking segments is possible. We use this equivalence to prove that, as a representation of $SU(1,1)$, any finite tensor product $\otimes_{i=0}^{n} \mathcal{H}_{k_i}^{i}$ can be decomposed by compositions of 'box diagrams'. These constructions are shown to carry over to $SU(p,q)$ and it is indicated, by means of examples, how they might be complemented to form complete sets of projections in the general case. This requires some explicit decomposition formulae which we give in a restricted case for $SU(2,1)$. For the decomposition of $\mathcal{H}_k \otimes \mathcal{H}_l$ the $SU(1,1)$ projections are almost sufficient and we extend them to a complete set for $SU(p,q)$. The translation of algebraic expressions for projections back into diagrams is in general found to be a simplification.

We then use our techniques to formulate a number of conformally invariant first order scattering amplitudes in terms of projections on tensor products of ladder representations and make a few remarks on extensions to conformally non-invariant cases. Apart from giving invariant descriptions of such amplitudes the use of orthogonal states and diagrams in higher dimensions also simplifies the calculational aspects of diagram integration.

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