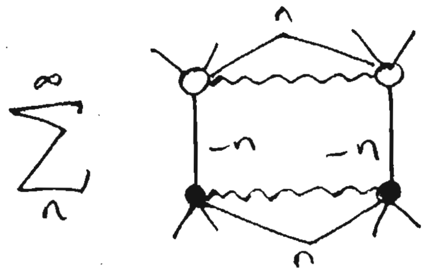
Progress with the massive propagator in twistor diagrams

In TN37 I outlined some new ideas on defining twistor diagrams for the massive Feynman propagator. The first key idea was that of replacing the original (1990) conjecture



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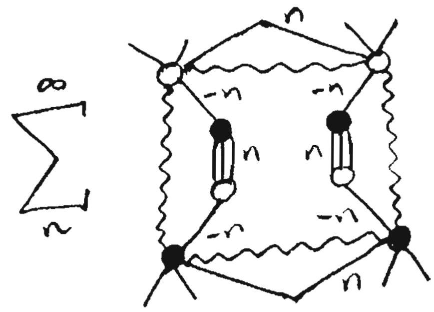
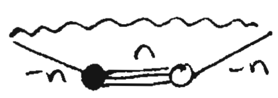


Some formulas were given which, it was stated, show that the new version must yield a solution of the inhomogeneous Klein-Gordon equation. But actually it is also necessary that a particular contour choice is made, namely a *double period* contour. At first sight one might think that introducing two boundaries, then taking two periods, will just get us back to where we began. But in fact we get back to something *almost* the same, but differing by lacking just the unwanted terms which arise (as shown by S.T.S.) in the originally conjectured integrals. But why should this be? A further observation helps give some intuitive feel, viz. that:

$-k\frac{\partial}{\partial k}$  $=$ 

Roughly, this means that making the replacement is like performing two operations of $(-k\frac{\partial}{\partial k})^{-1}$, which will transform $\log(W.Z/k)$ into $\frac{1}{6}\log^3(W.Z/k)$. On taking the double period, we regain $\log(W.Z/k)$, so that the 'wanted' term is left intact. However the 'unwanted' terms vanish under this sequence of operations. So the replacement acts almost like a projection operator for the wanted terms. One must say 'almost' because $\frac{1}{2}\log^2((W.Z/k)$ is transformed not to itself but to $\frac{1}{2}\log^2((W.Z/k) - \frac{1}{3}\pi^2$. We return to this point later.

This rough idea can be made exact. In doing so we are greatly helped by an observation about integrals where a boundary on $W.Z = k$ goes with an integrand of form $\log(W.Z/k)$. Hitherto, our methods (as in Lewis O'Donald's work) have required the expansion of the integrand as a power series in $(W.Z - k)$. In the present case these methods would lead to the summation of a formidable triple power series. However it turns out that it's quite unnecessary actually to do such a summation. Recall first that

has the property of vanishing when W.Z=k. Analogously, the spinor integral

$$\oint_{x.a=k,\ w.c=k} \frac{(w.c-k)^n\,(x.a-k)^n}{(w.x-k)^2}\, d^2w\wedge d^2x$$

vanishes when a.c=k; and it can be shown (by using power series) that this feature also extends to the more general

$$\oint_{\substack{x.a=k\\ w.c=k}} \frac{\left(\frac{x.a}{k}\right)^p\left(\frac{w.c}{k}\right)^q}{(w.x-k)^2}\, d^2w\wedge d^2x \qquad (*)$$

where p and q are any complex numbers. It follows also that

$$\oint_{\substack{x.a=k_1\\ w.c=k_2}} \frac{\left[\log\left(\frac{x.a}{k_1}\right)\right]^m\left[\log\left(\frac{w.c}{k_2}\right)\right]^n}{m!\ (w.x-\mu)^2\ n!}\, d^2w\wedge d^2x$$

has the property of vanishing when μ a.c = k_1k_2, for each m, n. Moreover, the value of this integral must be

$$\frac{\left[\log\left(\frac{\mu\, a.c}{k_1 k_2}\right)\right]^{m+n+1}}{(m+n+1)!}$$

(Proof: double induction on m and n, using repeated integration with respect to k_1, k_2, starting from the known result for m=n=0). Now write

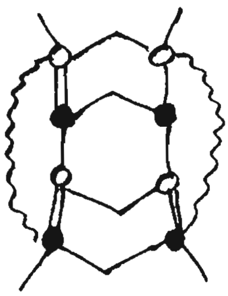
$$\left(\frac{x.a}{k}\right)^p = \sum_{m=0}^{\infty} \frac{p^m}{m!}\left[\log\left(\frac{x.a}{k}\right)\right]^m$$

and by summing the resulting double series we deduce

$$(*) = \frac{\left(\frac{a.c}{k}\right)^p - \left(\frac{a.c}{k}\right)^q}{p-q} = \frac{1}{2\pi i}\oint \frac{\left(\frac{a.c}{k}\right)^s ds}{(s-p)(s-q)}$$

By inserting a factor $(2i \sin\pi s)^2$ into the integrand we obtain the result of the double period contour. By straightfoward application and extension of this formula we can obtain all the results we need. See S.T.S's article in this TN for a statement of the conclusion, using the idea outlined above in a different way.

The same method ('integration with respect to parameters') can be used to simplify the calculations for the second of the key ideas introduced in TN37, that of replacing the $(-n)$-lines by 'ladders' with n rungs of the form



The overall picture is now that this line of development is firmly established and justified, thus taking us much closer to a scheme in which mass is generated by interaction with a Higgs field; i.e. the nth twistor diagram, with a ladder of n rungs, corresponds to n successive interaction with Higgs fields. However a new problem, at first overlooked, prevents us claiming that this is achieved. This is the $\frac{1}{3}\pi^2$ term that arises when the $\frac{1}{2}\log^2$ term is transformed to $\frac{1}{2}\log^2 - \frac{1}{3}\pi^2$. This is not fatal to the general programme, because we know that that the twistor diagrams so far arrived at are not actually quite of the form we want. They do not project out spin eigenstates, and they do not quite fit into the correct 'skeleton' pattern as required by my general dogma for the correspondence of twistor and Feynman diagrams. There is a reason for supposing that when the diagrams are yet again modified to meet these criteria, the $\frac{1}{3}\pi^2$ discrepancy will be eliminated.

Meanwhile there is no problem with getting the 'on-shell' (Hankel) functions from this general scheme, thus completing and superseding the formulation of my 1985 paper. In view of this I suggest that the two-twistor 'functions' of form

$$f(Z^\alpha)\,g(X^\alpha)\,(\overline{X\cdot Z}-\mu)^{-1}$$

should now be considered as primitive elements of the theory, corresponding to the measurement of a single free massive field rather than as two free massless fields, and should have a new cohomological interpretation. *Andrew Hodges*