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COMPLEX NUMBERS

in

QUANTUM MECHANICS

A One-Day Seminar will be held at the Sub-Faculty of Philosophy, University of Oxford, 10 Merton Street, Oxford on

Saturday, 3 June, 1995

The aim of the seminar is to survey the uses of complex numbers and complex structures in existing quantum theory and to consider the role they might play in quantum gravity. Some abstracts are given overleaf.

Programme

10:00 Registration and coffee (Lunch 12:45, Tea 16:30)

10:30 L. P. Hughston: "Complex Geometry and Quantum Probability"

11:30 J. S. Anandan: "A Geometric View of Quantum Theory"

14:15 L. J. Mason: "Generating Spacetime from Complex Numbers"

14.45 A. P. Hodges: "The Twistor Diagram Program"

15:15 R. F. Streater: "Complex Numbers and Complex Structures in Quantum Mechanics"

17:00 J. B. Barbour: "Why *i* and Is It in Quantum Gravity?"

17:15 G. W. Gibbons: "How the Complex Numbers Got into Physics"

Anyone interested is welcome to attend; time has been included in the programme for questions and comments from participants. Tea, coffee, and biscuits will be available at the Sub-Faculty from 10:00 and during the lunch and tea breaks. There are several restaurants near the Sub-Faculty, but participants are encouraged to bring sandwiches for a picnic lunch in the nearby Botanical Gardens. There is no registration fee but please advise either of the organizers if you plan to attend. **Organizers:**
Julian B. Barbour, College Farm, South Newington, Banbury, Oxon, OX15 4JG (Tel: 01295-720492; fax: 01295-721851). Dr Harvey Brown, Sub-Faculty of Philosophy, University of Oxford, 10 Merton Street, Oxford, OX1 4JJ (Tel: 01865-276930; fax: 01865-276932; e-mail: harvey.brown@philosophy.oxford.ac.uk.).

Abstracts and References

L. P. Hughston: "Complex Geometry and Quantum Probability." A survey of the basic constructions of ordinary quantum mechanics is presented by the use of the formalism of complex projective spaces and the Fubini–Study metric. In this way the various probabilistic assumptions that go into the interpretation of quantum mechanics can be formulated in a geometric language. The geometric view helps to clarify many aspects of the ordinary theory, but also highlights some of its problems, and forms a basis for possible generalisations. References:
L. P. Hughston (1995) "Geometric aspects of quantum mechanics," in *Twistor Theory* (S. A. Huggett, ed.), Marcel Dekker; L. P. Hughston (1995) "Geometry of stochastic state vector reduction" (preprint); L. P. Hughston, R. Josza, and W. K. Wooters (1993) "A complete classification of quantum ensembles having a given density matrix," *Phys. Letts.* A 183, 14–18.

J. S. Anandan: "A Geometric View of Quantum Theory." A new method of observing the wave function of a single quantum system and the geometric phase are used to justify operationally a geometric formulation of quantum theory in the quantum state space (the set of rays of the Hilbert space or the projective Hilbert space). Some generalizations and modifications of quantum theory are considered from this point of view.

J. B. Barbour: "Why *i* and is it in quantum gravity." My interest is this: Is there *one specific way* in which complex numbers enter quantum mechanics? If so, what is the corresponding physical phenomenon that makes this necessary? If, as Pauli argued, complex wave functions are needed to ensure unitarity, what happens in quantum gravity, in which there may be neither time nor unitarity? Does some other effect enforce the appearance of complex structures in quantum gravity? References: Chen Ning Yang, "Square root of minus one, complex phases and Erwin Schrödinger," in *Schrödinger: Centenary Celebration of a Polymath* (C. W. Kilmister, ed), CUP (1987); W. Pauli, Z. *Physik* **80**, 573 (1933); J. B. Barbour, *Phys. Rev.* D, 5422 (1993).

G. W. Gibbons: "How the Complex Numbers Got into Physics." I review the role that complex numbers play in setting up the usual quantum mechanical formalsim and I argue that there may be circumstances under which this is not possible, for example, if spacetime does not admit a global time orientation. I also argue that in currently popular models of quantum cosmology it makes sense to say that the complex numbers emerged contemporaneously with the emergence of a classical notion of time. References: G. W. Gibbons, *Nucl. Phys.* **B410** (1993) 117–142; *Nucl. Phys.* **B271** (1986) 495–508; *Int. J. Mod. Phys.* **D3** (1994) 61–70.