## The Rarita-Schwinger Equation for Einstein-Maxwell Space-times

It will be a familiar fact to readers of Twistor Newsletter that the Rarita-Schwinger equation (equation (1) below) has "as many" solutions in a space-time **M** as it has in flat space-time if **M** satisfies the vacuum equations. Also, the Rarita-Schwinger equation has gauge-solutions, generated by an arbitrary choice of spinor field $\alpha^A$, again provided **M** is vacuum. From these facts have arisen various attempts to use the Rarita-Schwinger equation as a means to generate vacuum solutions.

It is natural to ask (and the question arose after David Robinson's seminar in Oxford on 16/1/96) whether there is a generalisation of the Rarita-Schwinger equation which stands in the same relation to the Einstein-Maxwell equations as the original does to the vacuum equations. In fact there is, as is implicit in the theory of N=2 supergravity and as is therefore probably well-known to supergravity aficionados. My purpose here is simply to note, in the formalism of 2-component spinors, what these generalised Rarita-Schwinger equations are.

The Rarita-Schwinger equation for a spinor field $\psi^{BAA'}$ may be written as the pair of equations:

$$\nabla_{A(B'}\psi^{BA}{}_{A')} = 0 \qquad \text{1a}$$

$$\delta_A{}^C\,\nabla^{(B}{}_{B'}\psi_C{}^{A)B'} = 0 \qquad \text{1b}$$

and gauge solutions are obtained from an arbitrary spinor field $\alpha^A$ according to

$$\psi^B{}_{AA'} = \nabla_{AA'}\alpha^B \qquad \text{2}$$

It can be checked that (2), for arbitrary $\alpha^A$, solves equation (1a) iff the trace-free Ricci tensor vanishes, and satisfies (1b) iff the Ricci scalar vanishes. To generalise (1) we seek equations for a pair of Rarita-Schwinger fields (or a Dirac-spinor-valued 1-form) whose gauge solutions are generated by a pair of spinor fields $\alpha^A, \beta^{A'}$ (or a Dirac spinor) according to

$$\psi^B{}_{AA'} = \nabla_{AA'}\alpha^B - \kappa\,\phi_A{}^B\,\beta_{A'} \qquad \text{3a}$$

$$\chi^{B'}{}_{AA'} = \nabla_{AA'}\beta^{B'} + \kappa\,\bar{\phi}_{A'}{}^{B'}\,\alpha_A \qquad \text{3b}$$

Here $K$ is a constant to be determined and $\phi_{AB}$ is a Maxwell field. These equations are taken from my paper Phys.Lett.**121B** (1983) 241 where I obtained them by translating a 4-component-spinor expression given by Gibbons and Hull Phys.Lett.**109B** (1982) 190.

By eliminating the Maxwell field between (3a) (respectively (3b)) and the derivative of (3b) (respectively (3a)) we guess the candidate generalised Rarita-Schwinger equations:

$$\nabla_{A(B'}\psi^{BA}{}_{A')} + K\phi^{AB}\chi_{(A'B')A} = 0 \tag{4a}$$

$$\nabla_{A'(B}\chi^{B'A'}{}_{A)} - K\bar{\phi}^{A'B'}\psi_{(AB)A'} = 0 \tag{4b}$$

$$\delta_A{}^C\nabla^{(B}{}_{B'}\psi_C{}^{A)B'} + \frac{1}{2}K\phi_A{}^B\chi^{B'A}{}_{B'} = 0 \tag{4c}$$

$$\delta_{A'}{}^{C'}\nabla^{(B'}{}_B\chi_{C'}{}^{A')B} - \frac{1}{2}K\bar{\Phi}_{A'}{}^{B'}\psi^{BA'}{}_B = 0 \tag{4d}$$

Now the point is that (3a,b) are in fact the gauge solutions of the generalised Rarita-Schwinger equations (4a-d) *provided that the source-free Einstein-Maxwell equations are satisfied*, i.e. provided that

$$\Phi_{ABA'B'} = K^2\phi_{AB}\bar{\Phi}_{A'B'}; \; \Lambda = 0; \; \nabla_{A'}{}^A\phi_{AB} = 0 \tag{5}$$

(so that, in the usual conventions, $K^2 = 2G$).

The equations (4a-d) seem to be the translation into the 2-component-spinor language of equations given long ago in the supergravity literature by S.Ferrara and P.van Nieuwenhuizen (Phys.Rev.Lett.**37** (1977) 1669; see also T.Dereli and P.C.Aichelburg (Phy.Lett.**80B** (1979) 357) and P.C.Aichelburg and R.Güven (Phys.Rev.**D24** (1981) 2066).

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