Solutions to Puzzles in TN41

① Adding −12 to each term in

$$\ldots, 28, 0, 21, 4, 18, 0, ?, 24, 18, 20, 21, 24, 28, \ldots$$

and then multiplying the n^{th} term T_n by n ($?$ being the 0^{th} term), we get $12 F_n$, where F_n is the n^{th} Fibonacci number. Accordingly, the n^{th} term must be

$$T_n = 12\left(\frac{F_n}{n} + 1\right)$$

$$= \frac{12}{n}\left(\frac{\tau^n - \tilde{\tau}^n}{\tau - \tilde{\tau}}\right)$$

where $\tau = \frac{1+\sqrt{5}}{2}$ and $\tilde{\tau} = \frac{1-\sqrt{5}}{2} = -\frac{1}{\tau}$.

To get T_0 we use "l'Horpital's rule", and find

$$? = T_0 = \frac{12}{\sqrt{5}}\left\{\log\tau - \log\tilde{\tau}\right\} + 12$$

$$= \frac{24}{\sqrt{5}}\left(\log\tau + \frac{i\pi}{2}\right) + 12 \doteqdot \underline{17.16490729 + i\,16.85955535}$$

Of course, an alternative answer is the complex conjugate of this, but just taking the real part would be wrong. (But you could add any integer multiple of $i\,\frac{24\pi}{\sqrt{5}}$.)

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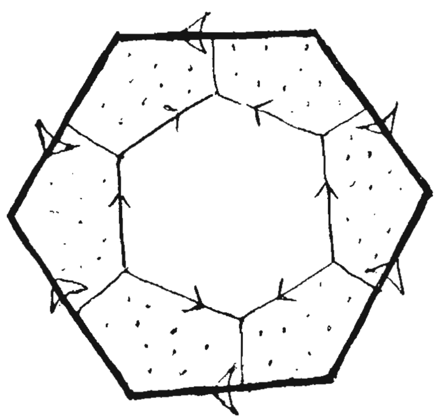
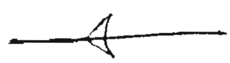
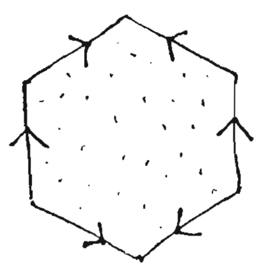
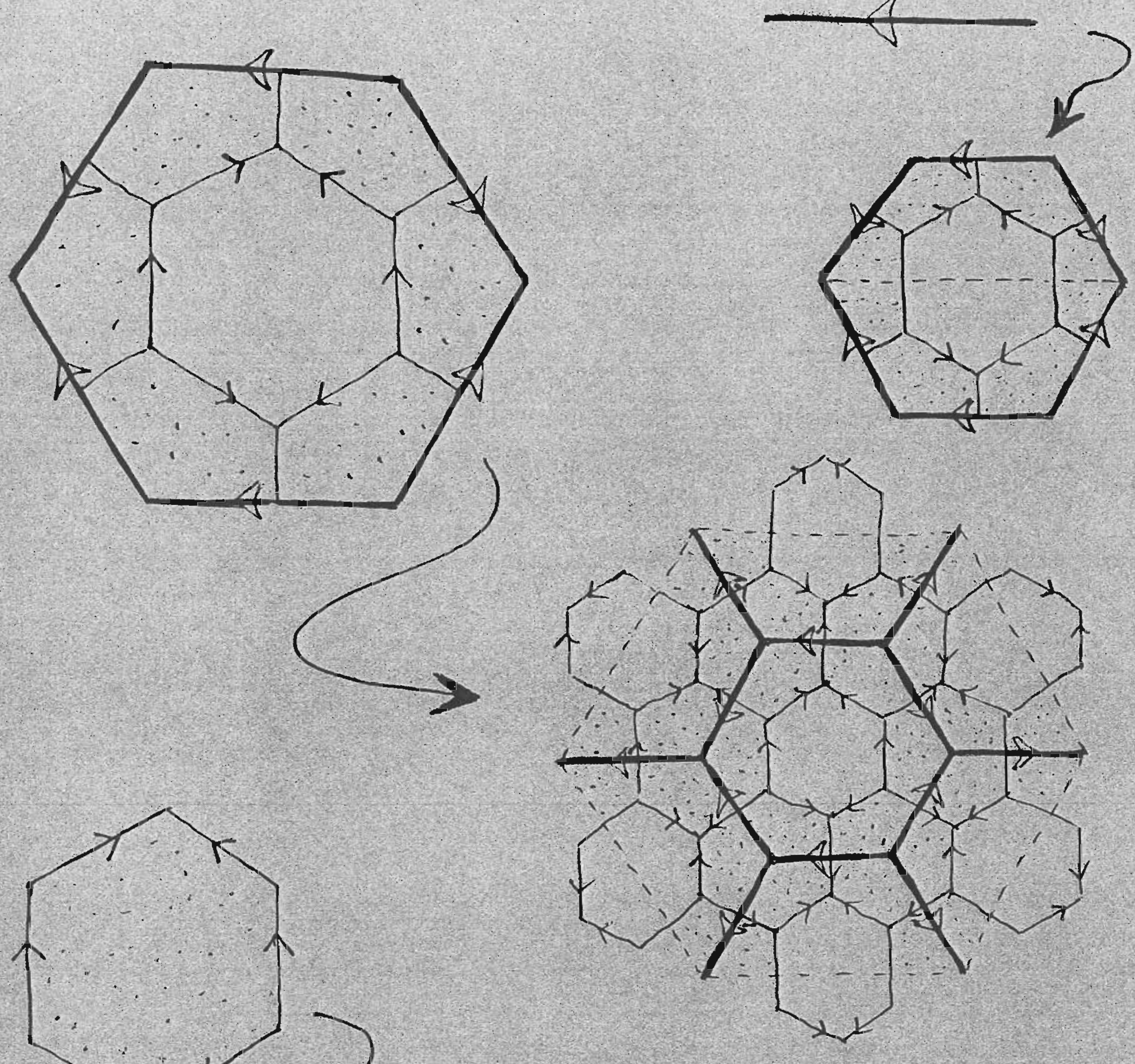
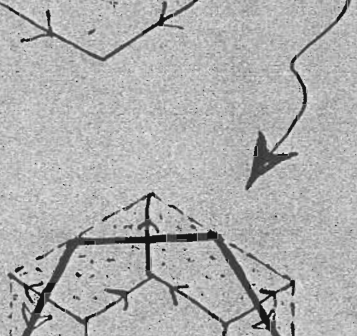
② Any tiling with , where the edges match 

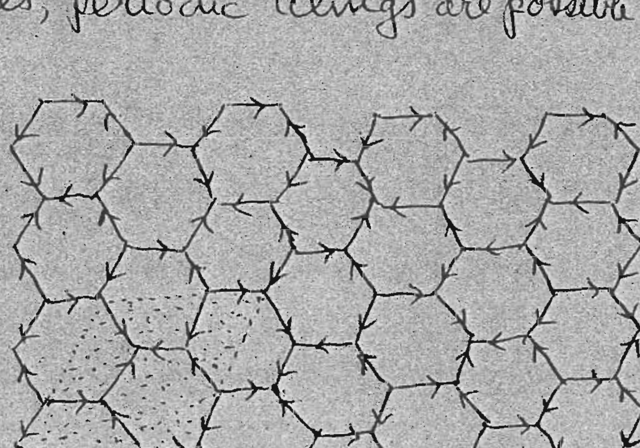
and the corners match 

must be <u>non-periodic</u>, and is composed according to the hierarchical scheme





NB: without the <u>double</u> matching rules, periodic tilings are possible, e.g.



Roger