Solutions to Puzzles in \textit{TN41}

1. Adding -12 to each term in

$$..., 28, 0, 21, 4, 18, 0, 24, 18, 20, 21, 24, 28, ...$$

and then multiplying the \textit{n}th term \(T_n\) by \(n\) (\(0\) being the 0th term), we get \(12F_n\), where \(F_n\) is the \(n\)th Fibonacci number. Accordingly, the \(n\)th term must be

\[
T_n = 12 \left( \frac{F_n}{n} + 1 \right)
\]

\[
= 12 \left( \frac{\tau^n - \overline{\tau}^n}{\tau - \overline{\tau}} \right)
\]

where \(\tau = \frac{1 + \sqrt{5}}{2}\)

and \(\tau = \frac{1 - \sqrt{5}}{2} = -\frac{1}{\tau}\).

To get \(T_0\) we use "l'Hôpital's rule", and find

\[
T_0 = \frac{12}{\sqrt{5}} \left\{ \log \tau - \log \overline{\tau} \right\} + 12
\]

\[
= \frac{24}{\sqrt{5}} \left( \log \tau + \frac{i\pi}{2} \right) + 12 = 17.16490729 + i\,16.85495335
\]

Of course, an alternative answer is the complex conjugate of this, but just taking the real part would be wrong. (But you could add any integer multiple of \(i\frac{24\pi}{45}\).)

2. Any tiling with \(\tau\), where the edges match and the corners match.
must be non-periodic, and is composed according to the hierarchical scheme.

NB: Without the double matching rules, periodic tilings are possible.

Regard.