The Geometry of Twistor Diagrams

This note is intended not to set forth any of the mass of detailed results about twistor diagrams for first and second-order scattering processes, but to air the question of how the theory should connect with other aspects of the twistor programme.

1. The refinement of the diagram calculus over the years has established that the fundamental element of the calculus is complex contour integration with boundaries on subspaces of form $W.Z = k$,

written 

We also need derivatives of this boundaries, namely simple, double, triple and quadruple poles which have the effect of restricting the integration to such subspaces: written

We also need conformal-symmetry-breaking boundaries of form

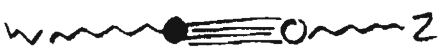
$$\overline{XZ} = m, \qquad \underline{WY} = m$$

At a secondary level, we bring massive fields into the picture with poles of form

$$(\overline{XZ} - m)^{-1}, \qquad (\underline{WY} - m)^{-1}$$

and the evaluation of the 'constant' field, the elementary state based at infinity, which corresponds to the Higgs field in the standard model:

2. There is a natural value of $|k|$, namely $\exp(-\gamma)$, where γ is Euler's constant, but the *sign* of k is not determined. It is obvious that there is also a sign ambiguity in specifying the conformal-symmetry-breaking elements. These questions of sign show up non-trivially in the evaluation of larger diagrams.
The simplest example comes from considering a chain of diagram elements:

 $= \log\left(\frac{W.Z}{k}\right)$

but

 $= \log\left(\frac{W.Z}{-k}\right)$

In the chains required for building up the massive progagator one also finds that one chain gives a term of form

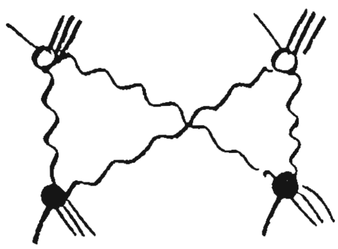
$$\frac{1}{2}\log^2\left(\frac{\begin{smallmatrix}W\ Y\\ \vdash\!\dashv\\ X\ Z\end{smallmatrix}\ m^2}{\underset{\sqcup}{W\,Y}\ \ \overset{\frown}{X\,Z}}\right)$$

but a longer one gives

$$\frac{1}{2}\log^2\left(\frac{\begin{smallmatrix}W\ Y\\ \vdash\!\dashv\\ X\ Z\end{smallmatrix}\ m^2}{\underset{\sqcup}{W\,Y}\ \ \overset{\frown}{X\,Z}}\right) - \pi^2/3$$

These differences are crucial to the problem that remains of getting exact agreement with the Feynman propagator, and seem to be related to the question of time-direction.

3. New progress has been made on the evaluation of such larger diagrams. Russ Ennis has seen how to describe in a more systematic way the building up of many-dimensional contours with boundary, and in particular has shown how to integrate the fundamental diagram



This in turn supplies a more elegant description of the integration of all 'single-box' diagrams. At the present time the parallel treatment of the 'double-box' diagram is not quite complete. This is of particular interest as it complicated enough for the sign of k to play a non-trivial role. R.E. has also seen applications of his new description to the long chains involved in the description of massive fields in which, as indicated above, signs are crucial.

4. There are now many directions in which to pursue the further extension of the diagram calculus. For instance, it is very striking how second-order gauge-theoretic interactions take on a simple form as twistor diagrams, with the many gauge-non-invariant Feynman diagrams reduced to one gauge-invariant twistor diagram. This is an area where one can foresee the emergence of a formalism with real advantages over Feynman diagram summation.

However, it seems that essential *geometrical ingredients* have emerged as fundamental, and that we now should be looking for connections with the new twistor geometry developed by R.P. for the description of curved space-time. From the point of view of the twistor programme, of course, such a unification of description is in any case the main point of pursuing the topic of twistor diagrams. We can ask:

(a) What is the generating principle which lies behind all the examples of twistor diagrams as so far evaluated?

(b) Can the non-projective twistor spaces employed be connected with the non-projective elements involved in R.P's extended twistor geometry? Can the special role of I, the line at infinity, be connected likewise?

(c) Could the values of k and mass parameters be involved in completing R.P.'s constructions, perhaps in the sense of taking some limit analogous to the regularisations of conventional QFT?

(c) In the calculus of twistor diagrams, twistor space and dual twistor space are on an equal footing, and as it stands this is goes against the 'googly' philosophy in R.P.'s programme of putting everything into twistor space. Can the dual spaces in twistor diagrams be understood through (inhomogeneous) twistor transforms which translate 'googly' constructions? As the simplest example, can the inner-product integral

i.e. $$\int_{W.Z>k} f_{-4}(Z^\alpha)\, g_{-4}(W_\alpha)\, d^4W \wedge d^4Z$$

be related geometrically to the googly description of spin-1 fields?

(d) If we could make progress with any of these questions, the outstanding questions of sign and time-direction should then make sense.

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