Applications of Scattering Amplitudes to Particle Physics

Z. Bern
UCLA

Oxford Twistor Strings Conference
January, 2005
Outline

- Brief overview of particle physics.
- Particle colliders and scattering amplitudes.
- Feynman diagrams.
- Status of the Standard Model scattering amplitude calculations.
- Simple structures
  (a) Helicity
  (b) Basics of twistor-space amplitudes
- QCD amplitudes and twistors
- Mixed QCD/Electroweak amplitudes and twistors.
Particle Physics

Higgs boson?
Gauge Forces in Standard Model

**SU(3)**: Quantum Chromodynamics or QCD. – Strong Nuclear Force

- Non-abelian color charge
- Gluons forces carriers. Quarks also carry color charge.
- Asymptotic freedom (2004 Nobel Prize). Forces become weaker at high energies
- Twistor-motivated methods can be applied at high energies.

**SU(2) × U(1)**: Electroweak Theory – Weak Nuclear force

- $W, Z$ Massive Vector bosons
- Quarks and leptons carry weak force.
- Twistor-motivated methods apply only indirectly.
Some crucial questions for colliders:

Where is the Higgs boson?

What dynamics underlies spontaneous symmetry breaking?

- Hierarchy of scales?
  - Electron mass $\sim 10^{-3}$ GeV
  - Electroweak $\sim 10^3$ GeV
  - Grand Unification $\sim 10^{15}$ GeV
  - Gravity $\sim 10^{19}$ GeV Planck Scale


The forthcoming collider experiments will shed light on this.
The answers will come from particle colliders.
The CMS Detector

CMS
A Compact Solenoidal Detector for LHC
1st Barrel Yoke Wheel Extracted (26 Oct 2000)
CDF

$Z \rightarrow ee$ and $Z \rightarrow \mu\mu$ Events

Sample Events at Fermilab Tevatron

$\mathbf{E_T} \approx 44 \text{ and } 36 \text{ GeV}$

$\mathbf{P_T} \approx 50 \text{ and } 32 \text{ GeV}$
Scattering is described theoretically by three separate processes.

1. Initial state. Parton distribution functions. Partons are quarks and gluons.

2. ‘Hard’ or high energy collision of partons. Described by Feynman diagrams. Part relevant to twistors.

Feynman diagrams are more than pictures. They represent algebraic formulas for the propagation and interaction of particles.

\[ g \left( \frac{\eta_{\mu \nu}}{p^2} + \gamma_{\mu} \right) \]

\[ g \left( (p_1 - p_2)_\rho \eta_{\mu \nu} + (p_2 - p_3)_\mu \eta_{\nu \rho} + (p_3 - p_1)_\nu \eta_{\rho \mu} \right) \]
Motivation

The quest for precision

- Uncover deviations from the Standard Model.
- Match experimental precision. From LEP: $\alpha_s = 0.121 \pm 0.001 \text{ (exp)} \pm 0.006 \text{ (theory)}$
- Need multi-leg scattering amplitudes because $\alpha_s$ is large (+ large logs).
- Constrain new physics: Higgs boson $M_H \leq 250$ GeV (95% CL)

$\Delta E \Delta t \sim \hbar$
Plot shows extreme difficulty of extracting the Higgs boson signal from background at Tevatron.

Precise theoretical understanding necessary. NLO necessary.

Unfortunately, luminosity problems at Tevatron.
LHC Higgs Search

Simulated $2\gamma$ mass plot for $10^5$ pb$^{-1}$ $m_H = 130$ GeV in the lead tungstate calorimeter

Situation will be much better at the LHC $\sim$ 2007

Precision calculations still required for understanding signal
Many of these are beyond our current capabilities and require new tools.
State of the Art at One Loop

Five point is state of the art for generic calculations.

Typical Examples:

\( pp \rightarrow W + 2 \text{ jets} \)

\( pp \rightarrow b\bar{b}H \) or \( pp \rightarrow t\bar{t}H \)

Bern, Dixon, Kosower
Dixon, Kunszt, and Signer
Campbell and Ellis: MCFM

Reina, Dawson, Jackson and Wackeroth
Beenakker, Dittmaier, Kramer, Plumper, Spira
Status of Higher Loop Computations

Some examples of well known impressive higher loop computations:

- Anomalous magnetic moments, 4 loops, Kinoshita and friends.

\[ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \approx O(\alpha_s^3) \text{ Gorishny, Kataev and Larin, etc} \]

- 4-Loop QCD $\beta$ function van Ritbergen, Vermaseren and Larin, 
  $\sim 50,000$ Feynman diagrams!

These are all in the class of zero or one kinematic variable.
Major Advance of Past Few Years

Computations involving more than 1 kinematic variable is a new art a few years old.

Key to progress

In the past few years the field of high loop computations has gotten a tremendous boost due to the influx of energetic bright young people.

Babis Anastasiou, Andrzej Czarnecki, Daniel de Florian, Thomas Gehrmann, Massimiliano Grazzini, Robert Harlander, Sven Heinemeyer, Kirill Melnikov, Pierpaolo Mastrolia, Sven Moch, Zoltan Nagy, Carlo Oleari, Matthias Steinhauser, M.E. Tejeda-Yeomans, Peter Uwer, Doreen Wackeroth, Stefan Weinzierl, and many others
The two-loop integrations are done in a rather brute force way:

Chetyrkin, Tkachov, Smirnov, Veretin, Gehrmann, Tausk, Remiddi, Anastasiou, Glover, Oleari, Laporta, Moch, Weinzierl, etc.

Typical Approach:

- Integration by parts identities.

\[ 0 = \int \frac{d^D p}{(2\pi)^D} \frac{\partial}{\partial p^\mu} (v^\mu f(p, k_i)) \]

- Solve the system of equations in terms of ‘master integrals’. Typically \(10^5\) equations.

- Construct differential equations for master integrals.

Also serious challenges with converting the amplitudes into physical predictions for colliders – but that’s another story.

Nevertheless, impressive progress in the last few years.
New Two-loop Scattering Amplitudes

After 4-point integration breakthrough many new two-loop calculations:

- Two-loop Bhabha scattering in QED, $e^+e^- \rightarrow e^+e^-$.  
  ZB, Dixon and Ghinculov

- All two-loop $2 \rightarrow 2$ QCD processes.  
  Anastasiou, Glover, Oleari and Tejeda-Yeomans

- $\gamma\gamma \rightarrow \gamma\gamma$  
  ZB, Dixon, De Freitas, A. Ghinculov and H.L. Wong

- $gg \rightarrow \gamma\gamma$. (Background to Higgs decay.)  
  ZB, De Freitas, Dixon

- $\bar{q}q \rightarrow \gamma\gamma$, $\bar{q}q \rightarrow g\gamma$, $e^+e^- \rightarrow \gamma\gamma$  
  Anastasiou, Glover and Tejeda-Yeomans

- $e^+e^- \rightarrow 3$ partons  
  Garland, Gehrmann, Glover, Kourkoutsakis and Remiddi

- DIS 2 jet and $pp \rightarrow W, Z + 1$ jet  
  Gehrmann and Remiddi
Example of NNLO Improvement

Anastasiou, Dixon, Melnikov and Petriello

Drell-Yan rapidity distribution

\[ pp \rightarrow (Z, \gamma^*) + X \]

\[ Y = \frac{1}{2} \log\left(\frac{E + p_z}{E - p_z}\right) \]

- Amazingly little uncertainty in the answer.
- Predictions good to \( \sim 1\% \).
- Clear example demonstrating why we want to go to higher orders.
What enters challenging calculations

Quantum Field Theory

Clever Ideas

Computational Rules

Computer Algebra

Calculation

FORM MAPLE etc.
Examples of Clever Ideas

Consider the five-gluon tree-level amplitude of QCD. Enters in calculation of multi-jet production at hadron colliders. Described by following Feynman diagrams:

\[ \text{\includegraphics[width=0.8\textwidth]{feynman_diagram.png}} \]

If you follow the textbooks you discover a disgusting mess.
Result of a brute force calculation:

\[ k_1 \cdot k_4 \varepsilon_2 \cdot k_1 \varepsilon_1 \cdot \varepsilon_3 \varepsilon_4 \cdot \varepsilon_5 \]
Vector polarizations

\[ \varepsilon_{\mu}^+(k; q) = \frac{\langle q^- | \gamma_\mu | k^- \rangle}{\sqrt{2} \langle q k \rangle}, \quad \varepsilon_{\mu}^-(k, q) = \frac{\langle q^+ | \gamma_\mu | k^+ \rangle}{\sqrt{2} [k q]} \]

More sophisticated version of circular polarization: \( \varepsilon_{\mu}^{\pm} = (0, 1, \pm i, 0) \)

All required properties of polarization vectors satisfied:

\[ \varepsilon_i^2 = 0, \quad k \cdot \varepsilon(k, q) = 0, \quad \varepsilon^+ \cdot \varepsilon^- = -1 \]

Notation

\[ \varepsilon^{ab} \lambda_{ja} \lambda_{lb} \leftrightarrow \langle j \ l \rangle = \langle k_j^- | k_l^+ \rangle = \sqrt{2 k_j \cdot k_l} \ e^{i \phi} \]
\[ \varepsilon_{\dot{a} \dot{b}} \tilde{\lambda}_{\dot{j}} \tilde{\lambda}_{\dot{l}} \leftrightarrow [j \ l] = \langle k_j^+ | k_l^- \rangle = -\sqrt{2 k_j \cdot k_l} \ e^{-i \phi} \]

Changes in reference momentum \( q \) are equivalent to gauge transformations.
Five Gluon Results with Helicity

Following contains the physical content of the messy formula:

\[ A_5(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0 \]

\[ A_5(1^-, 2^-, 3^+, 4^+, 5^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle} \]

These are color stripped amplitudes.

\[ A_5(1, 2, 3, 4, 5) = \sum_{\text{perms}} \text{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}T^{a_5}) A_5(1^-, 2^-, 3^+, 4^+, 5^+) \]

Motivated by the Chan-Paton factors of open string theory. Feynman diagrams scramble together kinematics and color.

Mangano and Parke
Feynman diagrams are not gauge invariant. Huge cancellations.

Consider a tree-level calculation

\[ k_i^2 = E_i^2 - \bar{k}_i^2 = 0 \]
\[ m_i^2 = 0, \text{ gluons massless} \]

Now consider a loop amplitude

The key to efficient computation is recycling.

- Recursive approach – mainly at tree-level
  - Berends and Giele; Kosower; Mahlon
  - Recent work by Bena, ZB and Kosower; Roiban, Spradlin, Volovich; Britto, Cachazo, Feng, Witten

- Unitarity approach – loops from trees.
  - ZB, Dixon, Dunbar and Kosower
  - Recent work by ZB, Dixon and Kosower; Britto, Cachazo, Feng
Unitarity Method for Loops

- **Key:** *Any* amplitude in any massless theory is fully determined from $D$-dimensional tree amplitudes to *all* loop orders. Off-shell Feynman diagrams unnecessary.

- **Four-dimensional cut constructibility:** At one-loop, any amplitude in a massless susy gauge theory is fully constructible from *four-dimensional* tree amplitudes. Use helicity and twistors.

The simplest of all gauge theories is the $N = 4$ maximally supersymmetric one. Relative of QCD.

N=4 theory in talks by Britto, Dixon, Dolan, Kosower, Roiban, Travaglini, Witten, etc
Generalized Unitarity

Two-particle cuts:

Intermediate legs on shell

\[ \ell_i^2 = 0 \]

Triple cuts:

Generalized double two-particle cut:

Tree-level properties induce loop-level properties.
In a beautiful paper Ed Witten demonstrated that "twistor space" can reveal hidden structures of scattering amplitudes. Inspiration from Nair.

Link to string theory is for $N = 4$ super-Yang-Mills theory, but at tree level it might as well be QCD.

Twistor space given by Fourier transform with respect to plus helicity spinors.

$$\tilde{A}(\lambda_i, \mu_i) = \int \prod_i \frac{d^2 \tilde{\lambda}_i}{(2\pi)^2} \exp\left(\sum_j \mu_j \tilde{\lambda}_j \tilde{\lambda}_{\dot{j}}\right) A(\lambda_i, \tilde{\lambda}_i)$$

Tree-level QCD scattering amplitudes $\leftrightarrow$ ‘Twistor-space’ $\leftrightarrow$ Topological String Theory

Witten observed that in twistor space external points lie on certain curves. Very constraining.
Motivated by twistor space structure Cachazo, Svrček and Witten (CSW) define an off-shell “MHV vertex” based on Parke-Taylor amplitudes

\[ V(1^-, 2^-, 3^+, \ldots, n^+, P^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \cdots \langle n-1, n \rangle \langle n P \rangle \langle P 1 \rangle} \]

Continue spinor off-shell \((P^2 \neq 0)\): \(\langle i \, P \rangle = \eta \sum_{j=1}^{n} \langle i^- \mid k_j \mid q^- \rangle\)

where \(P = k_1 + k_2 + \cdots k_n\) and \(q\) auxiliary, satisfying \(q^2 = 0\).

Non-MHV amplitudes obtained by sewing together MHV vertices.

Amplitudes are much simpler than anyone imagined.

non-MHV from MHV
Discovering the Higgs boson and the origin of electroweak symmetry breaking is a central issue in collider physics.

Is there any way to use the twistor-motivated developments to obtain electroweak scattering amplitudes?

Obvious problem: Twistor-motivated methods appear to apply only to massless theories. Electroweak vector bosons and Higgs bosons are massive.

But it is still possible to make progress:

- Higgs boson coupled to quarks and gluons. Badger, Dixon, Glover, and Khoze
  See Dixon’s talk
- Electroweak vector boson coupled to quarks. ZB, Forde, Kosower, Mastrolia
Consider mixed Electroweak/QCD process:

How do we use MHV vertices with massive electroweak vector bosons?

Idea: Split problem into two pieces.

Vector boson current
Using MHV vertices add arbitrary numbers of quarks and gluons

Feynman Methods CSW diagrams

Gauge invariance issues make this tricky.
Define a new set of vector boson current MHV vertices:

\[
J^\mu(1^-_q, 2^+, \ldots, (n-1)^+, n_{q^+}; P_V) = -\frac{i}{\sqrt{2}} \frac{\langle (-1)^- | \gamma^\mu P_V | (-1)^+ \rangle}{\langle (-1) 2 \rangle \langle 2 3 \rangle \ldots \langle (n-1) n \rangle}
\]

\[
J^\mu((-P_V)^-; P_V) = \frac{i}{\sqrt{2}} \frac{\langle \eta^+ | \gamma^\mu P_{V^b}^b \rangle}{[P_{V^b}^b \eta]} P_V^2
\]

**ZB, Forde, Kosower and Mastrolia**

- **V** are MHV vertices of CSW
- **J** are are MHV vector boson current vertices
- **W** leg can be coupled to arbitrary off-shell source

These may be coupled to any electroweak vector boson processes.

Numerical checks up to 10 legs against recursive Feynman diagram methods.

Mixed QCD/Electroweak amplitudes can take advantage of twistor-motivated ideas.
Summary

1. Scattering amplitude calculations are crucial in particle physics: Search for New Physics at electroweak symmetry breaking scale.

2. Detailed comparisons of Standard Model to experiments requires many particles and loop corrections. Difficult to calculate.

3. Key result from twistors: Gauge theory scattering amplitudes are much simpler than anyone imagined.

4. MHV vertices. Talks from Khoze, Svrček and Travaglini

5. Unitarity method: Tree amplitude simplicity transfers to loop amplitude simplicity. Talks from Britto, Dixon and Kosower

6. Initial applications of twistor-motivated methods to electroweak theory.

A deeper understanding of the underlying mathematical structures will surely lead to further progress in computing scattering amplitudes.