Applications of Scattering Amplitudes to Particle Physics

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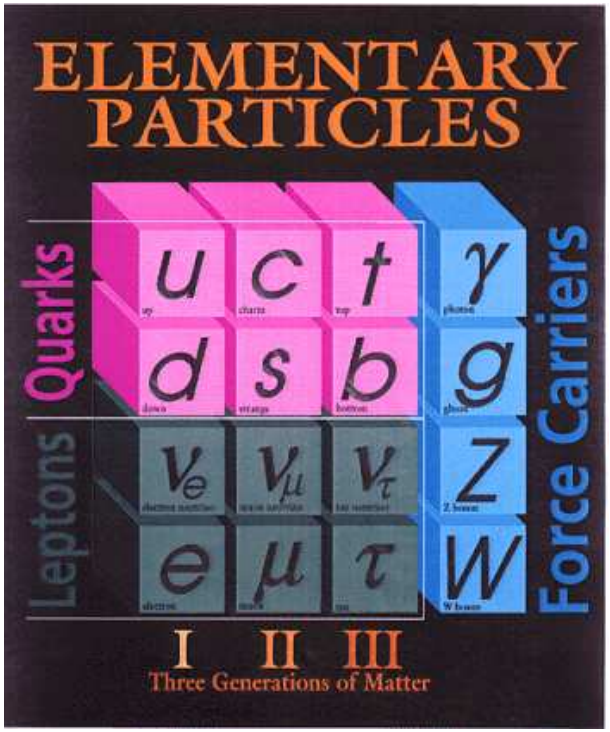
UCLA

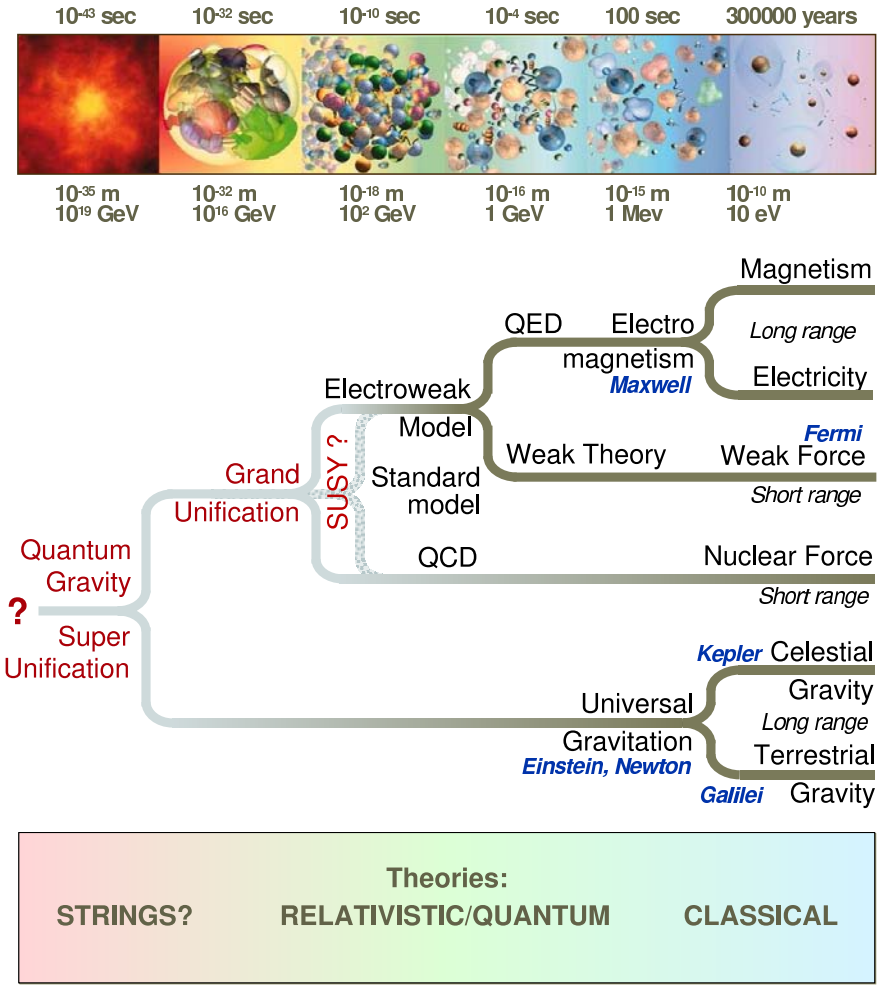
Oxford Twistor Strings Conference
January, 2005

Outline

- Brief overview of particle physics.
- Particle colliders and scattering amplitudes.
- Feynman diagrams.
- Status of the Standard Model scattering amplitude calculations.
- Simple structures
  (a) Helicity
  (b) Basics of twistor-space amplitudes
- QCD amplitudes and twistors
- Mixed QCD/Electroweak amplitudes and twistors.

Particle Physics





Higgs boson?

Gauge Forces in Standard Model

SU(3) : Quantum Chromodynamics or QCD. – Strong Nuclear Force

- Non-abelian color charge
- Gluons forces carriers. Quarks also carry color charge.
- Asymptotic freedom (2004 Nobel Prize). Forces become weaker at high energies
- Twistor-motivated methods can be applied at high energies.

SU(2)$\times$ U(1) : Electroweak Theory – Weak Nuclear force

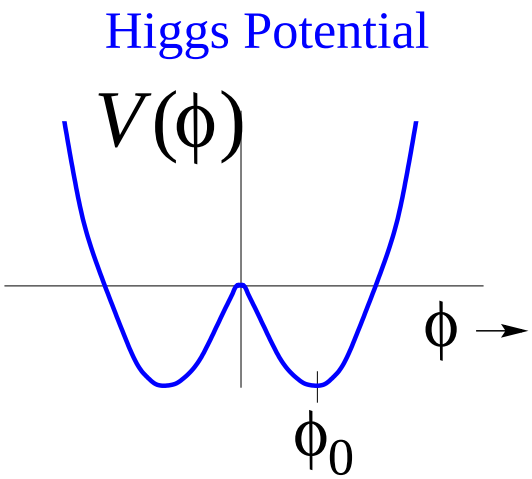
- W, Z Massive Vector bosons
- Quarks and leptons carry weak force.
- Twistor-motivated methods apply only indirectly.

Open Questions

Some crucial questions for colliders:

Where is the Higgs boson?

What dynamics underlies spontaneous symmetry breaking?



- Hierarchy of scales?
  Electron mass $\sim 10^{-3}$ GeV
  Electroweak $\sim 10^{3}$ Gev
  Grand Unification $\sim 10^{15}$ GeV
  Gravity $\sim 10^{19}$ GeV Planck Scale

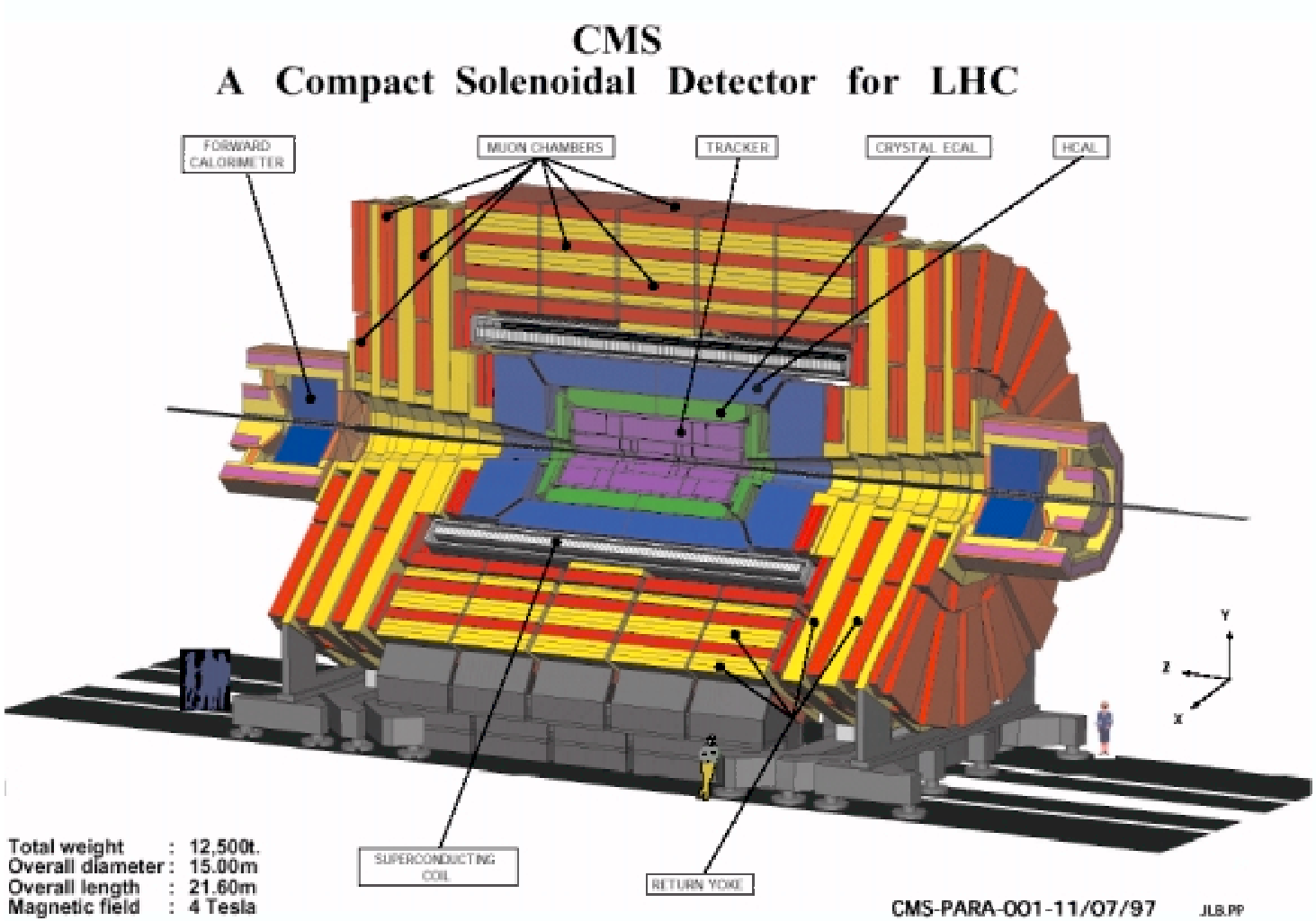
- What is beyond the Standard Model? Supersymmetry? Extra Dimensions? Strongly coupled theories? Other possibilities?

The forthcoming collider experiments will shed light on this.

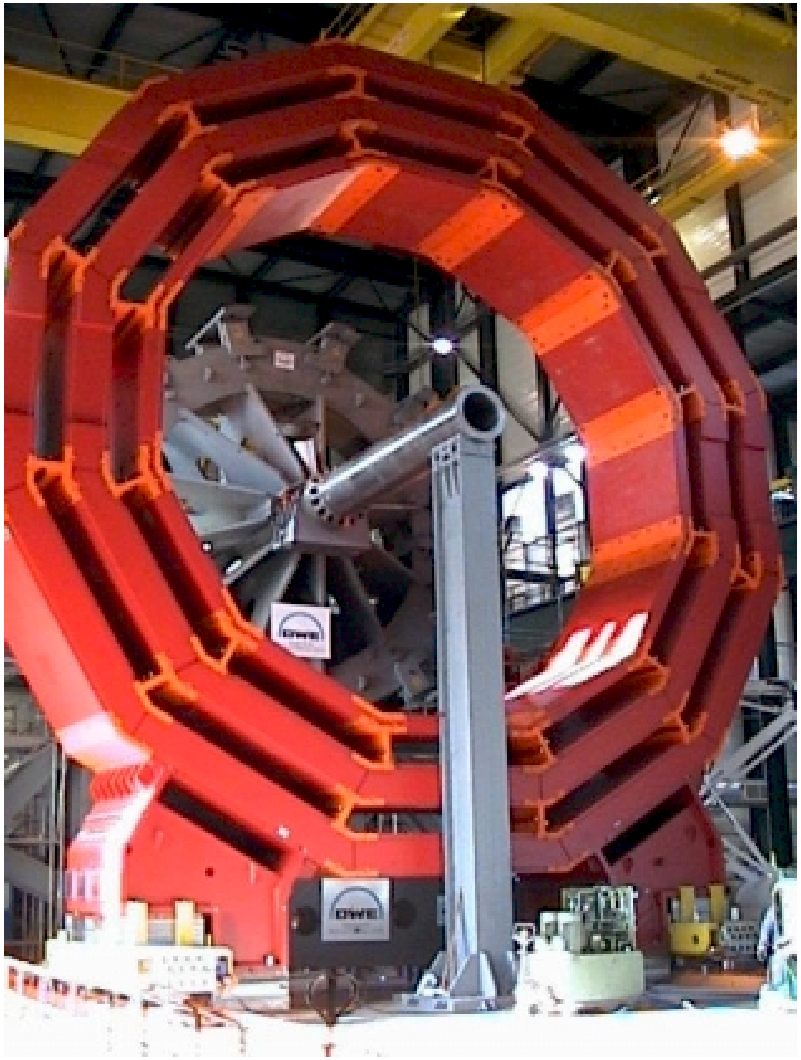
CERN LHC

The answers will come from particle colliders.

The CMS Detector

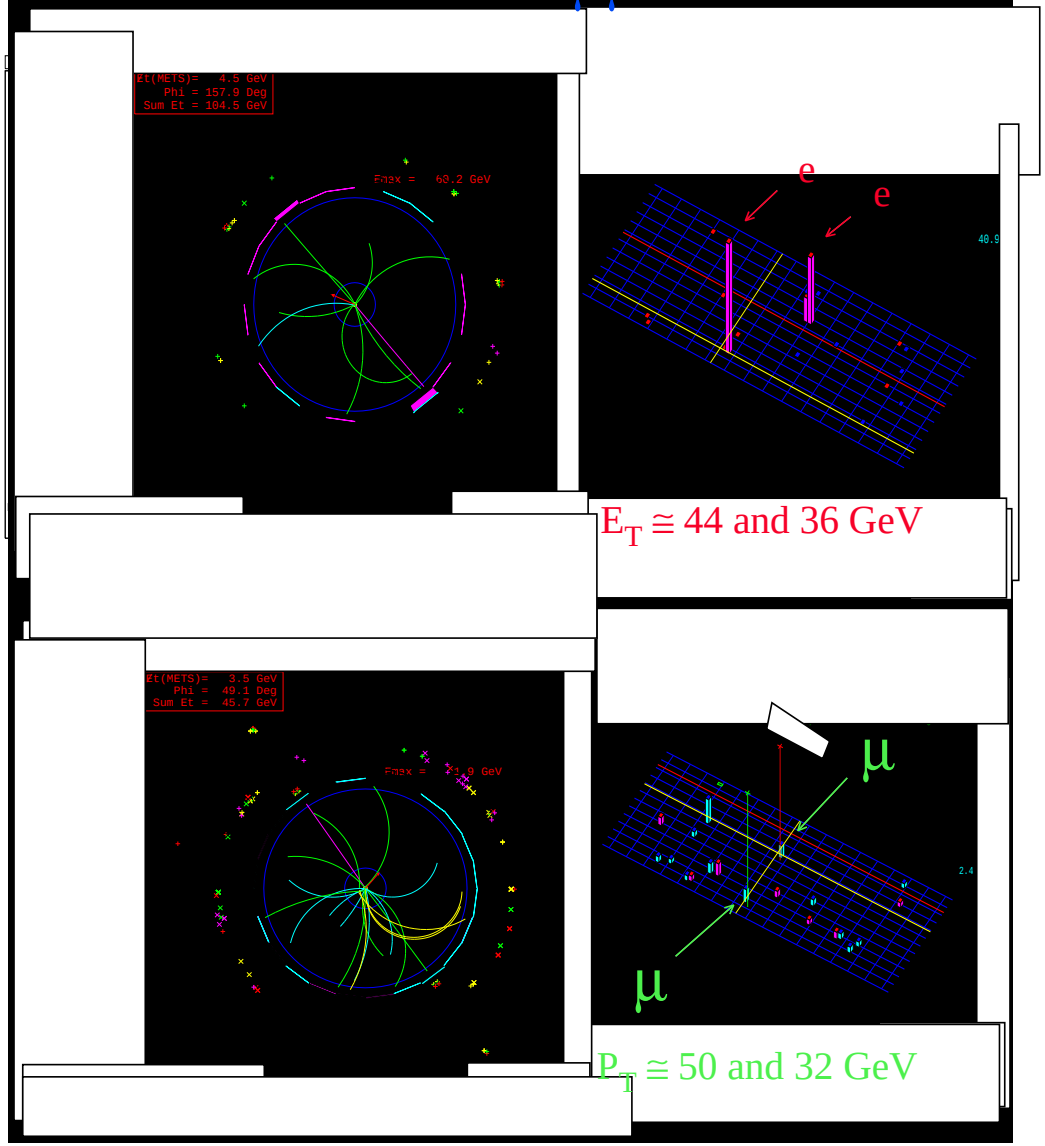


1st Barrel Yoke Wheel Extracted (26 Oct 2000)



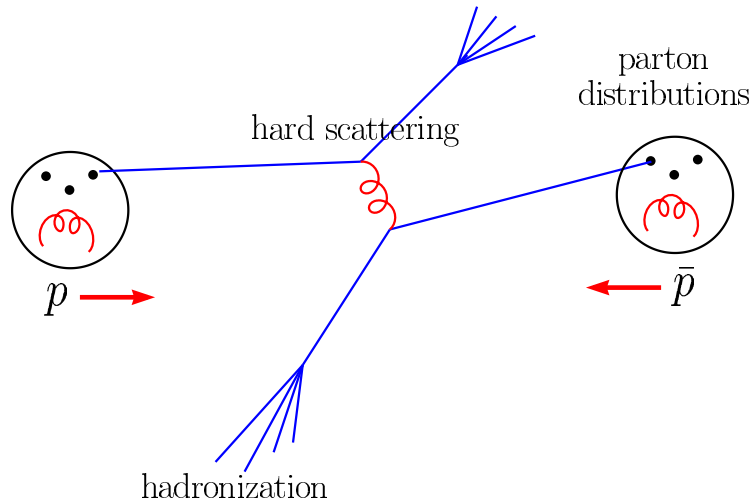
CDF

$Z \rightarrow ee$ and $Z \rightarrow \mu\mu$ Events



Sample Events at Fermilab Tevatron

Theorist's View of Scattering



Scattering is described theoretically by three separate processes.

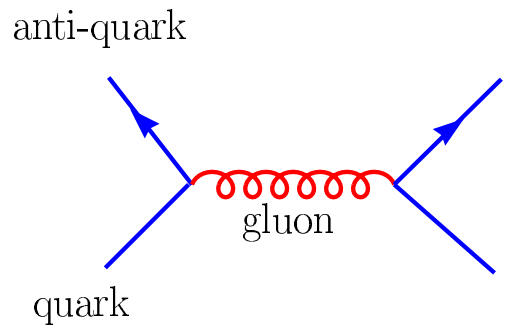
1. Initial state. Parton distribution functions. Partons are quarks and gluons.

2. 'Hard' or high energy collision of partons. Described by Feynman diagrams. Part relevant to twistors.

3. Showering and hadronization.

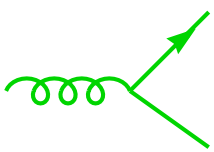
Feynman Diagrams

When forces are weak the basic tool for describing interaction of elementary particles is Feynman diagrams.

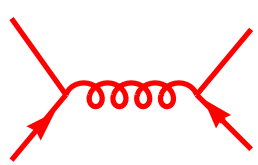


Feynman diagrams are more than pictures. They represent algebraic formulas for the propagation and interaction of particles.

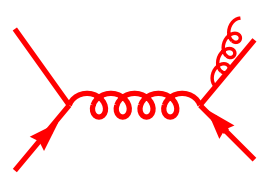
$$\frac{\eta_{\mu\nu}}{p^2}$$

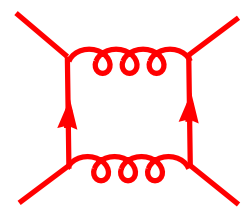


$$g\,\gamma_\mu$$

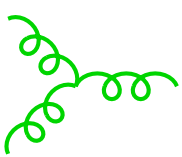


LO





NLO



$$g[\,(p_1-p_2)_\rho\,\eta_{\mu\nu}+(p_2-p_3)_\mu\,\eta_{\nu\rho}+(p_3-p_1)_\nu\,\eta_{\rho\mu}\,]$$

Motivation

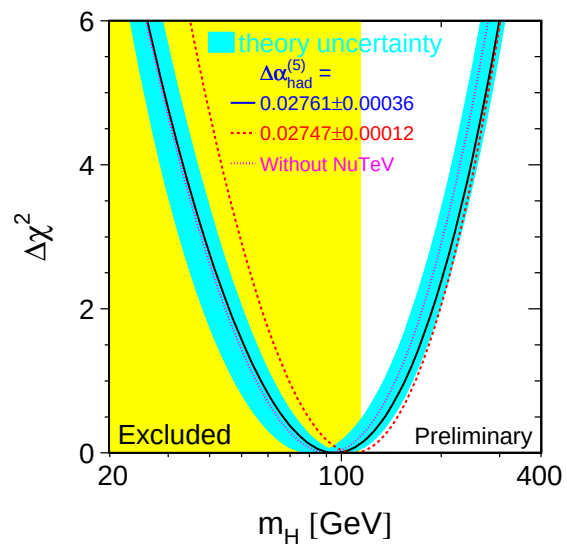
The quest for precision

- Uncover deviations from the Standard Model.

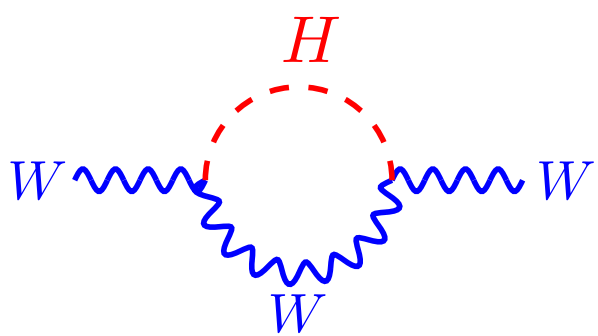
- Match experimental precision.
  From LEP: $\alpha_s = 0.121 \pm 0.001\,(\text{exp}) \pm 0.006\,(\text{theory})$ Bethke

- Need multi-leg scattering amplitudes because α_s is large (+ large logs).

- Constrain new physics:
  Higgs boson $M_H \leq 250$ GeV (95% CL)

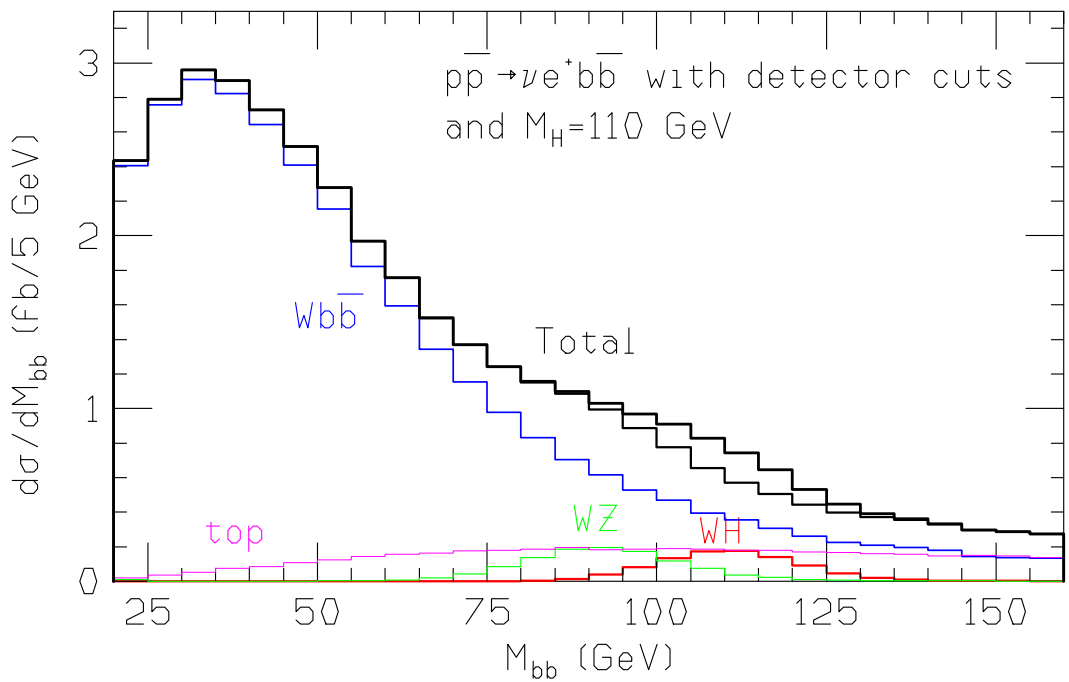


$\Delta E \Delta t \sim \hbar$

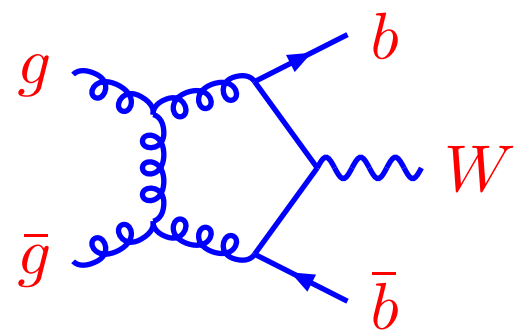


Tevatron Higgs Search

Campbell and Ellis



Best channel at Tevatron for Higgs search: $p\bar{p} \to HW$



Plot shows extreme difficulty of extracting the Higgs boson signal from background at Tevatron.

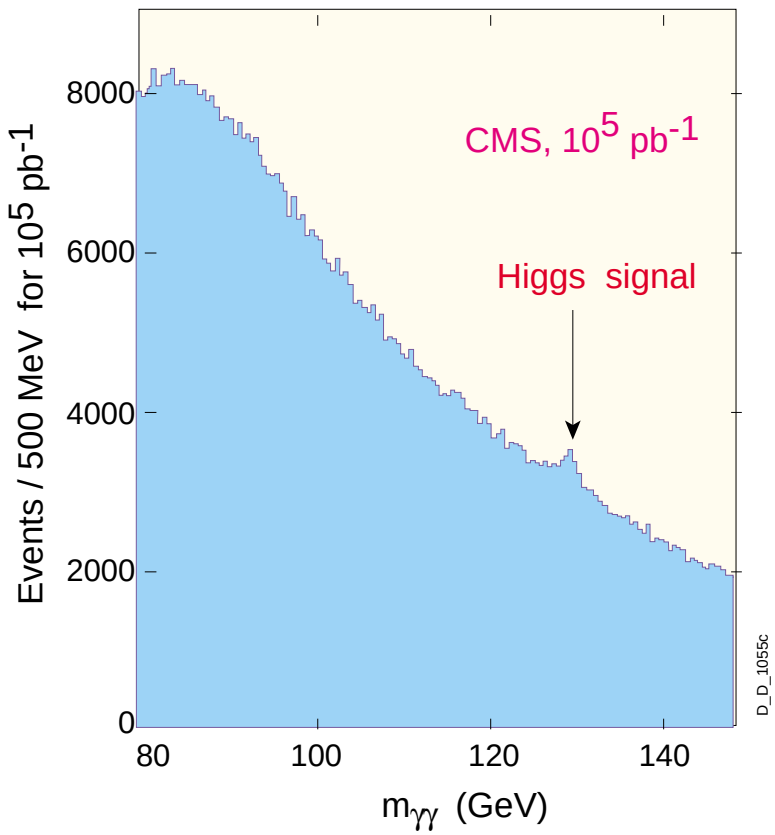
Precise theoretical understanding necessary. NLO necessary.

Unfortunately, luminosity problems at Tevatron.

LHC Higgs Search

$H_{SM} \longrightarrow \gamma\gamma$

Simulated 2γ mass plot
for 10^5 pb^{-1} m_H =130 GeV
in the lead tungstate calorimeter



Simulated Higgs boson data

Situation will be much better at the LHC ~ 2007

Precision calculations still required for understanding signal

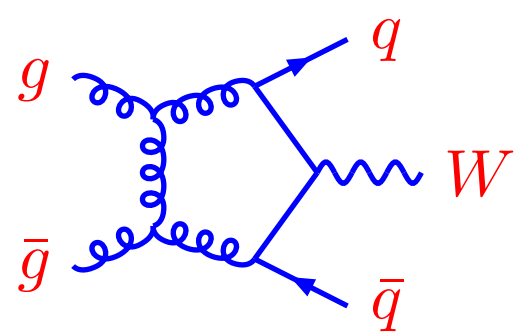
Experimenters NLO Wish List

John Campbell

Hadron collider cross-sections one would like to know at NLO

KITP Collider Physics Workshop, January-April 2004

Single boson	Diboson	Triboson	Heavy flavor
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		$b\bar{b}\,t\bar{t}$
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		



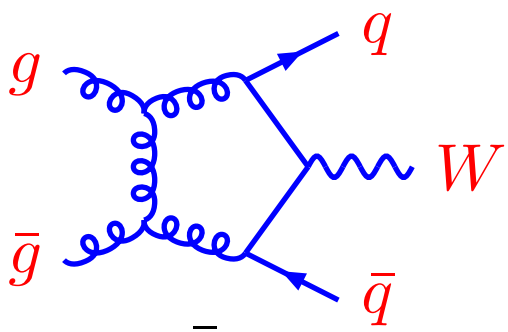
Many of these are beyond our current capabilities and require new tools.

State of the Art at One Loop

Five point is state of the art for generic calculations.

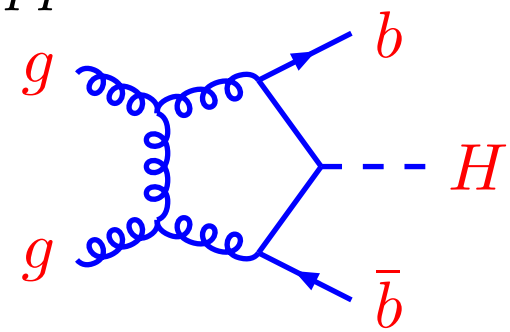
Typical Examples:

$pp \to W + 2$ jets



Bern, Dixon, Kosower
Dixon, Kunszt, and Signer
Campbell and Ellis: MCFM

$pp \to b\bar{b}H$ or $pp \to t\bar{t}H$



Reina, Dawson, Jackson and Wackeroth
Beenakker, Dittmaier, Kramer, Plumper, Spira

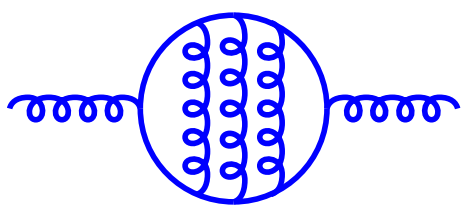
Status of Higher Loop Computations

Some examples of well known impressive higher loop computations:

- Anomalous magnetic moments, 4 loops, Kinoshita and friends.

- $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$, $O(\alpha_s^3)$ Gorishny, Kataev and Larin, etc

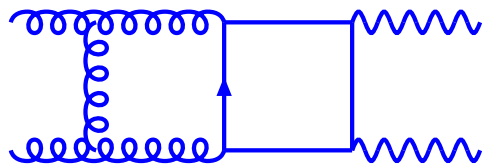
- 4-Loop QCD β function van Ritbergen, Vermaseren and Larin, $\sim 50,000$ Feynman diagrams!



These are all in the class of zero or one kinematic variable.

Major Advance of Past Few Years

Computations involving more than 1 kinematic variable is a new art a few years old.



Key to progress

In the past few years the field of high loop computations has gotten a tremendous boost due to the influx of energetic bright young people.

Babis Anastasiou, Andrzej Czarnecki, Daniel de Florian, Thomas Gehrmann, Massimiliano Grazzini, Robert Harlander, Sven Heinemeyer, Kirill Melnikov, Pierpaolo Mastrolia, Sven Moch, Zoltan Nagy, Carlo Oleari, Matthias Steinhauser, M.E. Tejeda-Yeomans, Peter Uwer, Doreen Wackeroth, Stefan Weinzierl, and many others

Current Situation

The two-loop integrations are done in a rather brute force way:

Chetyrkin, Tkachov, Smirnov, Veretin, Gehrmann, Tausk, Remiddi, Anastasiou, Glover, Oleari, Laporta, Moch, Weinzierl, etc.

Typical Approach:

- Integration by parts identities.

$$0 = \int \frac{d^D p}{(2\pi)^D} \frac{\partial}{\partial p^\mu} (v^\mu f(p, k_i))$$

- Solve the system of equations in terms of 'master integrals'. Typically 10^5 equations.

- Construct differential equations for master integrals.

Also serious challenges with converting the amplitudes into physical predictions for colliders – but that's another story.

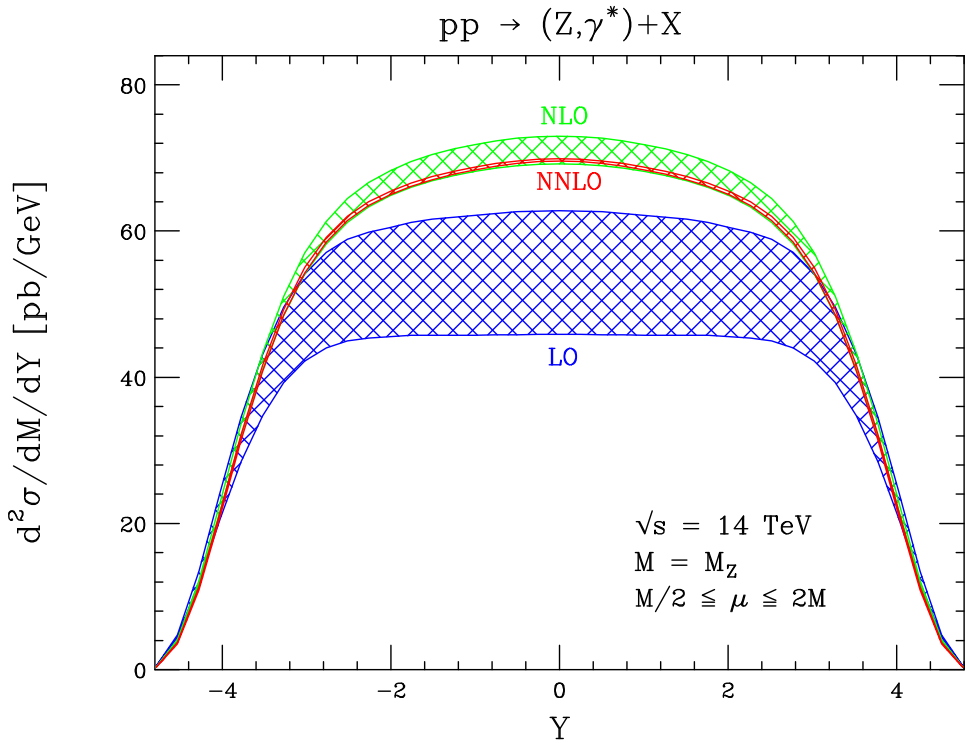
Nevertheless, impressive progress in the last few years.

New Two-loop Scattering Amplitudes

After 4-point integration breakthrough many new two-loop calculations:

- Two-loop Bhabha scattering in QED, $e^+e^- \to e^+e^-$. ZB, Dixon and Ghinculov
- All two-loop $2 \to 2$ QCD processes. Anastasiou, Glover, Oleari and Tejeda-Yeomans ZB,De Frietas, Dixon
- $\gamma\gamma \to \gamma\gamma$ ZB, Dixon, De Freitas, A. Ghinculov and H.L. Wong
- $gg \to \gamma\gamma$. (Background to Higgs decay.) ZB, De Freitas, Dixon
- $\bar{q}q \to \gamma\gamma$, $\bar{q}q \to g\gamma$, $e^+e^- \to \gamma\gamma$ Anastasiou, Glover and Tejeda-Yeomans
- $e^+e^- \to 3$ partons Garland, Gehrmann, Glover, Koukoutsakis and Remiddi Moch, Uwer, Weinzierl
- DIS 2 jet and $pp \to W, Z + 1$ jet Gehrmann and Remiddi

Example of NNLO Improvement



Anastasiou, Dixon, Melnikov and Petriello

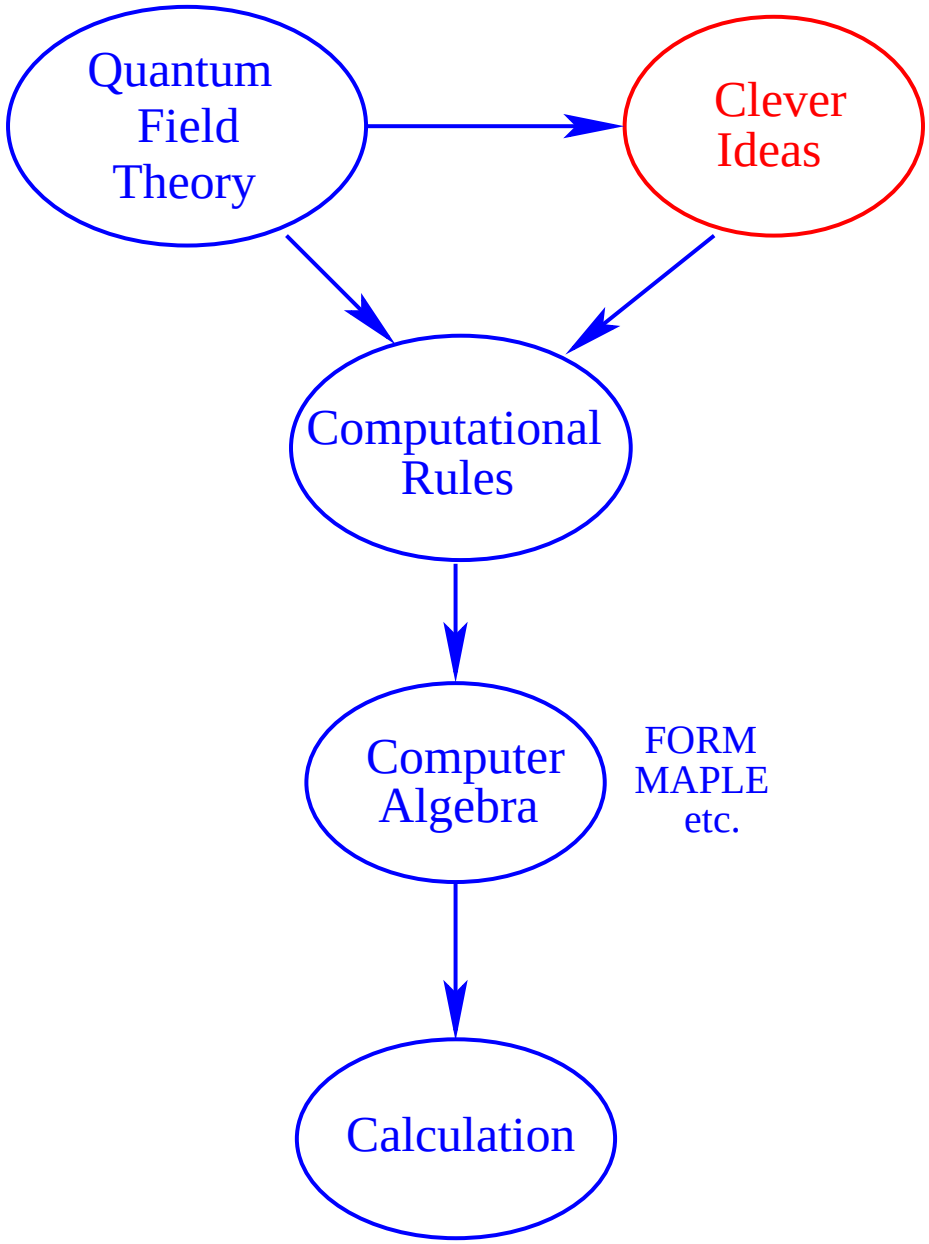
Drell-Yan rapidity distribution

$$pp \to (Z, \gamma^*) + X$$

$$Y = 1/2 \log((E + p_z)/(E - p_z))$$

- Amazingly little uncertainty in the answer.
- Predictions good to $\sim 1\%$.
- Clear example demonstrating why we want to go to higher orders.

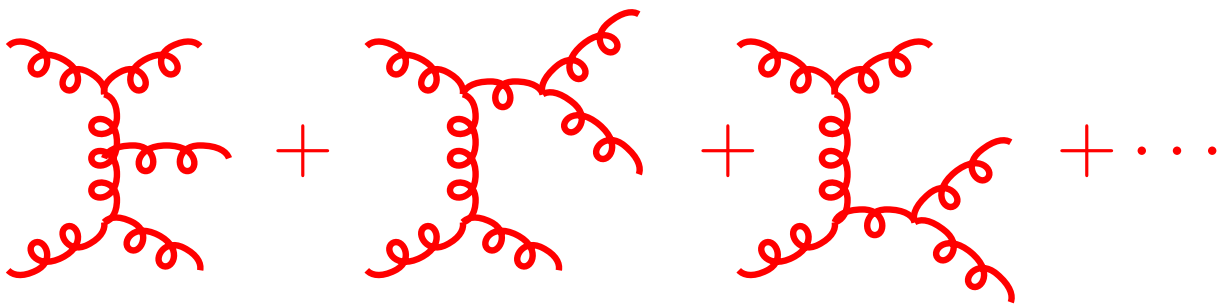
What enters challenging calculations



Examples of Clever Ideas

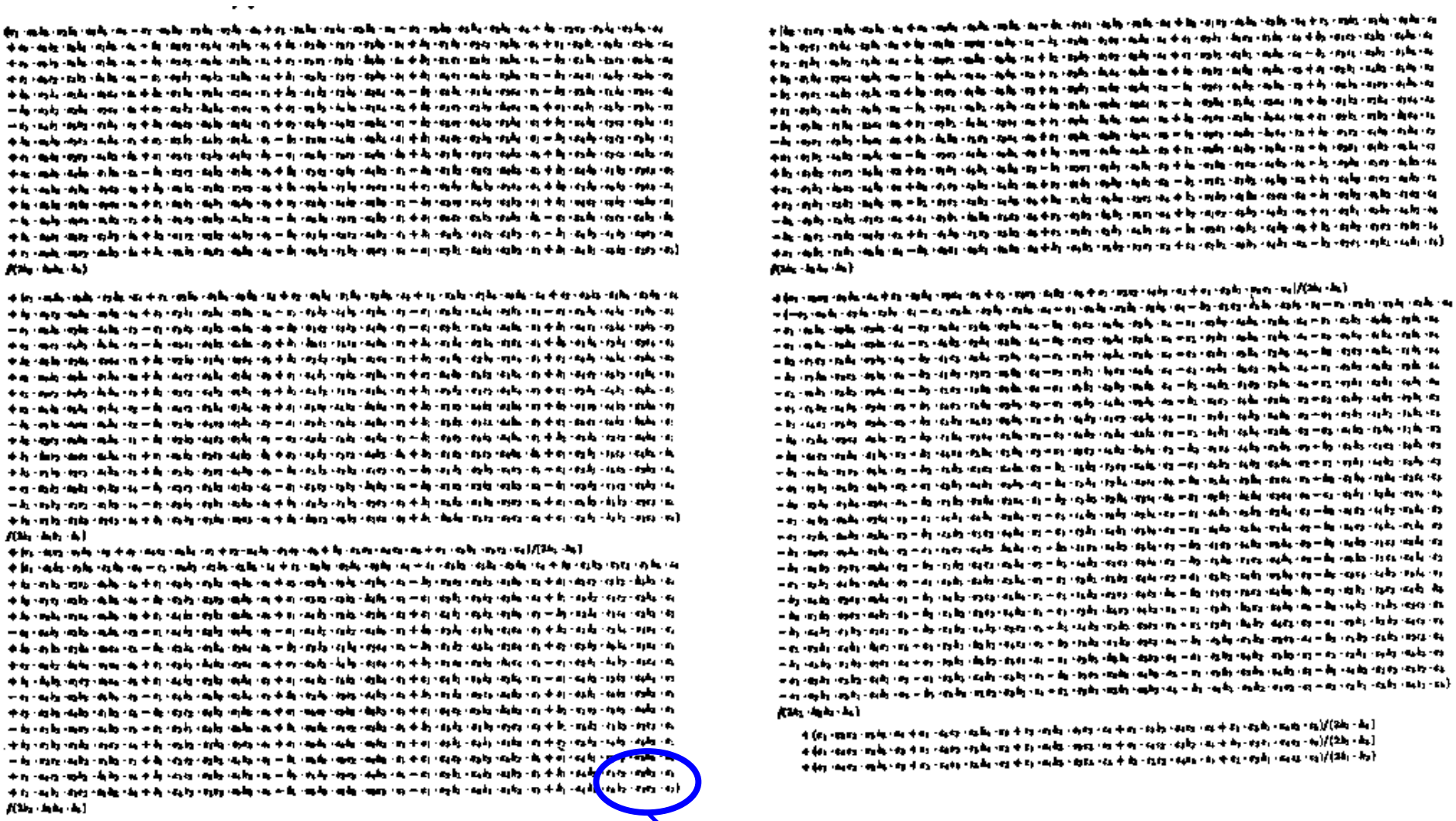
Consider the five-gluon tree-level amplitude of QCD. Enters in calculation of multi-jet production at hadron colliders.

Described by following Feynman diagrams:



If you follow the textbooks you discover a disgusting mess.

Result of a brute force calculation:



$$k_1 \cdot k_4\, \varepsilon_2 \cdot k_1\, \varepsilon_1 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5$$

Helicity

Xu, Zhang and Chang
F.A.Berends, R.Kleiss, P.De Causmaecker
R. Gastmans and T. T. Wu
J.F. Gunion and Z. Kunszt
& many others

Vector polarizations

$$\varepsilon_\mu^+(k;q) = \frac{\langle q^-|\,\gamma_\mu\,|k^-\rangle}{\sqrt{2}\,\langle q\,k\rangle}, \qquad \varepsilon_\mu^-(k,q) = \frac{\langle q^+|\,\gamma_\mu\,|k^+\rangle}{\sqrt{2}\,[k\,q]}$$

More sophisticated version of circular polarization: $\varepsilon_\mu^\pm = (0,1,\pm i,0)$

All required properties of polarization vectors satisfied:

$$\varepsilon_i^2 = 0\,, \qquad k\cdot\varepsilon(k,q) = 0\,, \qquad \varepsilon^+\cdot\varepsilon^- = -1$$

Notation

$$\epsilon^{ab}\lambda_{ja}\lambda_{lb} \longleftrightarrow \langle j\,l\rangle = \langle k_{j\_}|k_{l+}\rangle = \sqrt{2k_j\cdot k_l}\;e^{i\phi}$$

$$\epsilon_{\dot a\dot b}\tilde\lambda_j^{\dot a}\tilde\lambda_l^{\dot b} \longleftrightarrow [j\,l] = \langle k_{j+}|k_{l-}\rangle = -\sqrt{2k_j\cdot k_l}\;e^{-i\phi}$$

Changes in reference momentum q are equivalent to gauge transformations.

Five Gluon Results with Helicity

Following contains the physical content of the messy formula:

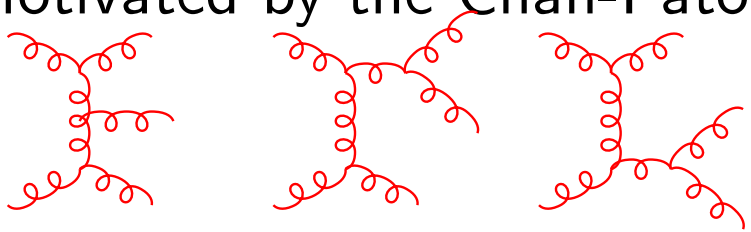
$$A_5(1^{\pm}, 2^+, 3^+, 4^+, 5^+) = 0$$

$$A_5(1^-, 2^-, 3^+, 4^+, 5^+) = i \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}$$

These are color stripped amplitudes.

$$\mathcal{A}_5(1,2,3,4,5) = \sum_{\text{perms}} \mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}T^{a_5})\, A_5(1^-, 2^-, 3^+, 4^+, 5^+)$$

Motivated by the Chan-Paton factors of open string theory. Mangano and Parke

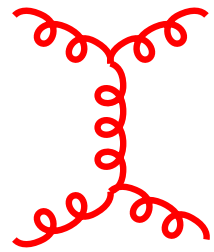


Feynman diagrams scramble together kinematics and color.

Feynman Diagram Problems at Loop Level

Feynman diagrams are not gauge invariant. Huge cancellations.

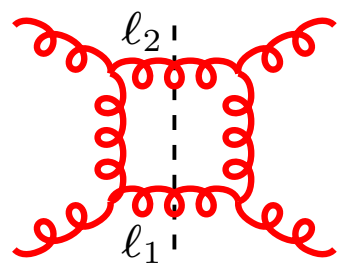
Consider a tree-level calculation



$$k_i^2 = E_i^2 - \vec{k}_i^2 = 0$$

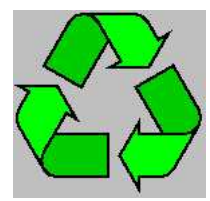
$m_i^2 = 0$, gluons massless

Now consider a loop amplitude



Trouble is $\ell_i^2 \neq 0$.

The key to efficient computation is recycling.



- Recursive approach – mainly at tree-level Berends and Giele; Kosower; Mahlon
  Recent work by Bena, ZB and Kosower; Roiban, Spradlin, Volovich; Britto, Cachazo, Feng, Witten
- Unitarity approach – loops from trees. ZB, Dixon, Dunbar and Kosower
  Recent work by ZB, Dixon and Kosower; Britto, Cachazo, Feng

See talks by Dixon and Kosower for further details

Unitarity Method for Loops

ZB, Dunbar, Dixon, Kosower
ZB and Morgan

- Key: *Any* amplitude in any massless theory is fully determined from D-dimensional tree amplitudes to *all* loop orders. Off-shell Feynman diagrams unnecessary.
- Four-dimensional cut constructibility: At one-loop, any amplitude in a massless susy gauge theory is fully constructible from *four-dimensional* tree amplitudes. Use helicity and twistors.

The simplest of all gauge theories is the $N = 4$ maximally supersymmetric one. Relative of QCD.

N=4 theory in talks by Britto, Dixon, Dolan, Kosower, Roiban, Travaglini, Witten, etc

See talks by Britto

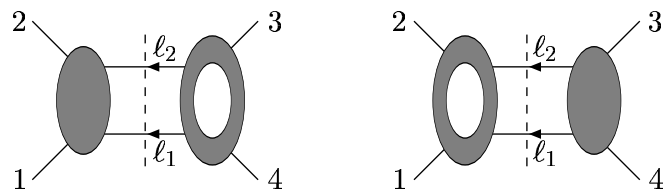
Dixon and Kosower

Generalized Unitarity

ZB, Dixon and Kosower

Britto, Cachazo and Feng

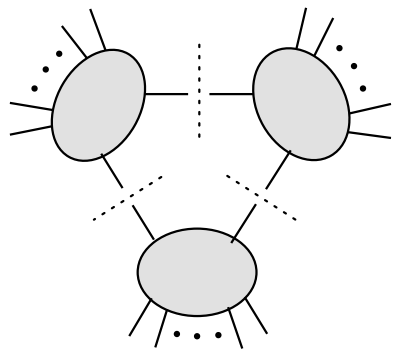
Two-particle cuts:

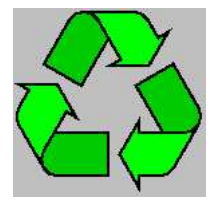


intermediate legs on shell

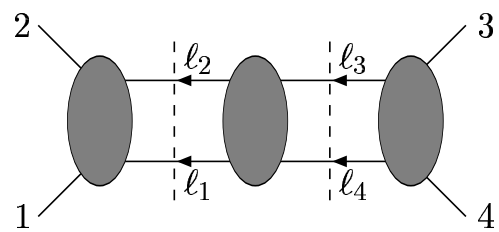
$$\ell_i^2 = 0$$

Triple cuts:





Generalized double two-particle cut:



Tree-level properties induce loop-level properties.

Twistor Space Scattering Amplitudes

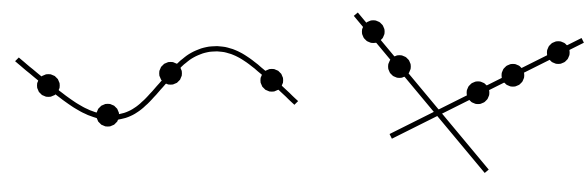
See talks by Witten and Spradlin

In a beautiful paper Ed Witten demonstrated that "twistor space" can reveal hidden structures of scattering amplitudes. Inspiration from Nair.

Link to string theory is for $N = 4$ super-Yang-Mills theory, but at tree level it might as well be QCD.

Twistor space given by Fourier transform with respect to plus helicity spinors.

$$\widetilde{A}(\lambda_i, \mu_i) = \int \prod_i \frac{d^2\widetilde{\lambda}_i}{(2\pi)^2} \exp\left(\sum_j \mu_j^{\dot{a}} \widetilde{\lambda}_{j\dot{a}}\right) A(\lambda_i, \widetilde{\lambda}_i)$$



Tree-level QCD scattering amplitudes $\leftrightarrow$ 'Twistor-space' $\leftrightarrow$ Topological String Theory

E. Witten; Roiban, Spradlin, and Volovich

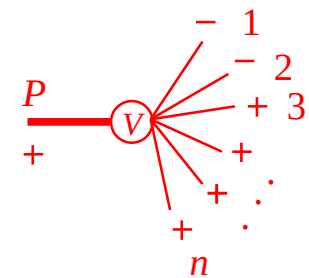
Witten observed that in twistor space external points lie on certain curves. Very constraining.

MHV Vertices

See talks from
Svrček, Khoze and Travaglini

Motivated by twistor space structure Cachazo, Svrček and Witten (CSW) define an off-shell "MHV vertex" based on Parke-Taylor amplitudes

$$V(1^-, 2^-, 3^+, \ldots, n^+, P^+) = \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle \cdots \langle n-1,\, n\rangle \langle n\,P\rangle \langle P\,1\rangle}$$

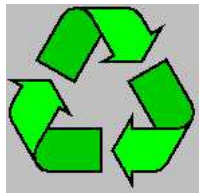


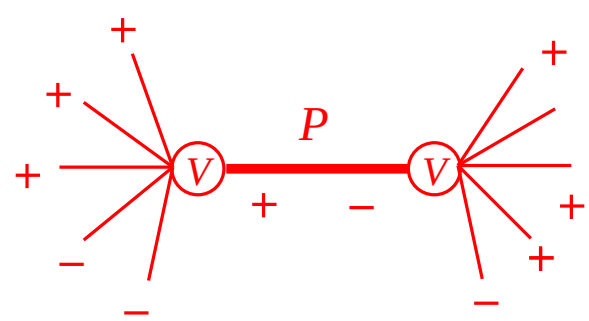
Continue spinor off-shell $(P^2 \neq 0)$: $\langle i\,P\rangle = \eta \sum_{j=1}^{n} \langle i^- | \not{k}_j | q^- \rangle$
where $P = k_1 + k_2 + \cdots k_n$ and q auxiliary, satisfying $q^2 = 0$.

Non-MHV amplitudes obtained by sewing together MHV vertices.

Amplitudes are much simpler than anyone imagined.

non-MHV from MHV





Towards Electroweak Phenomenology

Discovering the Higgs boson and the origin of electroweak symmetry breaking is a central issue in collider physics.

Is there any way to use the twistor-motivated developments to obtain electroweak scattering amplitudes?

Obvious problem: Twistor-motivated methods appear to apply only to massless theories. Electroweak vector bosons and Higgs bosons are massive.

But it is still possible to make progress:

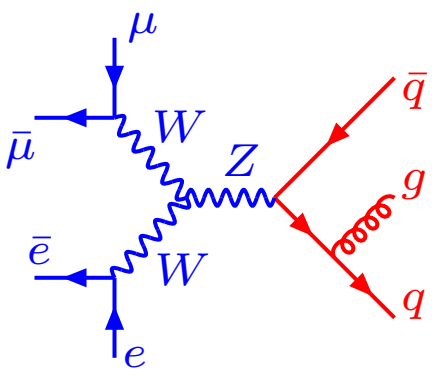
- Higgs boson coupled to quarks and gluons. Badger, Dixon, Glover, and Khoze

  See Dixon's talk

- Electroweak vector boson coupled to quarks. ZB, Forde, Kosower, Mastrolia

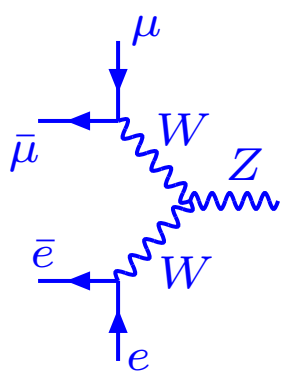
Electroweak Vector Bosons

Consider mixed Electroweak/QCD process:

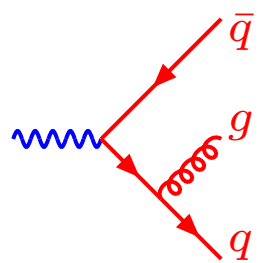


How do we use MHV vertices with massive electroweak vector bosons?

Idea: Split problem into two pieces.



$\times$



Feynman Methods CSW diagrams

Vector boson current

Using MHV vertices add arbitrary numbers of quarks and gluons

Gauge invariance issues make this tricky.

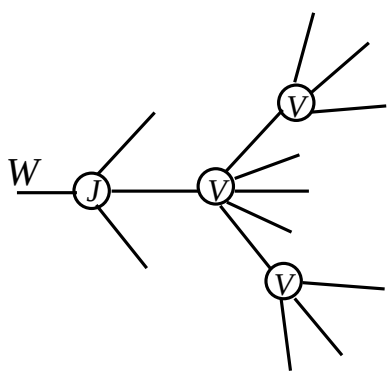
Electroweak Vector Bosons

Define a new set of vector boson current MHV vertices:

$$J^\mu(1_q^-, 2^+, \ldots, (n-1)^+, n_{\bar{q}}^+; P_V) = -\frac{i}{\sqrt{2}} \frac{\langle (-1)^- | \gamma^\mu \not{P}_V | (-1)^+ \rangle}{\langle (-1)\, 2 \rangle \langle 2\, 3 \rangle \ldots \langle (n-1)\, n \rangle}$$

$$J^\mu((-P_V)^-; P_V) = \frac{i}{\sqrt{2}} \frac{\langle \eta^+ | \gamma^\mu | P_V^{\flat\, +} \rangle}{[P_V^\flat\, \eta]} P_V^2$$

ZB, Forde, Kosower and Mastrolia



V are MHV vertices of CSW

J are are MHV vector boson current vertices

W leg can be coupled to arbitary off-shell source

These may be coupled to any electroweak vector boson processes.

Numerical checks up to 10 legs against recursive Feynman diagram methods.

Mixed QCD/Electroweak amplitudes can take advantage of twistor-motivated ideas.

Summary

1. Scattering amplitude calculations are crucial in particle physics: Search for New Physics at electroweak symmetry breaking scale.

2. Detailed comparisons of Standard Model to experiments requires many particles and loop corrections. Difficult to calculate.

3. Key result from twistors: Gauge theory scattering amplitudes are much simpler than anyone imagined.

4. MHV vertices. Talks from Khoze, Svrček and Travaglini

5. Unitarity method: Tree amplitude simplicity transfers to loop amplitude simplicity. Talks from Britto, Dixon and Kosower

6. Initial applications of twistor-motivated methods to electroweak theory.

A deeper understanding of the underlying mathematical structures will surely lead to further progress in computing scattering amplitudes.