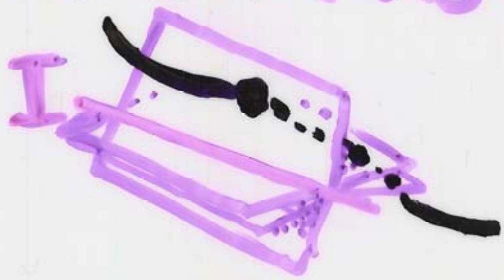
Questions for String Theorists

- Do we need the extra dims.? can $\mathbb{PT}$ take the place of Calabi-Yau ?
- Do we need the supersymm.? Is parameterization invar. needed in twistor theory?

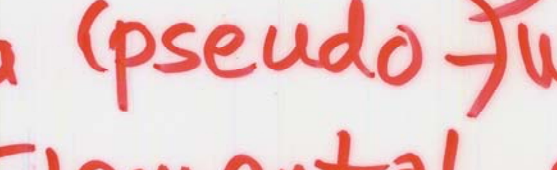


(conformal breaking)

- Do we really want (++−−) & a (pseudo-)Wick rot$\underline{\Delta}$? Elemental states?

} ← 2 "ears" ⊕ cohomolog

- No helicity for off-shell "gluing MHV's"



- Theory?? "Topological" QFT?

holomorphic

Some Remarks on Twistor/Spinor conventions

• In ++++ or ++−− signature we have a complex conjugation $Z^\alpha \rightsquigarrow \bar{Z}^\alpha$ rather than $\bar{Z}_\alpha$ (Lorentzian case) so does not agree with canonical conjugation so wavefunctions are not holomorphic.

n-twistor Symmetry Group

n=1: $A_{\alpha\beta} = 2\bar{Z}_{(\alpha} I_{\beta)\gamma} Z^\gamma$ $\quad \boxed{A_{\alpha\beta} = \begin{pmatrix} 0 & p_A{}^{B'} \\ p^{A'}{}_B & 2i\mu^{A'B'} \end{pmatrix}}$

$\bar{A}^{\alpha\beta} = 2 Z^{(\alpha} I^{\beta)\gamma} \bar{Z}_\gamma$ $\quad \boxed{\bar{A}^{\alpha\beta} = \begin{pmatrix} -2i\bar{\mu}^{AB} & p^A{}_{B'} \\ p_{A'}{}^B & 0 \end{pmatrix}}$

• In ++−− we can consider δ-fns on real α-planes. In $\mathbb{C}M$ these would be hyperfunctional objects ∴ cohomology elements in space-time

$M^{\alpha\beta} = \underbrace{\bar{\mu}^{AB}\varepsilon^{A'B'}}_{\text{ASD part}} + \underbrace{\mu^{A'B'}\varepsilon^{AB}}_{\text{SD part}}$

Compare with Hodges' elemental states

elementary elemental

General: $A_{\alpha\beta} = 2\bar{X}_{(\alpha} I_{\beta)\gamma} X^\gamma + \dots + 2\bar{Z}_{(\alpha} I_{\beta)\gamma} Z^\gamma$

$\bar{A}^{\alpha\beta} = 2 X^{(\alpha} I^{\beta)\gamma} \bar{X}_\gamma + \dots + 2 Z^{(\alpha} I^{\beta)\gamma} \bar{Z}_\gamma$

• Up/down nature of spinor parts $(\omega^A, \pi_{A'})$ fits in with conformal trans

Group: $\begin{pmatrix} X^\alpha \\ \vdots \\ Z^\alpha \end{pmatrix} = \Lambda U \begin{pmatrix} I^{\alpha\beta}\bar{X}_\beta \\ \vdots \\ I^{\alpha\beta}\bar{Z}_\beta \end{pmatrix}$

$\hat{g}_{ab} = \Omega^2 g_{ab}$ $\quad \hat{\varepsilon}_{AB} = \Omega\varepsilon_{AB}$ $\quad \hat{\varepsilon}^{AB} = \Omega^{-1}\varepsilon^{AB}$ $\quad \Upsilon_a = \nabla_a \log\Omega$ $\quad \hat{\omega}^A = \omega^A$ $\quad \hat{\pi}_{A'} = \pi_{A'} + i\Upsilon_{AA'}\omega^A$

Quantum Gravity

The application of standard procedures of quantum (field) theory to general relativity (or other gravitational theory)?

OR

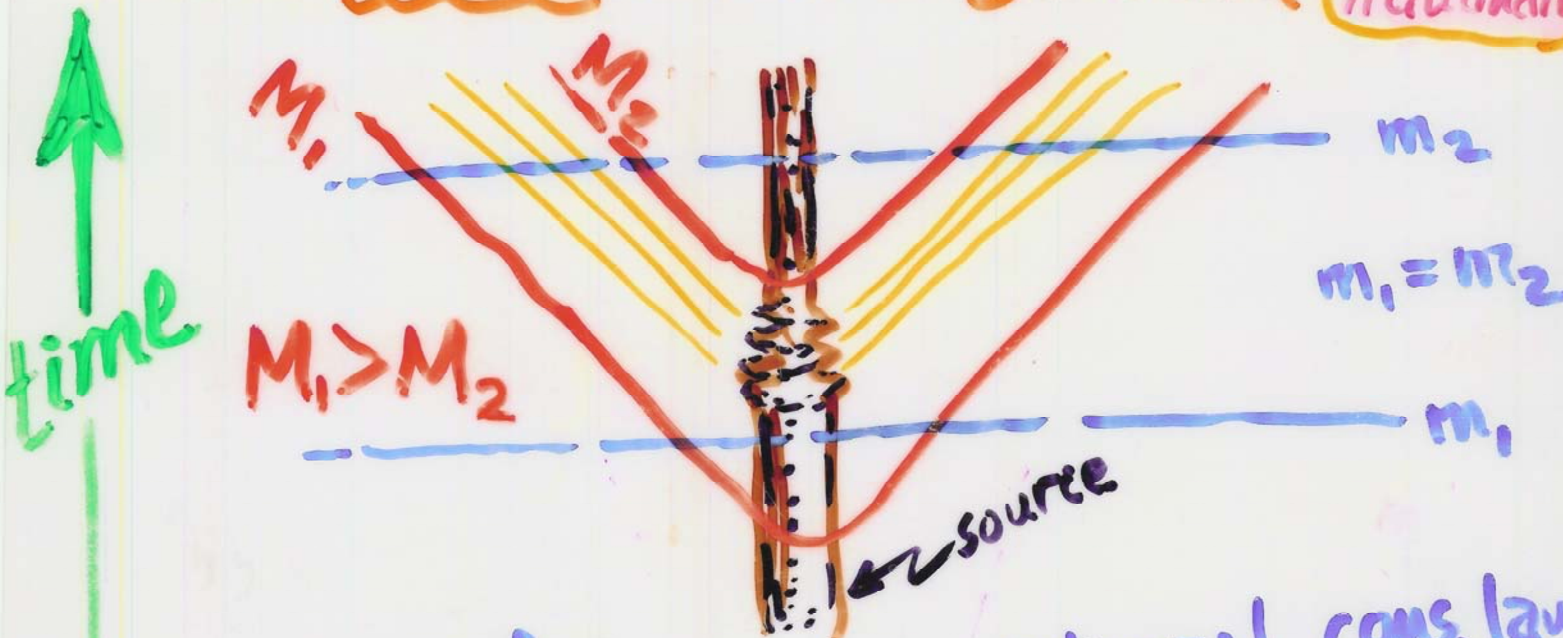
Some more even-handed marriage, with give on both sides?

⇒ modified quantum mechanics

Gravitational Radn.

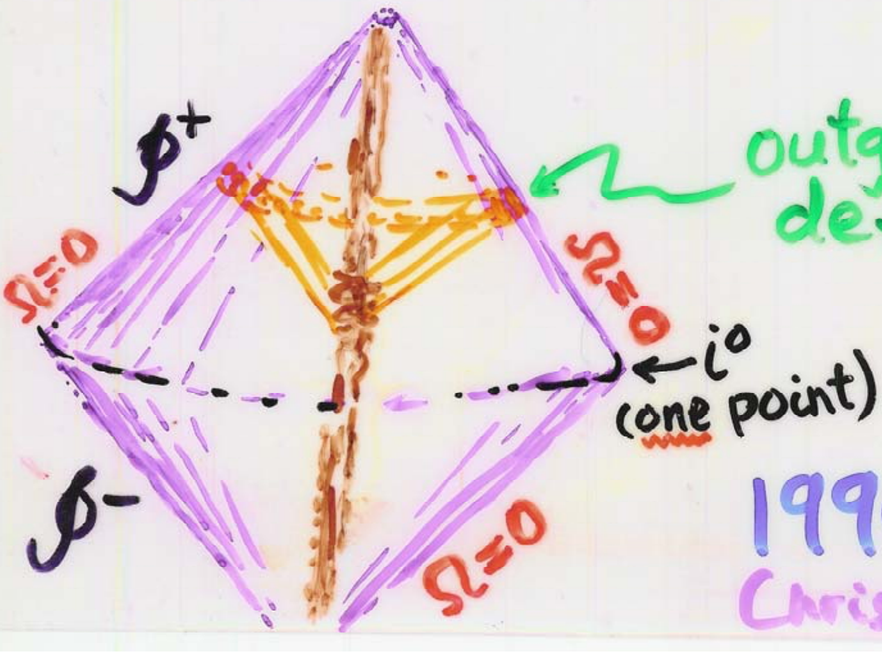
And earlier work by Trautman

Theory: Bondi 1960
use **NULL** ref. system



N.B. $\nabla_a T^{ab} = 0$ not an integral cons. law

~1960 Trautman
1962 Bondi, van der Burg, Metzner
1962 Sachs
1962 Newman, Unti; Newman, RP
1964/5 RP conformal approach



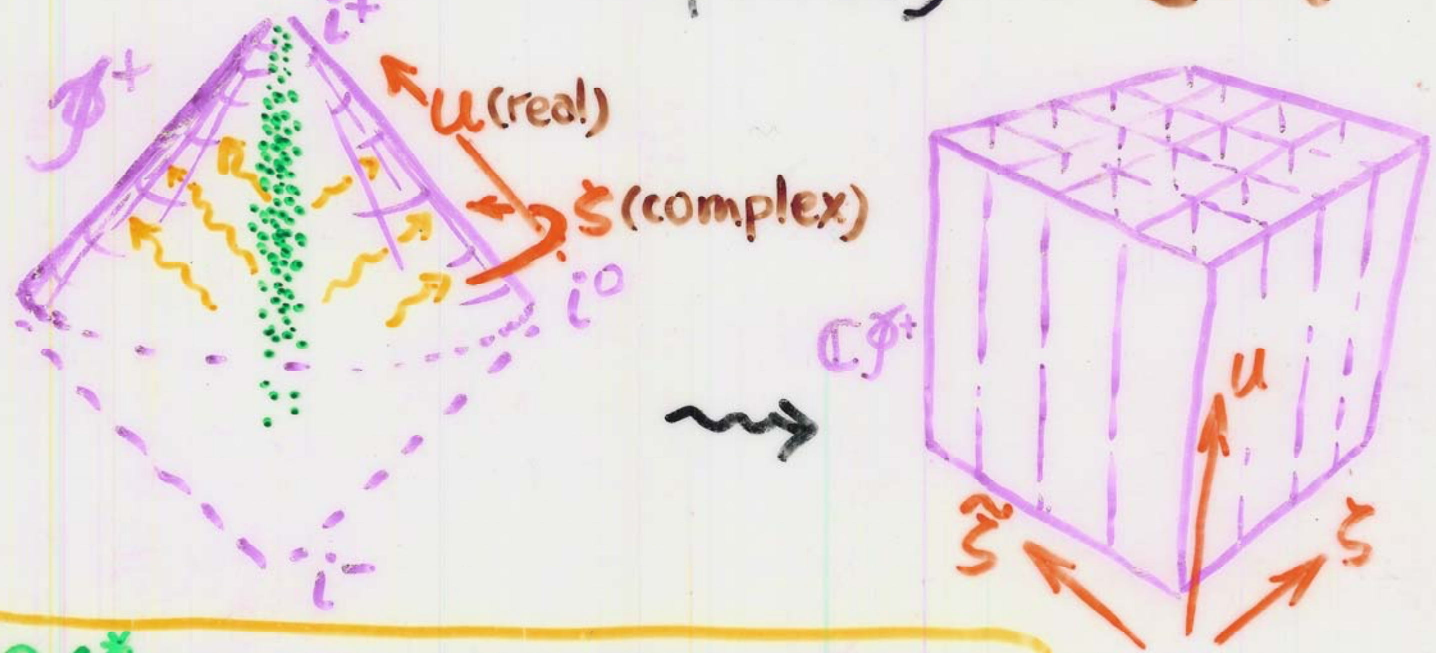
outgoing radiation defined here

$$\hat{g} = \Omega^2 g$$

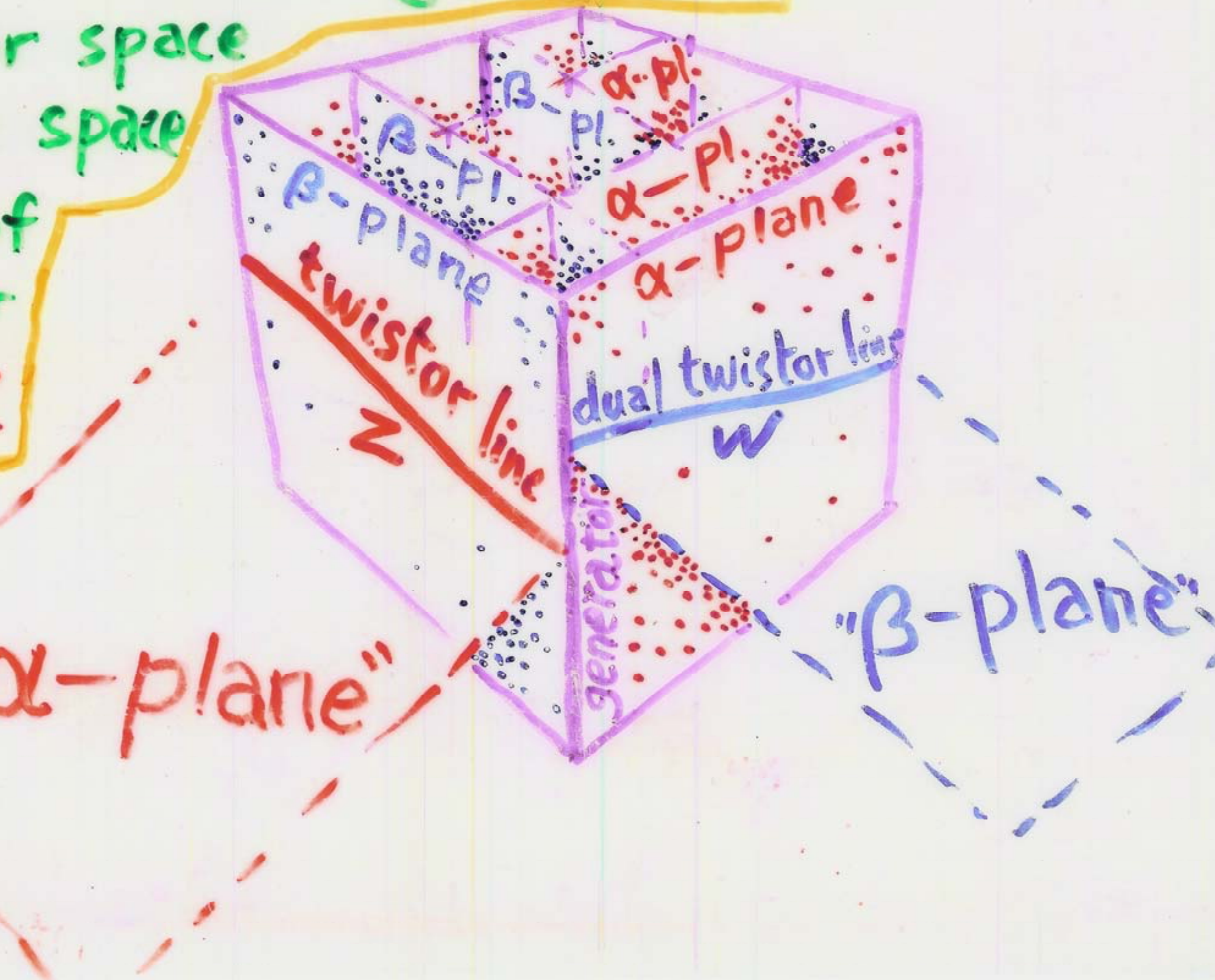
1990s Friedrich
Christodoulou - Kleinerman

Asymptotic twistors

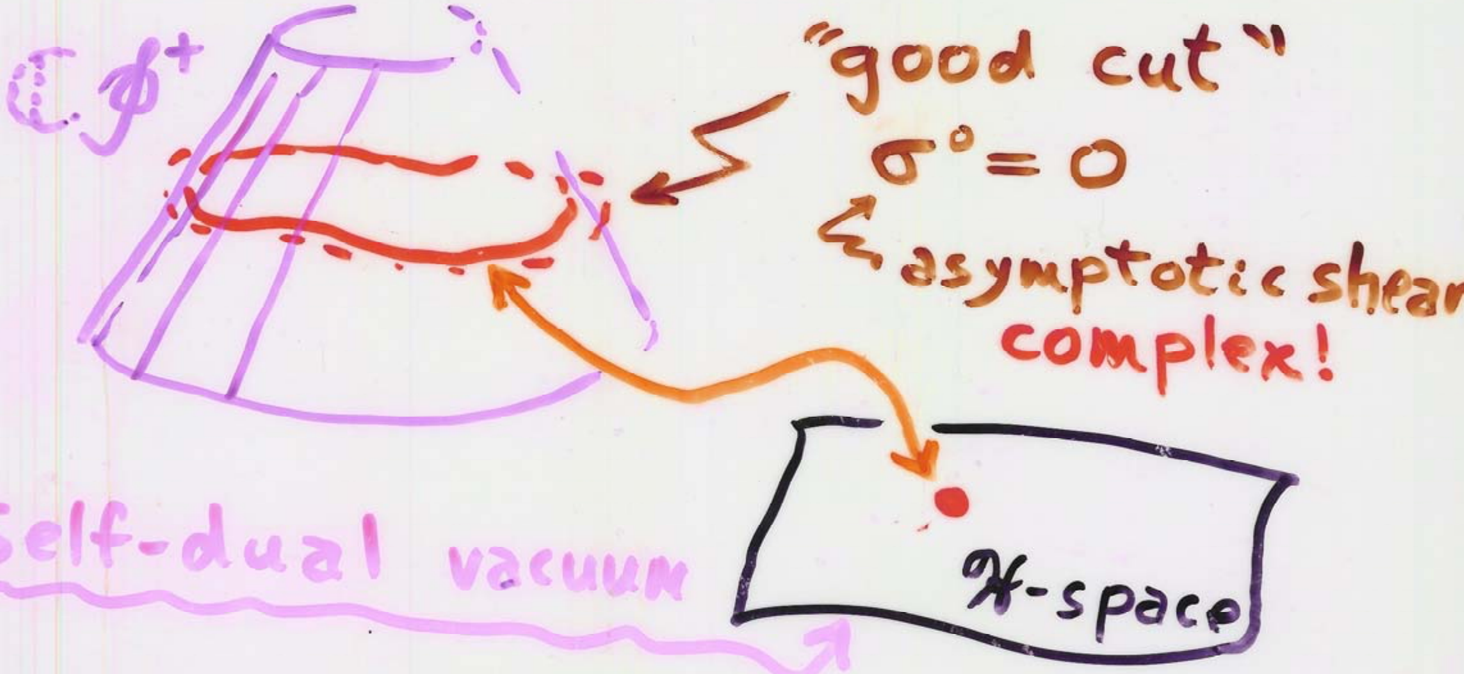
$\mathcal{M}$ asymptotically flat, radiating, analytic vacuum: complexify $\rightsquigarrow$ $\mathbb{C}\mathcal{M}$



$\mathcal{H}^*$-space = ASD "space-time" whose ("non-linear graviton") twistor space is the space $\mathbb{P}\mathcal{T}$ of twistor lines Z

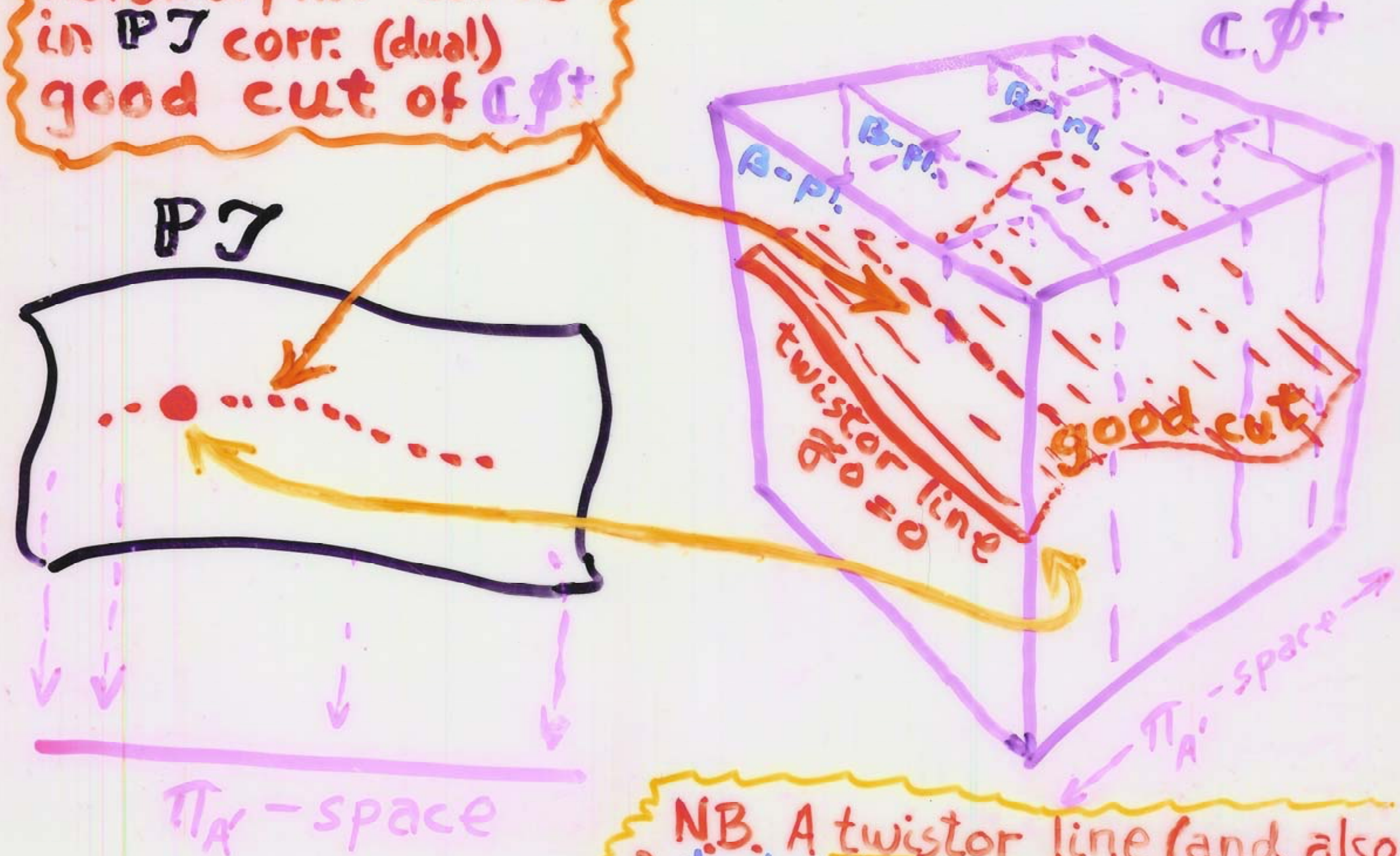


Newman's $\mathcal{H}$-space



Twistor view (take $\mathcal{H}^*$-space: $\tilde{\sigma}^0 = 0$)

Holomorphic curve in $\mathbb{P}\mathcal{T}$ corr. (dual) good cut of $\mathbb{C}\mathscr{I}^+$



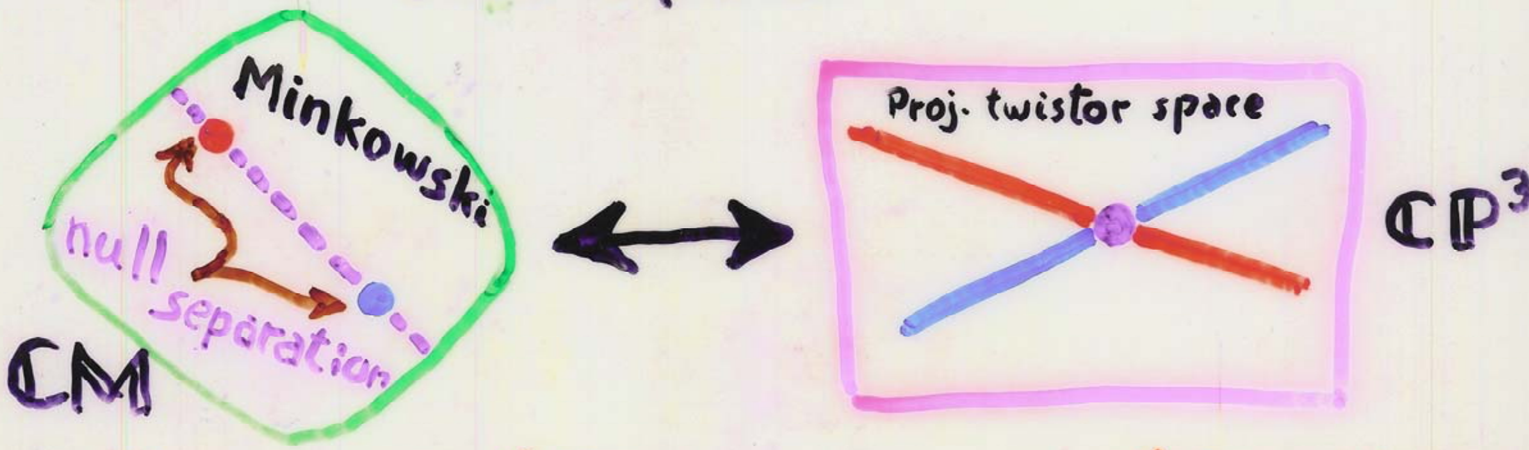
N.B. A twistor line (and also a dual twistor line and a generator) is a **NULL GEODESIC** on $\mathbb{C}\mathscr{I}^+$

General Relativity

Numerous special applications
(e.g. Woodhouse-Mason: stationary axi-symm

As part of general programme:
"non-linear graviton construction" [R.P. '7

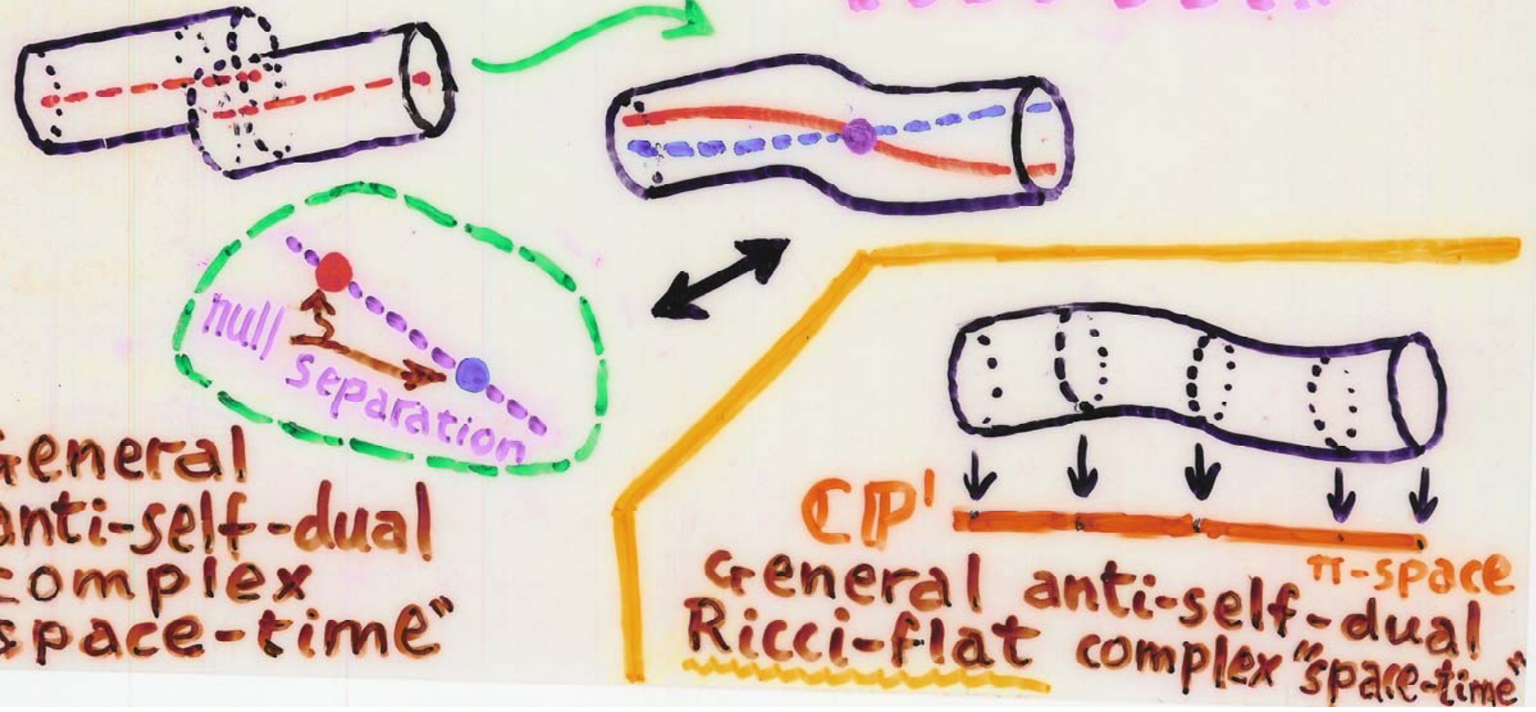
N.B. for flat space:



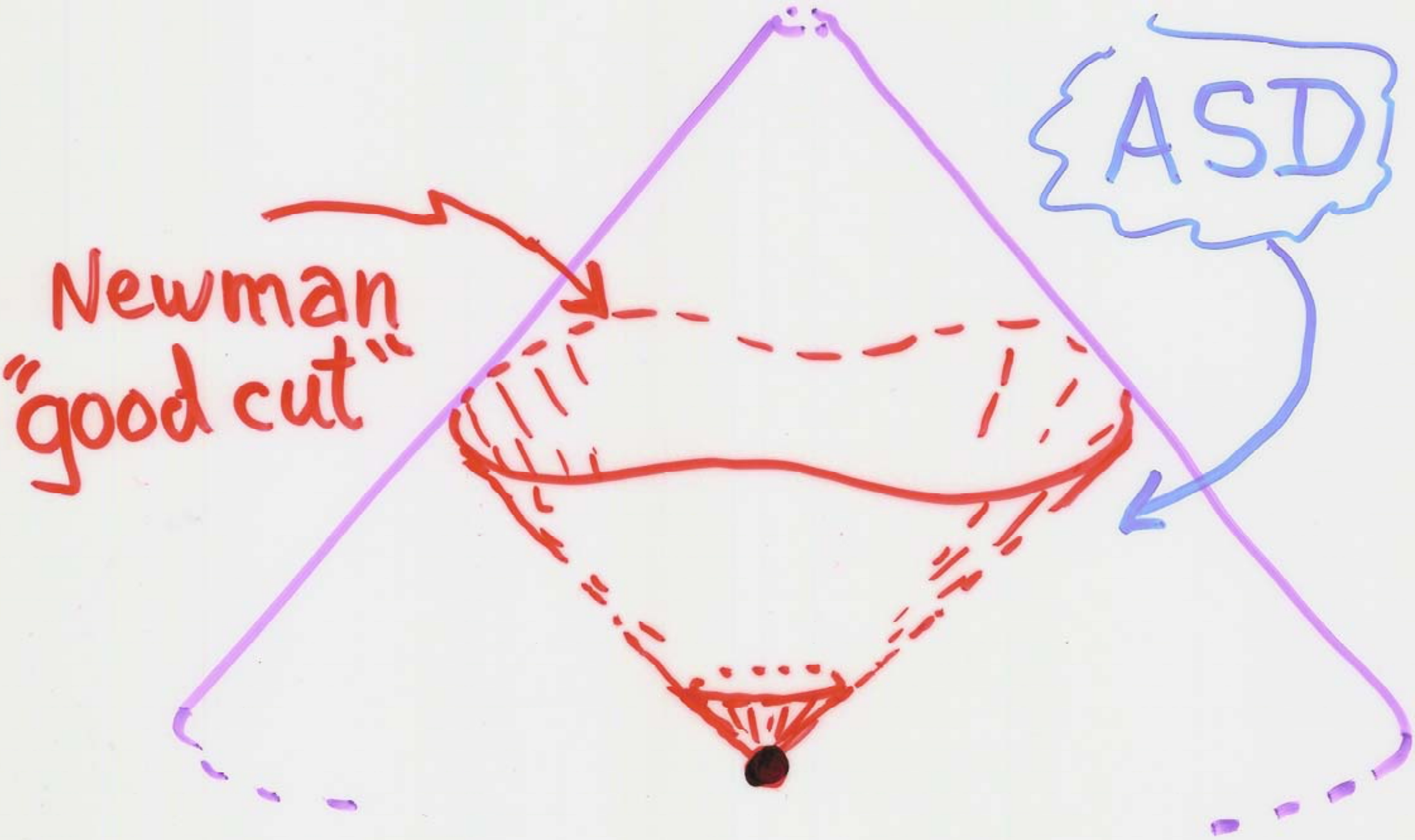
null separation ⟷ meeting lines

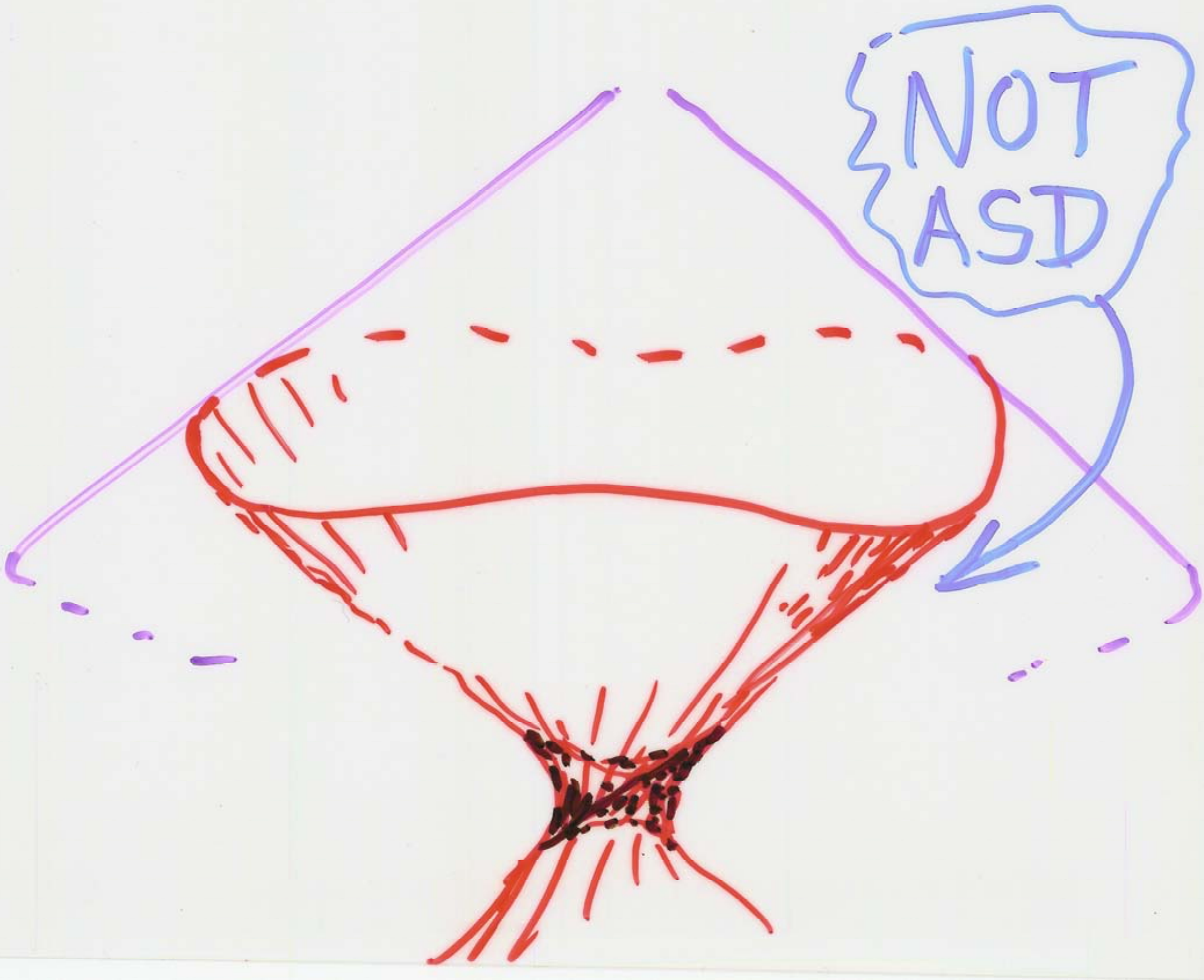
Deform:

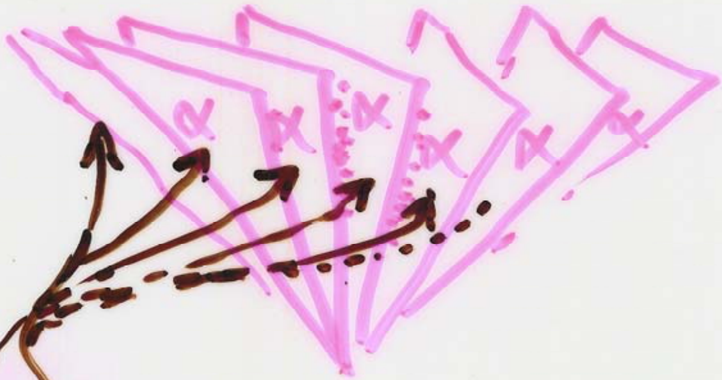
R.P. 1976



Light-cone cuts





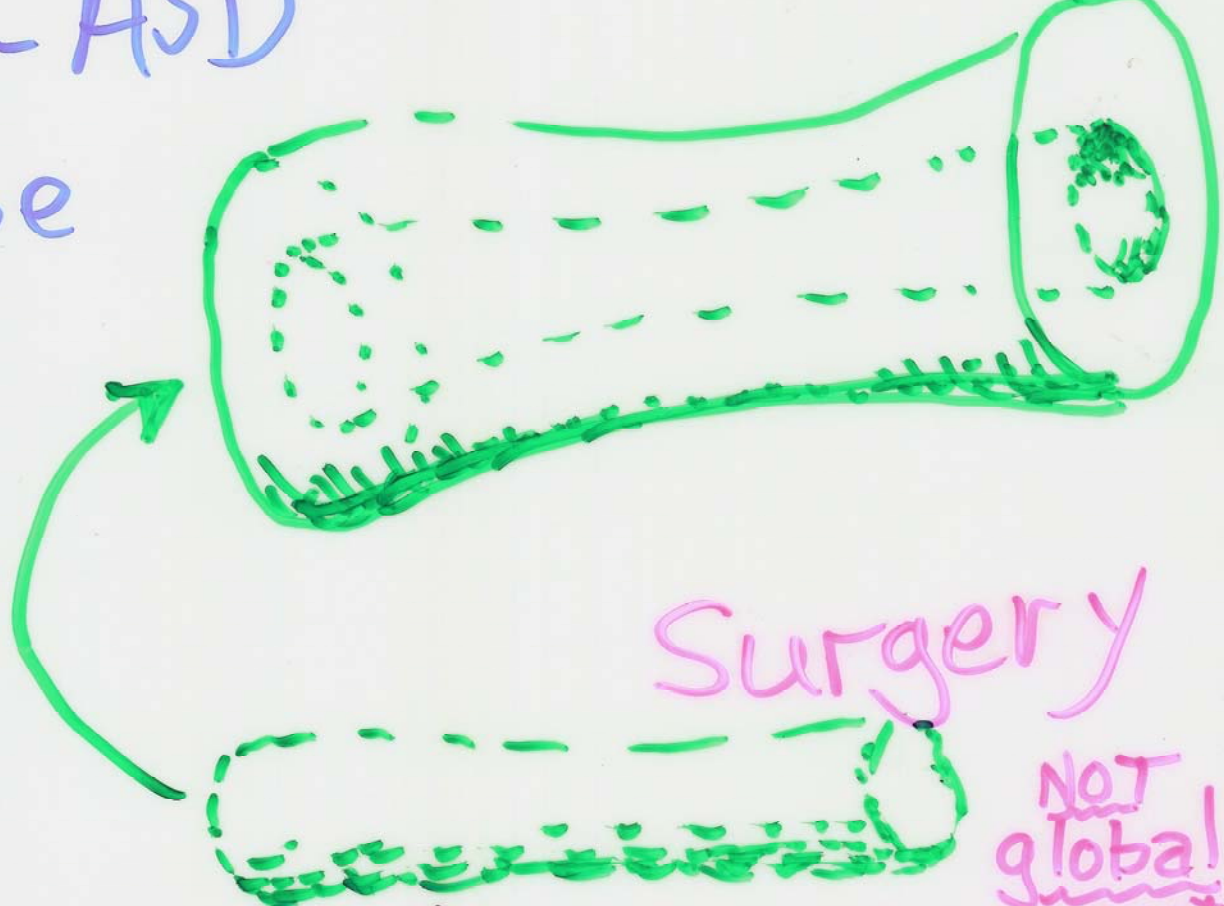


α-surfaces

Shearing
comes about
from SD curv.

ASD
case

non-ASD
case



NOT global
in a partic.
way.

Can we understand these surgeries in terms of a power series in strings (holomorphic curves) of higher and higher order?

Linearized gravity: (can be complex

$$K_{abcd} = \underbrace{\psi_{ABCD}\,\varepsilon_{A'B'}\,\varepsilon_{C'D'}}_{\text{anti-self-dual}} + \underbrace{\varepsilon_{AB}\,\varepsilon_{CD}\,\tilde{\psi}_{A'B'C'D'}}_{\text{self-dual}}$$

ψ_{ABCD}, $\tilde{\psi}_{A'B'C'D'}$ both symmetric; if K_{abcd} is real, they are complex conjugates

$$\nabla^{AA'}\psi_{ABCD} = 0, \qquad \nabla^{AA'}\tilde{\psi}_{A'B'C'D'} = 0$$

Twistor contour integrals:

$$\psi_{ABCD}(x) = \oint_{\omega = ix\pi} \frac{\partial}{\partial\omega^A}\frac{\partial}{\partial\omega^B}\frac{\partial}{\partial\omega^C}\frac{\partial}{\partial\omega^D} f(Z)\,\delta Z$$

hom. deg. +2

hom. deg. −6

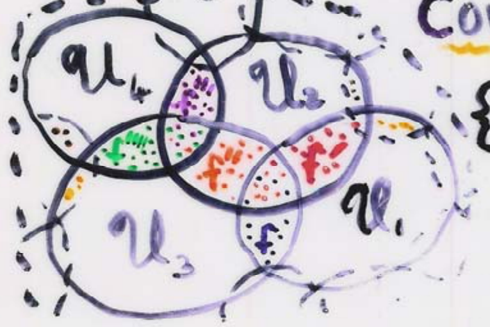
$$\tilde{\psi}_{A'B'C'D'}(x) = \oint_{\omega = ix\pi} \pi_{A'}\pi_{B'}\pi_{C'}\pi_{D'}\tilde{f}(Z)\,\delta Z$$

$$\boxed{\delta Z = \varepsilon^{A'B'}\pi_{A'}d\pi_{B'}}$$

Left-handed graviton: f hom. deg. +2
Right-handed graviton: $\tilde{f}$ hom. deg. −6

Really, f and $\tilde{f}$ are representatives of

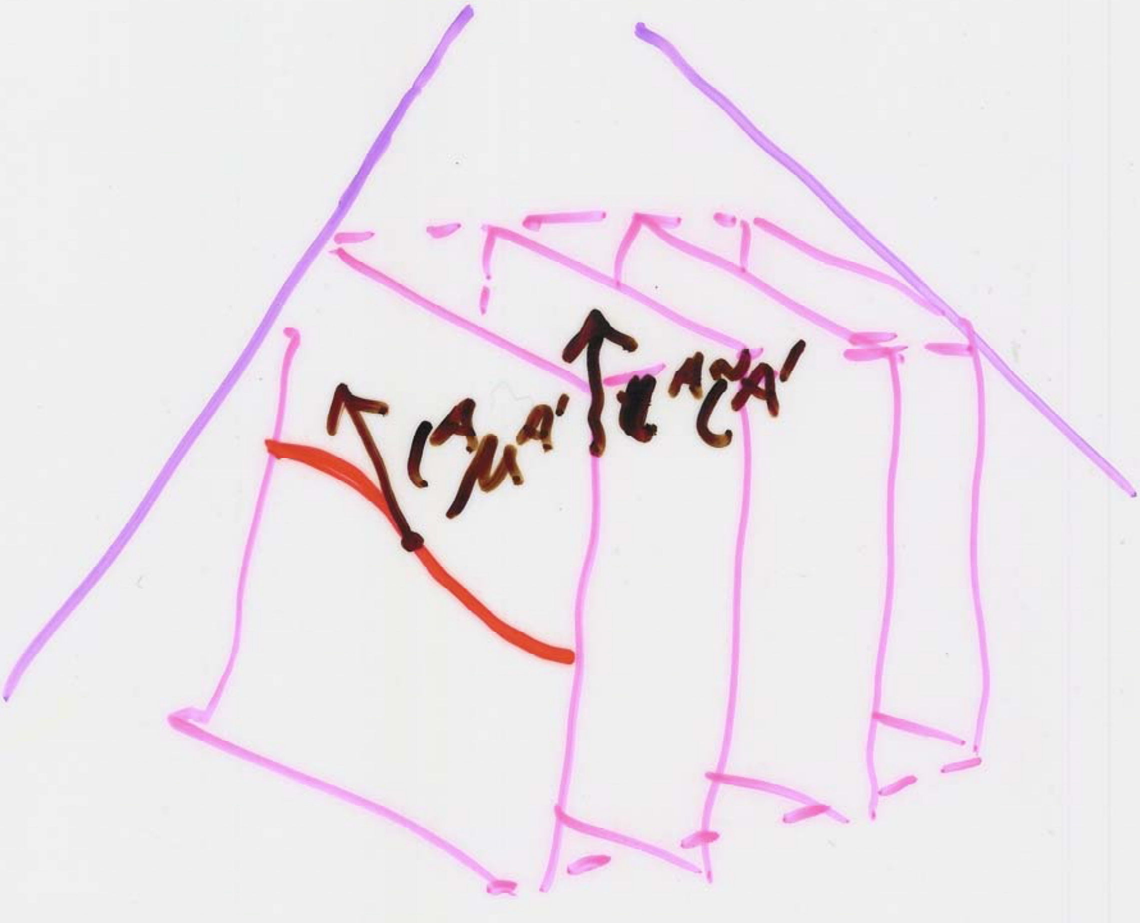
COHOMOLOGY



Covering $\{\mathcal{U}_i\}$

Encoding $\mathcal{O}(-6)$

$$\underset{i}{Z}^{\alpha} = \left(1 + f_{(-6)\,ij}\right)^{1/6} \underset{j}{Z}^{\alpha}$$



$$\mu^{B'} \nabla_{1B'} \mu_{A'} = K \mu_{A'} (\mu_1)^{-5} \eth_6 \tilde{\Psi}_{1'1'1'1'}$$

"1" means contract with $\tilde{\iota}^{A'}$ or ι^A

conf. invar. version of $\nabla_{M'}$

Twistor Space (New Def.)

Complex 4-manifold $\mathcal{T}$

Global structure encodes asymp. flat vacuums

Local structure (no continuous information):

1-form ι, 3-form θ,

satisfying:

$$\iota \wedge d\iota = 0, \qquad \iota \wedge \theta = 0,$$

known up to proportionality, s

Scaling, to preserve

$\Pi = d\theta \otimes d\theta \otimes \theta$, $\boxed{\Sigma = d\theta \otimes \iota = -2\theta \otimes d\iota}$

(where $\eta \otimes (dp \wedge dq) = \eta \wedge dp \otimes dq - \eta \wedge dq \otimes dp$)

Twistor-space forms | $\mathbb{M}$

1-form: ι ($=\delta z$) $= I_{\alpha\beta} Z^\alpha dZ^\beta = \varepsilon^{A'B'} \pi_{A'} d\pi_{B'}$

2-form: $\tau = \frac{1}{2} d\iota = \frac{1}{2} I_{\alpha\beta} dZ^\alpha \wedge dZ^\beta = d\pi_{0'} \wedge d\pi_{1'}$

3-form: $\theta = \frac{1}{6}\varepsilon_{\alpha\beta\gamma\delta} Z^\alpha dZ^\beta \wedge dZ^\gamma \wedge dZ^\delta$

$$= Z^0 dZ^1 \wedge dZ^2 \wedge dZ^3 - Z^1 dZ^0 \wedge dZ^2 \wedge dZ^3 + Z^2 dZ^0 \wedge dZ^1 \wedge dZ^3 - Z^3 dZ^0 \wedge dZ^1 \wedge dZ^2$$

4-form: $\phi = \frac{1}{4} d\theta = \frac{1}{24}\varepsilon_{\alpha\beta\gamma\delta} dZ^\alpha \wedge dZ^\beta \wedge dZ^\gamma \wedge dZ^\delta$

$$= dZ^0 \wedge dZ^1 \wedge dZ^2 \wedge dZ^3$$

Euler: $\Upsilon = \theta \div \phi = Z^\alpha \partial/\partial Z^\alpha$

$$da \wedge \theta = \Upsilon(a)\,\phi$$

$$\boxed{\iota \wedge \tau = 0, \quad \iota \wedge \theta = 0}$$

Homogeneity degrees

$$\pounds_\Upsilon \begin{bmatrix} \iota \\ \tau \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} 2\iota \\ 2\tau \\ 4\theta \\ 4\phi \end{bmatrix}$$

} $\Leftrightarrow$ $\theta \otimes \tau = -\phi \otimes \iota$

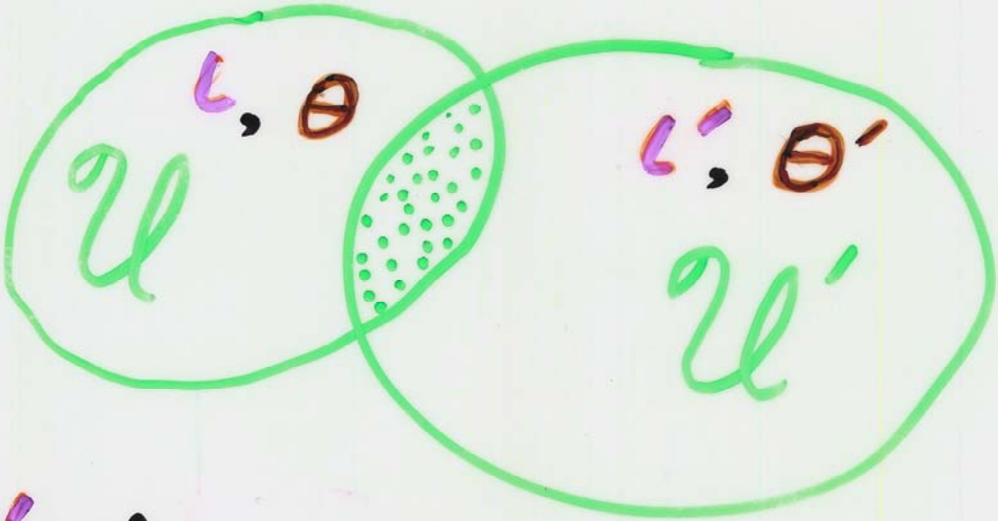
} automatic ($\theta \otimes \phi \equiv -\phi \otimes \theta$)

where $\alpha \otimes \beta = r\,\alpha_{[\ldots}\underbrace{\beta_{]}}_{r}{}_{\ldots}$ (β: r-form)

e.g.

$$\alpha \otimes dp \wedge dq = \alpha \wedge dp \otimes dq - \alpha \wedge dq \otimes dp$$

Googly scalings



$$\left.\begin{aligned} \iota' &= k\,\iota \\ \theta' &= k^2\theta \\ d\theta' &= k^{-1}\,d\theta \end{aligned}\right\}$$

preserves Π and Σ

and $\underbrace{d\theta\otimes\iota = -2\theta\otimes d\iota}_{\text{gives}}$

$$\boxed{\Upsilon(k) = 2k^{-2} - 2k}$$

[equiv.: $\Upsilon'(k^{-1}) = 2k^{2} - 2k^{-1}$ since $\Upsilon' = k^3\,\Upsilon$

recall: $\Upsilon = \theta \div (\tfrac{1}{4}\,d\theta)$]

Standard param. up Euler curves $\nearrow \Upsilon$

$$\Upsilon(z) = z$$

Find: $k^3 = 1 - F\,z^{-6}$ where F const. on curve

$$\therefore \boxed{k^3 = 1 - f_{-6}(Z^\alpha)}$$

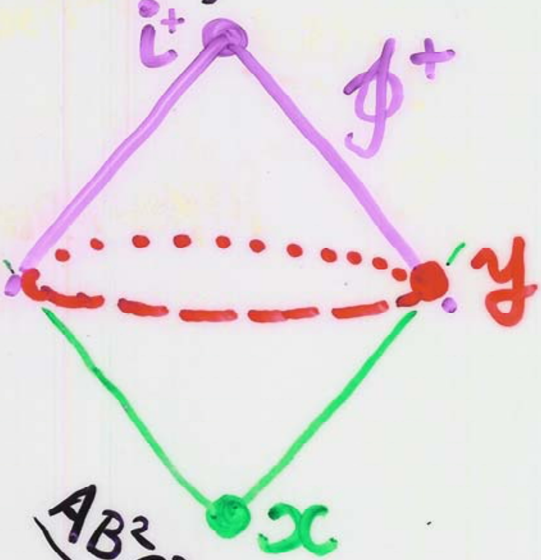
Googly contour integrals

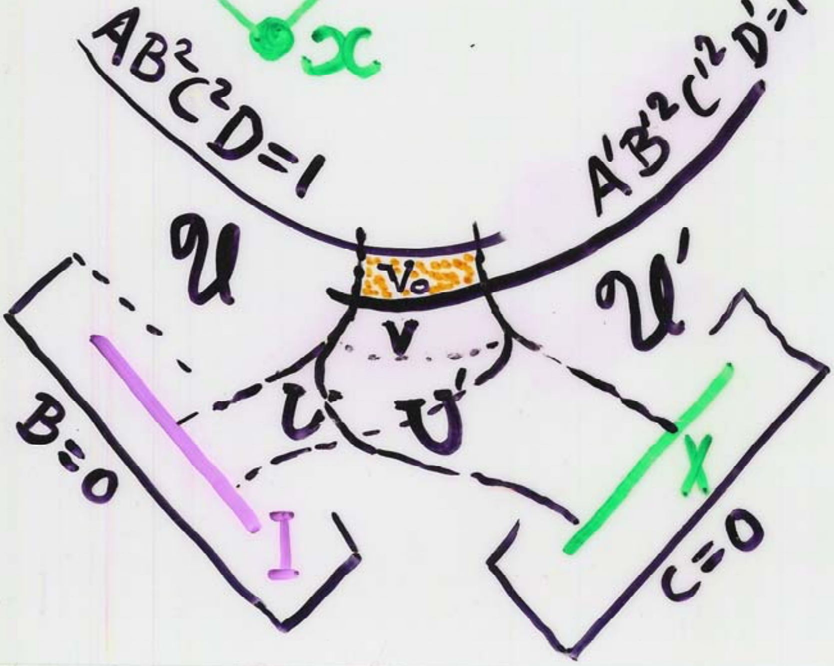
Supp. α, β, γ are 1-forms like ι, ξ, so $\left\{\begin{array}{l}\alpha' = k\alpha \\ \beta' = k\beta \\ \gamma' = k\gamma\end{array}\right\}$ on overlaps.

Then $d\{\alpha_\wedge \beta_\wedge \gamma\}$ is globally defined across patches, owing to special $1\text{-}f_{-6}(Z^\alpha)$ form of k.

Now, consider y, a variable point on $\mathbb{C}\mathscr{I}^+ \cap \mathscr{C}_x$ where η is the 1-form corr. to y. We wish to integrate

$$d\{\iota_\wedge \eta_\wedge \xi\}$$





$\iota = AdB - BdA$

$\eta = BdC - CdA$

$\xi = CdD - DdC$

$A' = kA,\ B' = kB,\ C' = kC,\ D' = kD$

$V_0 = U' - U = \oint f_{-6}\, \iota_\wedge \eta_\wedge \xi$

References (for googly construction)

[First three not very detailed]

R.P. (1999) The Central Programme of Twistor Theory
Chaos, Solitons & Fractals Vol. 10, no.2-3
pp. 581–611 (Elsevier Science Ltd.)

R.P. (1999) Twistor Theory and the Einstein Vacuum
Class. & Quantum Grav. 16
pp. A113 - A130

J. Frauendiener & R.P.
Twistors and General Relativity
in *Mathematics Unlimited – 2001 and Beyond*
(Pt. I) eds. B. Enquist & W. Schmidt (Springer, Berlin) 479–505

R.P. (2000) On Extracting the Googly Information
Twistor Newsletter 45, 1-24.

S.A. Huggett & K.P. Tod (1985) *An Introduction to Twistor Theory* (Camb. Univ. Press)

R.P & W. Rindler (1986) *Spinors and Space-Time Vol. 2* (Camb. U. Press)