FIRST, WHAT WAS THE MOTIVATION FOR TWISTOR-STRING THEORY?

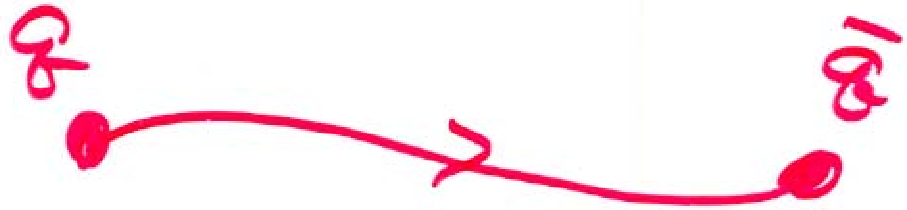
WE DON'T UNDERSTAND FOUR DIMENSIONAL QUANTUM GAUGE THEORY

i.e. QCD FOR GAUGE GROUP $SU(3)$

AS WELL AS WE'D LIKE.

FOR INSTANCE, WE DON'T REALLY UNDERSTAND QUARK CONFINEMENT

BUT IT SEEMINGLY IS RELATED TO "STRINGS"



FAT OR THIN STRING?

'T HOOFT PROPOSED (1974) THAT IT IS A "THIN STRING"... THAT 4D SU(N) GAUGE THEORY IS EQUIVALENT TO A STRING THEORY IN WHICH THE STRING COUPLING CONSTANT IS $1/N$ ... SO THE STRING DESCRIPTION IS USEFUL IF N IS LARGE ENOUGH

(EXPERIMENT AND COMPUTER SIMULATION APPEAR TO SHOW THAT $N=3$ IS LARGE ENOUGH)

BUT WHAT STRING THEORY IS EQUIVALENT TO GAUGE THEORY IN FOUR DIMENSIONS?

THERE WASN'T MUCH PROGRESS UNTIL MALDACENA (1997) SUGGESTED THE ANSWER FOR THE MAXIMALLY SUPERSYMMETRIC CASE, i.e.

$N=4$ SUPERSYMMETRY

$N=4$ SUPER YANG-MILLS ON

S^4 (= CONFORMAL COMPACTIFICATION OF MINKOWSKI 4-SPACE)

IS EQUIVALENT TO

TYPE IIB SUPERSTRING THEORY ON

$$AdS_5 \times S^5$$

hyperbolic 5-space

(OR H_5)

A REAL ROSETTA STONE... A DICTIONARY TRANSLATING FROM GAUGE THEORY TO GRAVITY AND BACK....

RANGE OF VALIDITY:

STRING COUPLING CONSTANT g_{st} $\longleftrightarrow$ $\dfrac{1}{N}$

RADIUS OF CURVATURE$^{4\text{th}}$ (i.e. INVERSE OF COSMOLOGICAL CONSTANT, IN PLANCK UNITS) $\longleftrightarrow$ $g^2 N$

g_{YM}

BECAUSE OF THE LAST POINT, THE GRAVITY INTERPRETATIO[N] IS MOST POWERFUL WHEN g^2N IS LARGE AND THE GAUGE THEORY INTERPRETION IS INTRACTABLE.

IF g^2N IS LARGE (ALONG WITH N) CURVATURES ARE SMALL AND WE CAN USE SUPERGRAVITY

BUT IF g^2N IS NOT LARGE THE CURVATURES ARE LARGE AND WE MUST USE THE FULL STRING THEORY

$$\text{MAPS}: \underset{\substack{\| \\ \text{2-d RIEMANN} \\ \text{SURFACE}}}{\Sigma} \longrightarrow AdS_5 \times S^5$$

A 2d QUANTUM FIELD THEORY WITH A RATHER SOPHISTICATED ACTION ... POSSIBLY INTEGRABLE ... WE'LL HEAR MORE

HERE WE GET TO MY MOTIVATION FOR TWISTOR-STRING THEORY ...

I WANTED TO FIND AN ALTERNATIVE DESCRIPTION OF THIS 2-D FIELD THEORY THAT WOULD BE USEFUL FOR SMALL g^2N.

FASCINATING QUESTION, AND MAYBE NECESSARY FOR THE "ORIGINAL" QUESTION OF QUARK CONFINEMENT SINCE IN QCD, BY ASYMPTOTIC FREEDOM, g^2N IS SMALL AT SHORT DISTANCES.

A QUANTUM FIELD THEORY OFTEN RADICALLY CHANGES ITS CHARACTER IF ONE GOES TO A NEW ASYMPTOTIC REGIME OF PARAMETERS ... MIRROR SYMMETRY, MONTONEN-OLIVE DUALITY, AND MANY OTHER EXAMPLES ....

SO THE THEORY THAT WE DESCRIBE FOR LARGE RADIUS BY

$$\text{MAPS}: \Sigma \rightarrow AdS_5 \times S^5$$

SHOULD PERHAPS HAVE A VERY DIFFERENT DESCRIPTION WHEN THE RADIUS IS SMALL

IT AT LEAST SHOULD HAVE THE SAME SYMMETRIES, THOUGH

$AdS_5 \times S^5$ HAS SYMMETRY

$PSU(4|4)$

OR

$PSU(2,2|4)$

$PSU(4|4)$ $\left(\begin{array}{c|c} \text{BOSE} & \text{FERMI} \\ \hline \text{FERMI} & \text{BOSE} \end{array}\right) \left(\begin{array}{c} 4 \\ \hline 4 \end{array}\right)$

WHY $PSU(4|4)$?

ONE $SU(4)$ ... CONFORMAL SYMMETRY IN FOUR DIMENSIONS

ONE $SU(4)$ ... MAXIMAL $N=4$ SUPERSYMMETRY

TRY TO REPLACE $AdS_5 \times S^5$

AS TARGET WITH SOME OTHER

SPACE OF THE SAME SYMMETRY

... IF WE DO THIS, WE'LL

GENERALLY NEED TO DO SOME SORT

OF "TOPOLOGICAL STRING THEORY"

AS OTHER HOMOGENEOUS SPACES OF

$PSU(2,2|4)$, OTHER THAN $AdS_5 \times S^5$,

HAVE THE WRONG SIGNATURE AND

WOULD GIVE UNPHYSICAL MODES

EVENTUALLY, ONE TRIES SUPER-TWISTOR SPACE

$$Z \to \lambda Z$$
$$\psi \to \lambda \psi$$
$$\lambda \in \mathbb{C}^*$$

$$\mathbb{CP}^{3|4}$$

PROJECTIVE SPACE OF FOUR BOSONS AND FOUR FERMIONS

$$(Z^1 \dots Z^4 ; \psi^1 \dots \psi^4)$$

WHY $\mathbb{CP}^{3|4}$ AND NOT SOME MORE GENERAL $\mathbb{CP}^{3|N}$?

THERE ARE A LOT OF STANDARD REASONS FOR $N=4$ (MAXIMAL SUPERSYMMETRY; AND QUANTUM CONFORMAL INVARIANCE)

BUT HERE WE FIND A NEW REASON:

$\mathbb{CP}^{3|N}$ IS A CALABI-YAU SUPER MANIFOLD IFF $N=4$

HOLOMORPHIC VOLUME FORM

$$(Z^1 \dots Z^4 | \psi^1 \dots \psi^N)$$

$$\Omega_0 = dZ^1 \wedge \dots \wedge dZ^4 \, d\psi^1 \dots d\psi^N$$

IS INVARIANT TO

$$Z \to \lambda Z \qquad \psi \to \lambda \psi$$

$$dZ \to \lambda dZ \qquad d\psi \to \lambda^{-1} d\psi$$

IFF $N=4$

WHAT GOOD IS THE

CALABI-YAU CONDITION?

IT ENABLES US TO DEFINE A

TOPOLOGICAL B-MODEL WITH

TARGET $\mathbb{CP}^{3|4}$.

THE TOPOLOGICAL B-MODEL, WITH

ANY CALABI-YAU TARGET, DESCRIBES

FOR OPEN STRINGS HOLOMORPHIC

BUNDLES, AND MORE GENERAL

SHEAVES, AND THEIR MODULI ...

FOR CLOSED STRINGS IT DESCRIBES

VARIATIONS OF COMPLEX STRUCTURE IN THE TARGET

SO IF ONE IS FAMILIAR w/ B-MODEL WITH THE PENROSE TRANSFORM, IT ISN'T TOO HARD TO SEE THAT, FOR $\mathbb{CP}^{3|4}$, THE OPEN STRINGS (FOR "SPACE-FILLING BRANES") REPRODUCE THE PERTURBATIVE SPECTRUM OF N=4 SUPER YANG-MILLS

(IT IS HARDER TO SEE HOW TO GET THE INTERACTIONS, AND DESPITE THE STRIKING RESULTS OBTAINED THIS WAY, I DON'T CONSIDER ANY OF THE DERIVATIONS REALLY SATISFACTORY....)

ONE CAN ALSO SEE (NB & EW)

THAT THE <u>CLOSED</u> <u>STRINGS</u>

GIVE <u>CON</u>FORMAL <u>SUPER</u>GRAVITY

... A RESULT THAT I

DEFINITELY DON'T LIKE

U(N)

IN THE B-MODEL, WE START WITH N SPACE-FILLING D-BRANES ON $\mathbb{CP}^{3|4}$

THE GAUGE GROUP WILL HENCE BE U(N) OR GL(N) AND THE BASIC FIELD IS A GAUGE-FIELD a ON $\mathbb{CP}^{3|4}$ ... IT IS OF TYPE (0,1) AS THE TOPOLOGICAL SYMMETRY REMOVES THE (1,0) PART.

MOREOVER, THE ACTION IN THE PERTURBATIVE B-MODEL IS REALLY THE ONLY ACTION ONE CAN WRITE FOR THIS FIELD:

$$I = \int_{\mathbb{CP}^{3|4}} \Omega \cdot \mathrm{Tr}\left(a\,\bar{\partial}\,a + \frac{2}{3}\, a \wedge a \wedge a\right)$$

A SORT OF CHERN-SIMONS (0,3)-FORM ACTION.

Ω IS THE HOLOMORPHIC CALABI-YAU MEASURE OF $\mathbb{CP}^{3|4}$

$$0 = F_a^{(0,2)} = \bar{\partial} a + a \wedge a$$

$$\delta a = \bar{\partial}_a \epsilon = \bar{\partial} \epsilon + [a, \epsilon]$$

I CLAIM THAT THIS ACTION, WHEN INTERPRETED IN MINKOWSKI SPACE VIA THE PENROSE-WARD TRANSFORM, REPRODUCES THE SPECTRUM AND PART OF THE INTERACTIONS OF $N=4$ SUPER YANG-MILLS ...

TO GET THE SPECTRUM, LET US JUST EXPAND IN POWERS OF ψ (THE FERMIONIC HOMOGENEOUS COORDINATE OF $\mathbb{C}P^{3|4}$)

$a(z,\bar{z},\psi)$

$$
\begin{aligned}
a = \tilde{A}(z,\bar{z}) &+ \tilde{\chi}_A(z,\bar{z})\,\psi^A \\
&+ \tilde{\phi}_{AB}(z,\bar{z})\,\frac{\psi^A\psi^B}{2!} \\
&+ \tilde{\chi}^A(z,\bar{z})\,\frac{\varepsilon_{ABCD}\,\psi^A\psi^B\psi^C}{3!} \\
&+ \tilde{G}(z,\bar{z})\,\frac{\varepsilon_{ABCD}\,\psi^A\psi^B\psi^C\psi^D}{4!}
\end{aligned}
$$

EQNS OF MOTION

$$\bar{\partial} a = 0$$

GAUGE INVARIANCE $\quad a \to a + \bar{\partial}\epsilon$

AT THE LINEAR LEVEL,

EACH FIELD

$21\frac{1}{2}$

$$\tilde{\hat{A}}, \tilde{\chi}, \ldots, \tilde{G}$$

HAS ITS OWN EQN OF MOTION

$$\bar{\partial} \tilde{\hat{A}} = 0, \quad \text{etc}$$

AND ITS OWN GAUGE INVARIANCE

$$\tilde{\hat{A}} \to \hat{A} + \bar{\partial} \varphi$$

SO THEY REPRESENT $\bar{\partial}$-COHOMOLOGY CLASSES IN

$$H^1(\mathbb{CP}^{3|4}, \mathcal{L})$$

WHERE $\mathcal{L}$ IS DIFFERENT FOR EACH FIELD

THE WHOLE IDEA OF THE PENROSE TRANSFORM (FOR LINEAR WAVE EQUATIONS) IS THAT SUCH $\bar{\partial}$ COHOMOLOGY CLASSES CORRESPOND IN SPACETIME TO SOLUTIONS OF MASSLESS WAVE EQUATIONS ...

WHICH DEPEND ON $\mathcal{L}$.

THE FUNCTIONS APPEARING HERE ARE (0,1)-FORMS ON $\mathbb{CP}^{3|4}$ WITH VALUES IN

$$\mathcal{O} \quad \mathcal{O}(-1) \quad \mathcal{O}(-2) \quad \mathcal{O}(-3) \quad \mathcal{O}(-4)$$

$$\tilde{A} \quad \tilde{\chi}_A \quad \tilde{\phi}_{AB} \quad \tilde{\chi}^A \quad \tilde{G}$$

SO THEY DESCRIBE AT THE LINEARIZED LEVEL SOLUTIONS OF THE MASSLESS LINEAR WAVE EQNS OF HELICITY

$$1 \quad \tfrac{1}{2} \quad 0 \quad -\tfrac{1}{2} \quad -1$$

$$A \quad \chi_A \quad \phi_{AB} \quad \chi^A \quad G$$

STANDARD SPECTRUM OF $N=4$ SUPER YANG-MILLS!

THE FACT THAT THIS WORKS SHOULD NOT REALLY COME AS A SURPRISE, SINCE IT IS LARGELY DETERMINED BY THE PSU(4|4) SYMMETRY.

WHAT INTERACTIONS DO WE GET IN SPACETIME FOR THE VARIOUS TWISTOR SPACE FIELDS?

THE ANSWER TO THIS QUESTION IS THAT WE ONLY GET SOME, AND NOT ALL, OF THE DESIRED $\mathcal{N}=4$ SUPER YANG-MILLS INTERACTIONS.

I WILL EXPLAIN WHY IN TWO WAYS:

(I) LEFT-RIGHT ASYMMETRY

$$\mathcal{A} = \underbrace{\tilde{A}}_{\substack{\text{helicity} \\ 1}} + \dots + \underbrace{\tilde{G}\,\psi^4}_{\substack{\text{helicity} \\ -1}}$$

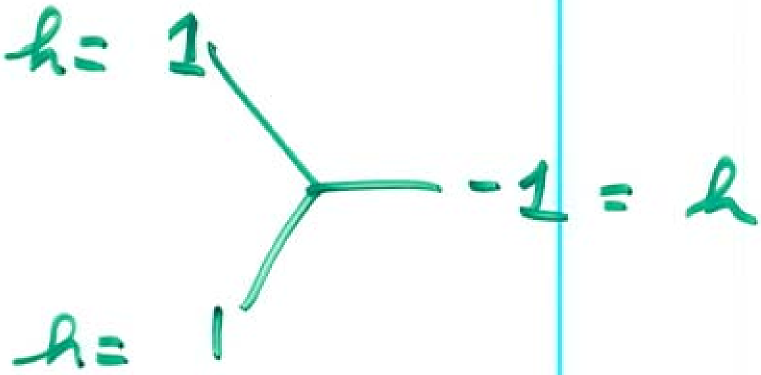
IF WE DROP ALL OTHER FIELDS, THE TWISTOR SPACE ACTION BECOMES

$$\int_{\mathbb{CP}^{3|4}} \Omega \, \mathrm{Tr}\left(a \bar{\partial} a + \frac{2}{3} a^3\right)$$

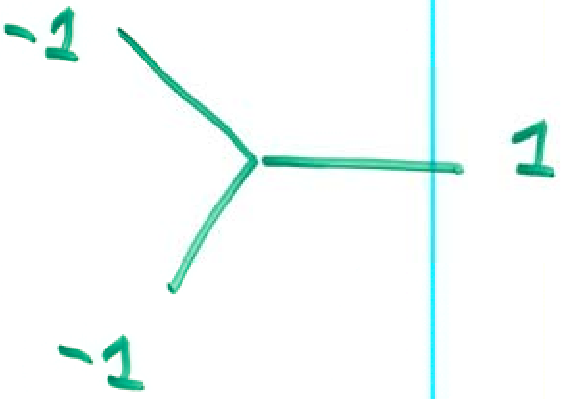
$$= 2 \int_{\mathbb{CP}^3} \Omega \, \mathrm{Tr}\left(\tilde{G} \bar{\partial} \hat{A} + \tilde{G} \wedge \hat{A} \wedge \hat{A}\right)$$

WE SEE THE ASYMMETRY – STANDARD IN TWISTOR THEORY – BETWEEN HELICITY 1 AND -1

WE HAVE ONE INTERACTION



AND NOT THE OTHER



$$0 = \bar{\partial} a + a \wedge a$$

THE EQUATIONS OF MOTION IN TWISTOR SPACE ARE

1) $0 = \bar{\partial} \tilde{A} + \tilde{A} \wedge \tilde{A}$