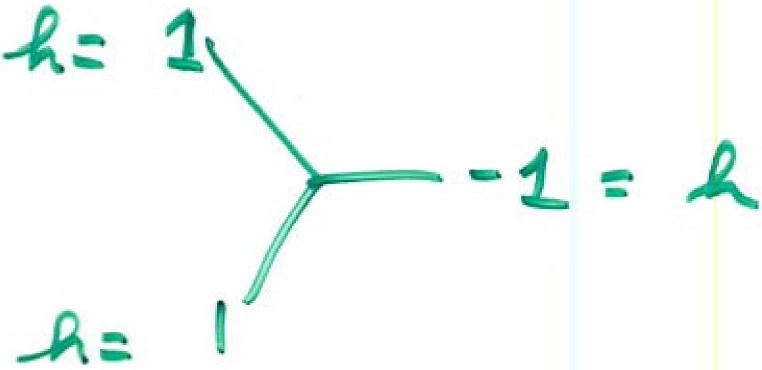
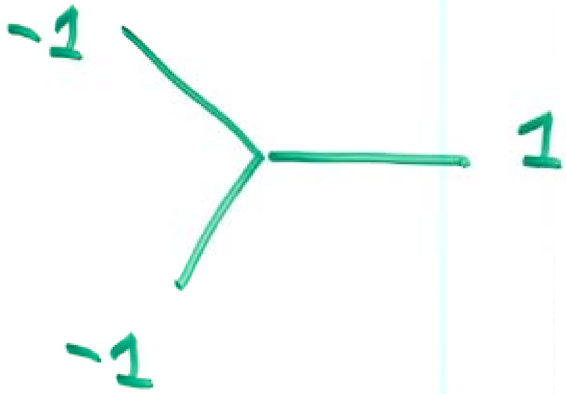
WE HAVE ONE INTERACTION



AND NOT THE OTHER



$$0 = \bar{\partial} a + a \wedge a$$

THE EQUATIONS OF MOTION IN TWISTOR SPACE ARE

1) $$0 = \bar{\partial} \tilde{A} + \tilde{A} \wedge \tilde{A}$$

2) $$0 = \bar{\partial}_{\tilde{A}} \tilde{G} = \bar{\partial} \tilde{G} + [\tilde{A}, \tilde{G}]$$

THE PENROSE-WARD TRANSFORM TELLS HOW TO INTERPRET THESE IN MINKOWSKI SPACETIME

1) MEANS $\tilde{A}$ DEFINES A HOLOMORPHIC BUNDLE OVER TWISTOR SPACE, SAY $\tilde{E}$

$\Rightarrow$ $F^+(A)=0$, THE CORRESPONDING FIELD A IN SPACETIME IS ANTI-SELF-DUAL

2) $\tilde{G} \in H^1(\text{TWISTOR SPACE}, \tilde{E})$

$\Rightarrow$ $G \in \Omega^{2,+}(\mathrm{ad}\ E)$

$$d_A G = 0.$$

THE ACTION THESE COME FROM IS

$$\int_{M^4} \mathrm{Tr}\, G \wedge F^+(A)$$

THIS ISN'T YANG-MILLS THEORY.

TO GET YANG-MILLS WE NEED ALSO

$$-\frac{1}{2}\int_{M^4} \mathrm{Tr}\, G^2$$

FROM THE SUM, INTEGRATE OUT G

TO GET

$$\frac{1}{2}\int_{M^4} \mathrm{Tr}\, F^+(A)^2$$

$$= \frac{1}{4}\int_{M^4} \mathrm{Tr}\, F^2 + \text{topological invariant}$$

(II) OTHER WAY TO SEE THE PROBLEM: SYMMETRIES OF $\mathbb{CP}^{3|4}$

I EMPHASIZED $PSU(4|4)$

$$Z^1 Z^2 Z^3 Z^4 \ \psi^1 \psi^2 \psi^3 \psi^4$$

BUT $\mathbb{CP}^{3|4}$ HAS THE BIGGER SYMMETRY $PU(4|4)$,

THE EXTRA GENERATOR BEING (SAY)

$$Z^i \to Z^i, \quad \psi^A \to t\,\psi^A$$

WHAT DISTINGUISHES THE PSU(4|4) SUBGROUP IS THAT IT IS A SYMMETRY OF

$$\Omega = dz^1 \cdots dz^4 \, d\psi^1 \cdots d\psi^4$$

AND HENCE OF THE B-MODEL

UNDER THE EXTRA SYMMETRY

$$\Omega \to t^{-4} \Omega$$

SO

$$I = \int \Omega \, \mathrm{Tr}\left(a \bar{\partial} a + \frac{2}{3} a a a\right)$$

DOES THE SAME

$$I \to t^{-4} I$$

THE YANG MILLS ACTION DOES **NOT** SIMPLY SCALE AS t^{-4}...

IT IS A SUM OF TERMS THAT SCALE AS t^{-4} AND t^{-8}.

$$a = \underset{t^{0}}{\tilde{A}} + \dots + \underset{t^{-4}}{\tilde{G}} \; \underset{t^{4}}{\psi^{4}}$$

SO

$$\int \mathrm{Tr}\, \underset{-4}{G} \; \underset{0}{F^{+}(A)}$$

$$-\frac{1}{2} \int \mathrm{Tr}\, \underset{-8}{G^{2}}$$

(AND LIKEWISE FOR THE REST OF THE SUPER YANG MILLS ACTION)

WHERE DOES THE REST COME FROM?

THE ANSWER IS

"D-INSTANTONS", WHICH ARE

HOLOMORPHIC CURVES IN $CP^{3|4}$

OVER WHICH ONE MUST INTEGRATE.

BY INTEGRATING OVER SUCH CURVES,

ONE CAN ... AT LEAST IN THE GENUS

ZERO CASE, WHICH CORRESPONDS TO

$\hbar \to 0$, ... CALCULATE SCATTERING

AMPLITUDES OF SUPER YANG-MILLS

IN PERTURBATION THEORY... A SPECIAL

CASE BEING THE EFFECTIVE ACTION

WITH THE $\int \mathrm{Tr}\, G^2$ TERM THAT

WE JUST DISCUSSED.

to finish...

I HAVE TO EXPLAIN BOTH A 33 QUESTION (WHAT PHYSICAL QUANTITY IN GAUGE THEORY TO CALCULATE) AND AN ANSWER (HOW TO CALCULATE IT). I'LL DESCRIBE THE ANSWER FIRST.

LET C BE A HOLOMORPHIC CURVE IN TWISTOR SPACE AND LET $\mathcal{M}$ BE THE MODULI SPACE OF CURVES CONTAINING C.

LET $\mathcal{a}$ BE A TWISTOR SPACE FIELD, i.e. A (0,1)-FORM ON $\mathbb{CP}^{3|4}$ WITH VALUES IN $GL(N,\mathbb{C})$

WE CAN RESTRICT IT TO A CURVE C AND TAKE THE $\bar{\partial}$ OPERATOR

$\bar{\partial}_{A|C}$ IN THE FUNDAMENTAL REPRESENTATION OF $GL(N,\mathbb{C})$

(some inspiration from Nair)

THEN ROUGHLY WE CALCULATE

$$\int_{\mathcal{M}} d\mu \quad \det \bar{\partial}_{A|C}$$

WHERE $d\mu$ SHOULD BE A NATURAL MEASURE ON $\mathcal{M}$ AND THE INTEGRAL IS ON A REAL SLICE IN $\mathcal{M}$.

THIS SHOULD GIVE "PERTURBATIVE YANG MILLS SCATTERING AMPLITUDES," WHERE THE DEGREE OF C DETERMINES WHICH PARTICLES ARE SCATTERED

AND THE GENUS OF C DETERMINES THE "ORDER" IN PERTURBATION THEORY, i.e. THE NUMBER OF LOOPS.

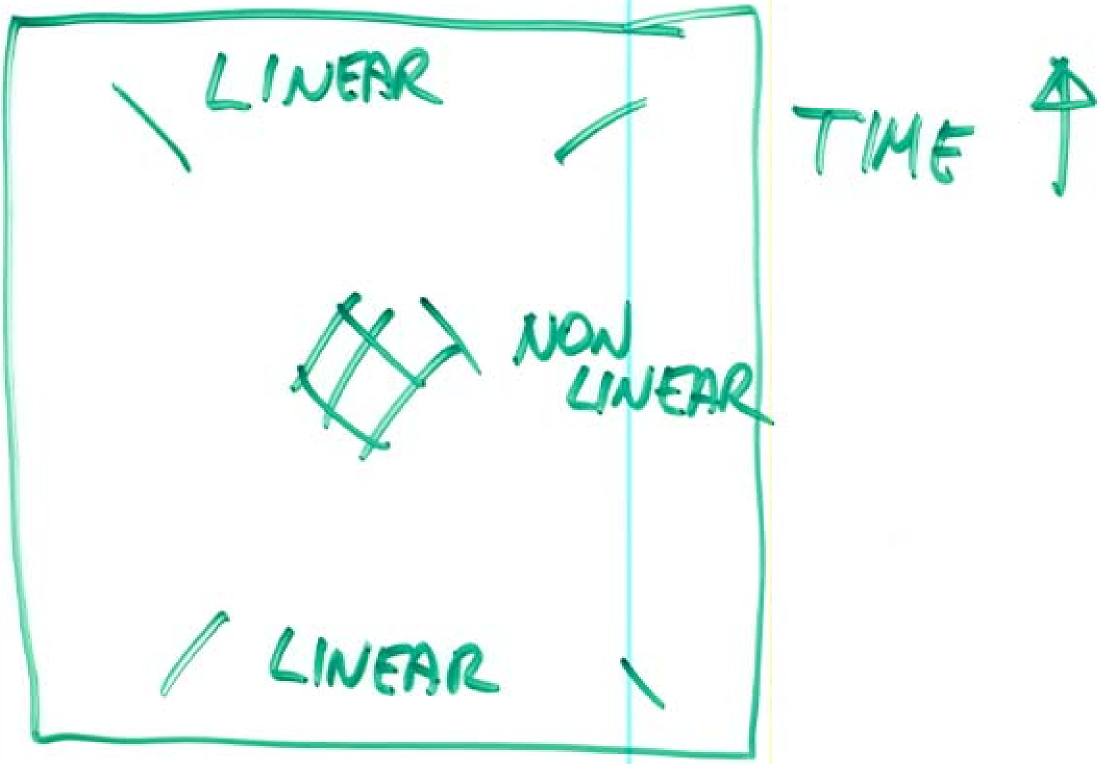
IN PRACTICE, THIS HAS BEEN CARRIED OUT SUCCESSFULLY, BUT WITH SOME SLEIGHT OF HAND, FOR GENUS ZERO ... GIVING FORMULAS THAT ARE ELEGANT AND NEW AND MAYBE EVEN USEFUL.

BUT WHAT ARE WE CALCULATING?

IN CLASSICAL YANG MILLS THEORY

WE SOLVE THE NONLINEAR WAVE

EQUATIONS $\quad 0 = d_A * F$



THE SOLUTIONS ARE FREE, OR LINEAR,
IN THE PAST AND THE FUTURE
BUT NONLINEAR IN BETWEEN

THE NONLINEAR EVOLUTION FROM PAST TO FUTURE GIVES A SYMPLECTOMORPHISM S_{cl} FROM LINEAR DATA IN THE PAST TO LINEAR DATA IN THE FUTURE

QUANTUM MECHANICALLY S_{cl} BECOMES A UNITARY OPERATOR MATRIX S WHICH IS CALLED THE "S-MATRIX"

QUANTUM MECHANICALLY, THE INITIAL AND FINAL STATES ARE DESCRIBED IN PERTURBATION THEORY NOT AS NONLINEAR WAVES

BUT AS A COLLECTION OF PARTICLES. MATRIX ELEMENTS OF S BETWEEN INITIAL AND FINAL COLLECTIONS OF PARTICLES ARE CALLED

"PERTURBATIVE SCATTERING AMPLITUDES"

THE PERTURBATIVE SCATTERING AMPLITUDES OF YANG-MILLS THEORY ARE IMPORTANT FOR REAL PHYSICS, IN PARTICULAR THEY ARE IMPORTANT FOR UNDERSTANDING HIGH ENERGY proton-proton AND proton-antiproton COLLISIONS.

THESE SCATTERING AMPLITUDES HAVE REMARKABLE PROPERTIES, DISCOVERED IN INCREASING GENERALITY BY de WITT (1968), PARKE & TAYLOR (mid 1980's) BERN, DIXON, KOSOWER (1990 → ...)

SOME OF THOSE PROPERTIES HAVE BEEN EXPLAINED, BETTER UNDERSTOOD, OR GENERALIZED BY THE INTERPRETATION VIA TWISTOR-STRING THEORY AND CURVES IN $\mathbb{CP}^{3|4}$.

THIS IS THE MAIN EVIDENCE THAT TWISTOR-STRING THEORY HAS SOME SUBSTANCE.