THE MOMENTUM p OF A MASSLESS PARTICLE IS LIGHTLIKE

$$p_\mu p^\mu = 0$$

SO p IS IN A CONE

OVER A QUADRIC ... A COMPLEX QUADRIC AS WE'LL TAKE OUR MOMENTA TO BE COMPLEX.

IN FOUR ~~DIMENSIONS~~ VARIABLES THE

PROJECTIVE QUADRIC IS

$$Q = P^1 \times P^1$$

AND THE MOMENTUM, WHEN WRITTEN IN TERMS OF SPINORS BY

$$p_{a\dot{a}} = \sigma^{\mu}_{a\dot{a}} \, p_{\mu}$$

FACTORS AS

$$p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$$

WHERE $\lambda_a, \tilde{\lambda}_{\dot{a}}$ $\quad a, \dot{a} = 1, 2$

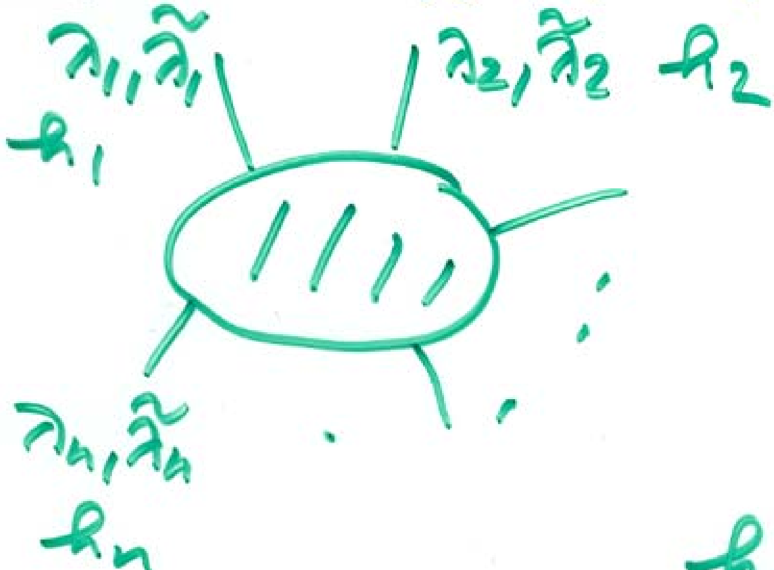
ARE HOMOGENEOUS COORDINATES ON THE TWO $\mathbb{P}^1$'s

DETERMINED UP TO

$$\lambda \to t\lambda, \quad \tilde{\lambda} \to t^{-1}\tilde{\lambda}, \quad t \in \mathbb{C}^*$$

INTRODUCING THE λ's IS USEFUL BECAUSE IT ENABLES US TO NEATLY INCORPORATE GLUON (OR NEUTRINO OR GRAVITON) <u>HELICITY</u> IN A SCATTERING PROCESS

A SCATTERING AMPLITUDE



$h_n = \pm 1$

IS A FUNCTION

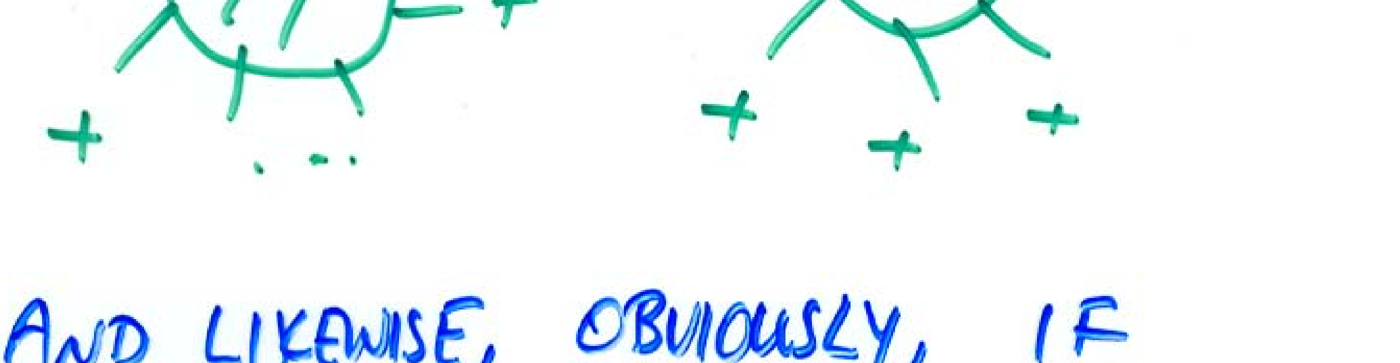
$$A(\lambda_1, \tilde{\lambda}_1; \dots; \lambda_n, \tilde{\lambda}_n)$$

THAT SCALES UNDER

$\lambda_i \to t_i \lambda_i$, $\tilde{\lambda}_i \to t_i^{-1} \tilde{\lambda}_i$ AS $t_i^{-2h_i}$

(THIS FOLLOWS FROM THE FACT THAT $\lambda, \tilde{\lambda}$ TRANSFORM WITH ANGULAR MOMENTUM $\mp\frac{1}{2}$ UNDER ROTATION AROUND $\vec{p}$ AXIS.)

NOW, TREE LEVEL YANG-MILLS SCATTERING AMPLITUDES FOR CERTAIN CASES



AND LIKEWISE, OBVIOUSLY, IF

$$+ \Longleftrightarrow -$$

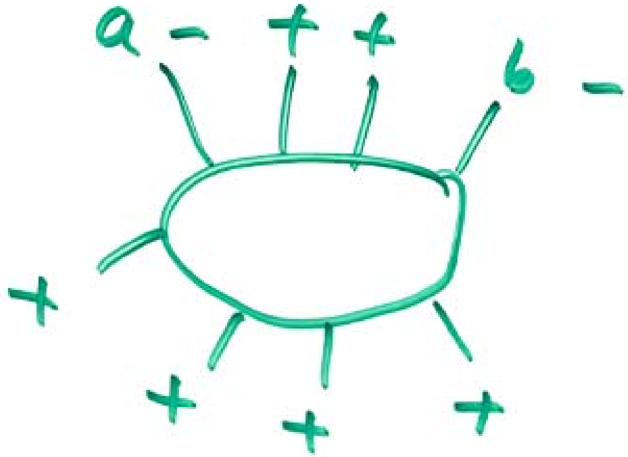
THIS FOLLOWS FROM THE FACT THAT THE SELFDUAL AND ANTISELFDUAL EQNS $F^+ = 0$ AND $F^- = 0$ ARE CONSISTENT TRUNCATIONS OF THE FULL YANG-MILLS EQUATIONS.

$$D_\mu F^{\mu\nu} = 0$$
$$d_A * F = 0$$

FROM THIS POINT OF VIEW THE FIRST NONVANISHING AMPLITUDE IS THE "MHV" OR MAXIMALLY HELICITY VIOLATING AMPLITUDE WITH HELICITIES SOME PERMUTATION OF

$$- \; - \; + + \; \ldots \; +$$

THE PARKE-TAYLOR FORMULA (1980's) IS 6



$$A = (2\pi)^4 \delta^4\left(\sum p_i\right) \cdot \frac{\langle \lambda_a, \lambda_b \rangle^4}{\prod_{i=1}^{n} \langle \lambda_i, \lambda_{i+1} \rangle}$$

WHERE $\lambda_i \in S \simeq \mathbb{C}^2$

AND $\langle\ ,\ \rangle$ IS THE SKEW FORM ON $\mathbb{C}^2$ DEFINED BY

$$\langle \lambda, \lambda' \rangle = \epsilon_{ab} \lambda^a (\lambda')^b$$

$$\epsilon_{12} = -\epsilon_{21} = 1$$

INDEPENDENT OF $\tilde{\lambda}$, i.e.

"HOLOMORPHIC IN λ" (AS $\tilde{\lambda} = \pm \bar{\lambda}$ IF p IS REAL IN LORENTZ SIGNATURE)

NAIR SHOWED THAT THE NUMERATOR $\langle \lambda_a, \lambda_b \rangle^4$ IS A SUPERSYMMETRIC EXTENSION OF THE δ FUNCTION $\delta^4(\Sigma p_i)$

AND INTERPRETED THE DENOMINATOR AS A "CURRENT CORRELATION FUNCTION" ON $\mathbb{CP}^1$.

I INTERPRETED THIS $\mathbb{CP}^1$ AS A CURVE IN $\mathbb{CP}^{3|4}$ AND INTERPRETED "HOLOMORPHY" (INDEPENDENCE OF $\tilde{\lambda}$) AS A CONSEQUENCE OF THE CURVE HAVING DEGREE [redacted] ONE.

(MOREOVER, FOR DEGREE ONE, A SHORT THOUGH NOT QUITE RIGOROUS COMPUTATION RECOVERS THE FORMULA, WITH NAIR'S INTERPRETATION OF THE DENOMINATOR.)

WE CAN SEE WHY THE CURVE SHOULD HAVE DEGREE ONE BY PURSUING THE REASONING OF THE FIRST LECTURE.

RECALL THAT $\mathbb{CP}^{3|4}$, WITH HOMOGENEOUS COORDINATES

$$Z^1 \dots Z^4 \quad \psi^1 \dots \psi^4$$

HAS A SYMMETRY $PSU(4|4)$ THAT LEAVES FIXED THE MEASURE

$$\Omega = dZ^1 \dots dZ^4 d\psi^1 \dots d\psi^4$$

AND AN ADDITIONAL $\mathbb{C}^*$ SYMMETRY

U: $\quad Z \to Z \quad \psi \to \lambda \psi$

THAT DOES NOT: $\Omega \to \lambda^{-4} \Omega$

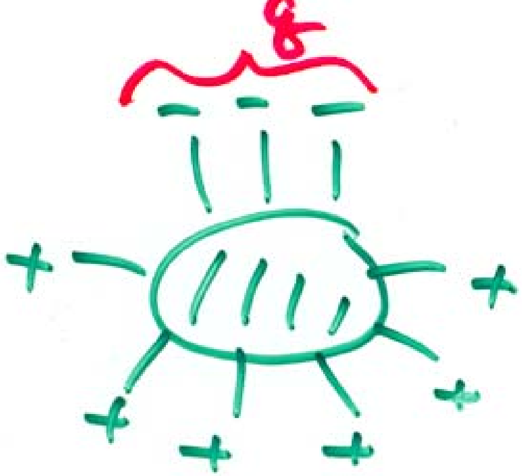
THE TWISTOR SPACE FIELD

$$a = \underset{h=1}{\hat{A}} + \quad \dots \quad + \underset{h=-1}{\hat{G}}\,\psi_1 \cdots \psi_4$$

TRANSFORMS AS

$$\hat{A} \to \hat{A} \qquad \hat{G} \to \lambda^{-4}\,\hat{G}$$

SO TO GET AN AMPLITUDE



WITH PRECISELY q NEGATIVE HELICITY GLUONS (NOT NECESSARILY CONSECUTIVE, WE NEED SOMETHING THAT SCALES AS

$$\lambda^{-4q}$$

THE STRATEGY IS WHAT I SAID: LET $C \subset \mathbb{CP}^{3|4}$ BE A CURVE OF GENUS g AND DEGREE d.

FOR THE MOMENT, TAKE C CONNECTED.

LET $\mathcal{M}$ BE THE MODULISPACE OF CURVES CONTAINING C.

IDEALLY WE FIND A HOLOMORPHIC MEASURE $d\mu$ ON $\mathcal{M}$ AND THE CONTRIBUTION OF $\mathcal{M}$ TO A SCATTERING AMPLITUDE IS OBTAINED BY EXPANDING

$$\int_{\mathcal{M}} d\mu \ \det \bar{\partial}_{A|C}$$

IN GENUS ZERO, THIS IS ACTUALLY CARRIED OUT SUCCESSFULLY THOUGH NOT RIGOROUSLY, AS SPRADLIN EXPLAINED. (GENUS ZERO $\leftrightarrow$ CLASSICAL LIMIT)

THE KEY FIRST STEP IS TO DEFINE THE MEASURE. IN FACT, ON THE SPACE OF GENUS ZERO CURVES IN $\mathbb{CP}^{3|4}$ THERE IS A NATURAL HOLOMORPHIC MEASURE.

TO DESCRIBE THE MODULI SPACE OF CURVES OF GENUS ZERO AND DEGREE d IN $\mathbb{CP}^{3|4}$, WE TAKE AN ABSTRACT $\mathbb{CP}^1$ WITH HOMOGENEOUS COORDINATES (u,v) AND LET

$$\left.\begin{aligned} Z^i &= P^i(u,v) \\ \psi^A &= \alpha^A(u,v) \end{aligned}\right\}$$

P^i, α^A ARE HOMOGENEOUS POLYNOMIALS OF DEGREE d

THE MEASURE IS CONSTRUCTED FROM

$$d\mu_0 = \prod_i dP_i \prod_A d\alpha^A$$

MORE EXPLICITLY IF WE

PICK A BASIS f_γ $\gamma = 1, \ldots, d+1$

OF THE SPACE OF DEGREE d ON ||

POLYNOMIALS IN u, v

$\mathbb{C}^{4(d+1)|4(d+1)} / GL(2)$

WE WRITE

$$p^i = \sum_\gamma p^i{}_\gamma f_\gamma$$

$$\alpha^A = \sum_\gamma \alpha^A{}_\gamma f_\gamma$$

AND

$$d\mu_0 = \prod_{\gamma, i} dp^i{}_\gamma \prod_{\gamma, A} d\alpha^A{}_\gamma$$

$d\mu_0$ IS INVARIANT TO GL(2) ACTING ON u, v SO DESCENDS TO A MEASURE $d\mu$ ON THE MODULI SPACE.

NOW WE CAN SEE HOW

$d\mu$ TRANSFORMS UNDER $\quad \Psi = d(u,v)$

$$U: \; Z \to Z \,, \quad \Psi \to \lambda \Psi$$

EACH OF THE $4d+4$ COEFFICIENTS

$\alpha^{A}{}_{\gamma}$ SCALES AS $\alpha \to \lambda \alpha$

SO $\quad d\alpha \to \lambda^{-1} d\alpha$ AND

$$d\mu \to \lambda^{-4(d+1)} d\mu$$

THE NUMBER OF NEGATIVE HELICITY GLUONS IN A SCATTERING AMPLITUDE TO WHICH A DEGREE d, GENUS ZERO CURVE CONTRIBUTES IS HENCE

λ^{-4q} $\qquad q = d+1$

AND ROIBAN, SPRADLIN, AND VOLOVICH EXTRACTED THE TREE AMPLITUDES OF ANY q FROM THE FORMULA

$$\int d\mu \, \det \bar{\partial}_{a|c}$$

I REGARD THIS AS THE BEST EVIDENCE THAT YANG-MILLS THEORY CAN BE DESCRIBED BY TWISTOR STRINGS ... WHILE THE PARALLEL ANALYSIS OF DISCONNECTED DIAGRAMS (SVRCEK'S LECTURE) IS THE BEST EVIDENCE THAT THIS DESCRIPTION IS USEFUL.

MORE ON THAT LATER.

I'LL DEVOTE THE REST OF THE LECTURE TO SURVEYING WHAT I CONSIDER <u>NOT</u> SATISFACTORY WHICH I'LL TRY TO SUMMARIZE (MOST OF) IN FOUR TOPICS.

0 IN GENERAL, THE DERIVATIONS LACK THE CRISPNESS AND PRECISION OF STANDARD STRING THEORY ... HOPEFULLY BECAUSE WE AREN'T DOING IT RIGHT RATHER THAN BECAUSE THE THEORY DOESN'T EXIST.

1 THE DEFINITION OF THE MEASURE DOESN'T WORK IN GENUS $g > 0$.

START WITH ABSTRACT CURVE C AND LINE BUNDLE $\mathcal{L} \to C$

DESCRIBE MAP $\Phi : C \to \mathbb{CP}^{3|4}$

BY

$$Z^{i} \in H^0(C, \mathcal{L})$$

$$\psi^A \in \Pi H^0(C, \mathcal{L})$$

(Π : bose $\leftrightarrow$ fermi)

THERE IS AS IN GENUS ZERO

A NATURAL MEASURE

$dZ\,d\Psi$ FOR FIXED $C, \mathcal{L}$

BUT THERE IS NO WAY

TO INTEGRATE OVER THE

CHOICE OF $C, \mathcal{L}$.

IN THE B-MODEL, I CANNOT SEE A CURE FOR THIS!

(IN THE BERKOVITS MODEL, ONE APPEARS TO GET A MEASURE FROM THE $C=28$ MATTER SYSTEM)

ONE THOUGHT THAT HAS CROSSED MY MIND IS THAT MAYBE THE TARGET SPACE OF THE B-MODEL SHOULD BE NOT ALL OF $\mathbb{CP}^{3|4}$

BUT ONLY THE "+" PART (OR THE "−" PART)

$$\mathbb{C}P^{+3/4} = \{ Z^i, \psi^A \mid |Z^1|^2 + |Z^2|^2 - |Z^3|^2 - |Z^4|^2 > 0 \}$$

(WITH A "FLOP"-LIKE TRANSITION FROM $\mathbb{C}P^{+3/4}$ TO $\mathbb{C}P^{-3/4}$)

ARE CURVES IN $\mathbb{C}P^{+3/4}$ A REASONABLE MATHEMATICAL ENTITY AND MIGHT THEY ADMIT A HOLOMORPHIC MEASURE?

I ALSO SOMETIMES WONDER

IF IT IS MISLEADING TO CONSIDER

"ABSTRACT CURVE C + MAP

TO $\mathbb{C}P^{3|4}$"

MAYBE THE SINGULARITIES OF

C AS A CURVE IN $\mathbb{C}P^{3|4}$ ARE

IMPORTANT.

(BUT I HAVE NO GOOD IDEA.)

2 CONFORMAL SUPERGRAVITY

IN THE B-MODEL, IT APPEARS RATHER LIKE YANG-MILLS FIELDS... VIA DEFORMATIONS OF COMPLEX STRUCTURE OF (REGIONS OF) $\mathbb{CP}^{3|4}$

IN BERKOVITS MODEL... VIA OPEN STRINGS

IT'S UNUSUAL TO SEE CONFORMAL SUPERGRAVITY IN STRING THEORY ... BECAUSE PHYSICAL STRING THEORY GENERATES SENSIBLE THEORIES!

MAYBE IT MEANS SOMETHING, BUT HOW?

CAN IT?

A THEORY THAT IS A SMALL g^2N STRING DUAL OF YANG-MILLS THEORY... AND THUS A STRONG CURVATURE COUSIN OF THE MALDACENA CORRESPONDENCE ... SHOULDN'T HAVE GRAVITY.

IN THE PRESENCE OF CONFORMAL SUGRA, ONLY CERTAIN GAUGE GROUPS ALLOWED ... HOW TO SEE THIS?

MY BASIC – SO FAR UNSUCCESSFUL – STRATEGY TO REMOVE CONFORMAL SUGRA IS TO ADD MORE DIMENSIONS, FOR INSTANCE REPLACE

$$\mathbb{CP}^{3|4}$$

WITH

$$\mathbb{CP}^{3|4} \times \mathbb{C}$$

BY ANALOGY WITH VARIOUS CONSTRUCTIONS IN PHYSICAL STRING THEORY.

AT ANY RATE, WHETHER OR NOT CONFORMAL SUPERGRAVITY IS IMPORTANT TO STUDY, THERE SHOULD BE A THEORY RELATED TO THE SMALL g^2N LIMIT OF AdS-CFT... IT WOULD BE NICE IF IT IS A TWISTOR-STRING THEORY OF SOME KIND.

(3) FINALLY, I SOMETIMES THINK THE BEST CLUE IS A POINT THAT CAME UP IN SPRADLIN'S LECTURE: IN GENUS ZERO THE INTEGRAL

$$\int_{\mathcal{M}} d\mu \quad \det \bar{\partial}_{A|C}$$

HAS MOTIVATED A FORMULA THAT WORKS... BUT THERE IS A SMALL SLEIGHT OF HAND

TO MOTIVATE THE RIGHT FORMULA, ONE INTERPRETS THE INTEGRAL AS A "CONTOUR INTEGRAL" ON THE REAL AXIS

THIS LEADS TO A MULTIDIMENSIONAL VERSION OF

f = A POLYNOMIAL

$$\int_{-\infty}^{\infty}\!\!\int dy\, dx\, \exp i x f(y) = 2\pi \int_{-\infty}^{\infty} dy\, \delta(f(y))$$

$$= 2\pi \sum_{f(y_i)=0} \frac{1}{|f'(y_i)|}$$

TO GET THE RIGHT FORMULA FOR TREE AMPLITUDES ONE NEEDS TO REMOVE THE ABSOLUTE VALUE AND USE

$$2\pi \sum_{f(y_i)=0} \frac{1}{f'(y_i)}$$

WHERE THE SUM GOES OVER ALL ROOTS, REAL OR NOT. EVEN WHEN THE ROOTS ARE ALL REAL, $f'(y_i)$ IS NOT POSITIVE DEFINITE

WHAT KIND OF INTEGRAL GENERATES A SUM OVER COMPLEX ROOTS OF A POLYNOMIAL

$$\sum_{f(y_i)=0} \frac{1}{f'(y_i)} \qquad ?$$

BEASLEY AND EW (IN RELATION TO HETEROTIC STRING WORLDSHEET INSTANTONS)

$$\int dy\, d\bar{y}\, d\psi\, d\chi\, \exp\left(-\bar{f}f - \frac{\partial \bar{f}}{\partial \bar{y}}\psi\chi\right)$$

$$= c\sum_{f(y_i)=0} \frac{1}{f'(y_i)}$$

TOPOLOGICAL SYMMETRY

$\delta\bar{y} = \chi \quad \delta\chi = 0$

$\delta\psi = f(y) \quad = \delta y$

THIS GENERALIZES IN $\mathbb{C}^N$

WITH COORDINATES $y_1 \cdots y_N = \vec{y}$

AND EQUATIONS $\underbrace{f_1(y) \cdots = f_N(y)}_{\vec{f}} = 0$

AND A POLYNOMIAL

$g(y)$ TO A "TOPOLOGICAL"

INTEGRAL THAT COMPUTES

$$\sum_{\vec{f}(\vec{y}_\alpha)=0} \frac{g(y_\alpha)}{\det \frac{\partial f_j}{\partial y_i}\Big|_{\vec{y}=\vec{y}_\alpha}}$$

WHICH IS OF THE RIGHT FORM

MOREOVER

$\mathbb{C}^N \to$ COMPLEX N-FOLD $\mathcal{M}$

$\vec{f} \to$ SECTION

$f \in H^0(M, V)$

V OF RANK N

$g \to$ SUITABLE POLYNOMIAL

IF $\mathcal{M}$ IS COMPACT AND ONE INCLUDES ALL ROOTS

$$\sum \frac{g(\vec{y}_\alpha)}{\det df|_{\vec{y}=\vec{y}_\alpha}} = 0$$

"RESIDUE THEOREM" - see GRIFFITHS & HARRIS

(OR BEASLEY & EW)

SO FOR GENUS ZERO

"SUM OVER FINITE ROOTS" = − "SUM OVER ROOTS AT INFINITY"

I HOPE THIS IS A GOOD WAY TO INTERPRET THE RELATION (GUKOV-NIETZKE-MOTL) BETWEEN CONNECTED AND DISCONNECTED GRAPHS

FINALLY A LAST THOUGHT

$\square \phi = 0$ IN MINKOWSKI SPACE

$\Updownarrow$ Penrose

$\tilde{\phi} \in H^1(\mathbb{CP}^3, \mathcal{O}(-2))$

GREEN'S FUNCTION OF $\phi \Rightarrow$ WHAT?

THIS WAS ANSWERED 25 YEARS AGO (SEE ATIYAH'S BOOK "Geometry of Yang-Mills Fields") BUT ANSWER SEEMED FRUSTRATINGLY ABSTRACT ... TO ME.

EASIER NOW FOR PHYSICISTS -

VIEW $\tilde{\phi}$ IN $\bar{\partial}$ COHOMOLOGY

$$\bar{\partial}\tilde{\phi} = 0 \, , \quad \tilde{\phi} \to \tilde{\phi} + \bar{\partial}\epsilon$$

$$\tilde{\phi} \in H^1(\mathbb{CP}^3, \mathcal{O}(-2))$$

LAGRANGIANS

$$\Box\phi = 0$$

MINKOWSKI $\int d^4x \, \phi \, \Box \, \phi$

$$\Updownarrow$$

$$\bar{\partial}\Omega = 0$$

$$\int \Omega \, \tilde{\chi} \, \bar{\partial} \, \tilde{\phi}$$

TWISTOR $\displaystyle\int_{\mathbb{CP}^3} \Omega \, \tilde{\phi} \, \bar{\partial} \, \tilde{\phi}$

$$\varepsilon_{ijklm} Z^{i} dZ^{k} dZ^{l} dZ^{m}$$

$\cancel{\Omega \in \Omega^{3}(\mathbb{CP}^3, \mathcal{O}(4))}$

Ω = (3,0)-FORM VALUED IN $\mathcal{O}(4)$

HAVING WRITTEN THE LAGRANGIAN

ANY PHYSICIST WOULD SAY

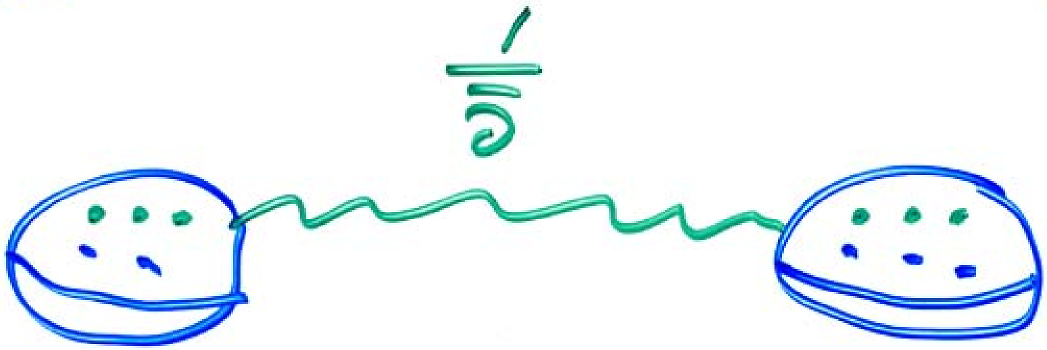
$$\frac{1}{D} \longleftrightarrow \frac{1}{\partial}\Big|_{+}$$

AND THIS GIVES US THE

PROPAGATOR USED (HEURISTICALLY,

I REGRET) IN DERIVING MHV

TREE DIAGRAMS (SURCEK'S TALK)



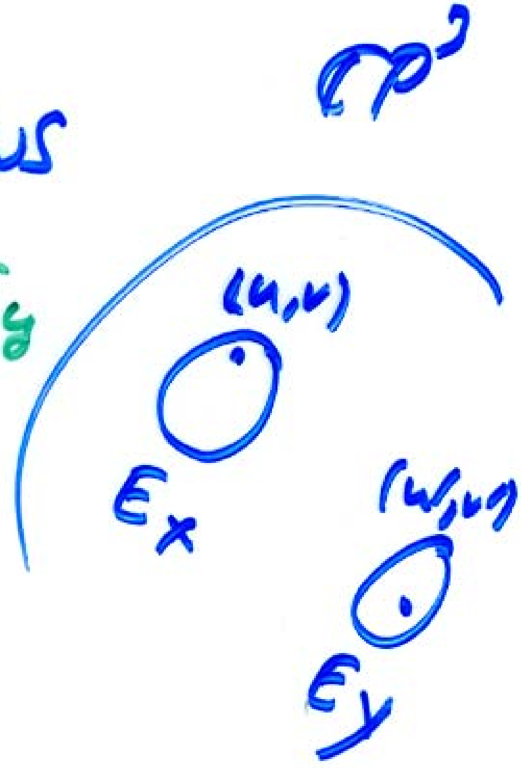
THE DOTS ARE POINTS WHERE GLUONS ARE INSERTED.

(REMOVE THE DOTS AND YOU GET WHAT ATIYAH CALLED HOLOMORPHIC LINKING - A COUSIN OF WHAT PENROSE CALLED HOLOMORPHIC LINKING)

LITERALLY SPEAKING THE PROPAGATOR $\frac{1}{\bar\partial}$ $\Omega^{0,1}(\mathcal{O}(-2))$

$$\phi(x,y) = \langle y | \frac{1}{\Box} | x \rangle$$

IS COMPUTED AS FOLLOWS



$x \Leftrightarrow E_x \quad y \Leftrightarrow E_y$

$E_x = \mathbb{CP}^1$ w/ (u,v)

$E_y = \mathbb{CP}^1$ w/ (u',v')

$$\phi(x,y) = \int_{E_x \times E_y} (u\,dv - v\,du)(u'dv' - v'du') \langle u,v | \frac{1}{\bar\partial} | u',v' \rangle$$

MAKES SENSE FOR $\bar\partial \in H^1(\mathcal{O}(-2))$