Neck pinches in Lagrangian mean curvature flow of surfaces

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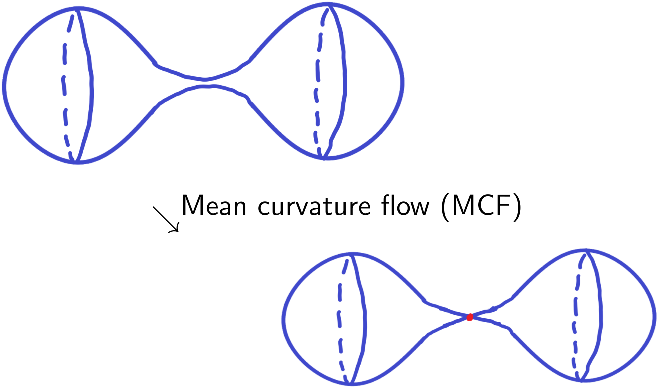
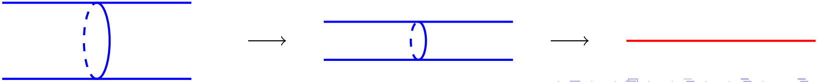
(Joint work with Felix Schulze and Gábor Székelyhidi)

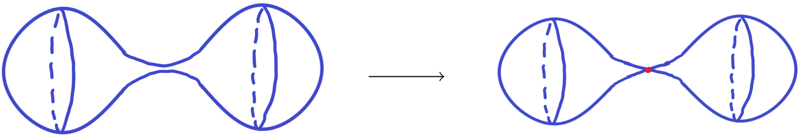
Neck pinch singularity



$\Sigma_t^2 \subseteq \mathbb{R}^3$ MCF $\rightsquigarrow$ neck pinch $\sim$ shrinking cylinder & generic



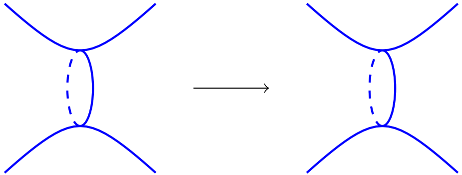
Lagrangian neck pinch singularity



$L_t^2 \subseteq \mathbb{C}^2$ Lagrangian mean curvature flow (LMCF) $\rightsquigarrow$

Conjecture (Joyce)

Neck pinch $\sim$ *Lawlor neck (special Lagrangian) & generic*



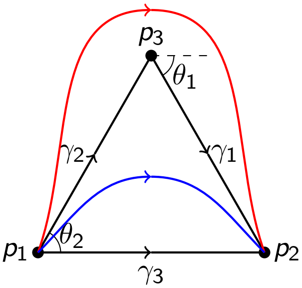
Note: Lawlor neck asymptotic to union of two transverse planes

Lagrangian angle and stability

$L^2 \subseteq \mathbb{C}^2$ Lagrangian $\rightsquigarrow$

- mean curvature $H = J\nabla\theta \rightsquigarrow \theta$ Lagrangian angle
- special Lagrangian $\Leftrightarrow \theta$ constant
- almost calibrated $\Leftrightarrow \sup\theta - \inf\theta < \pi$

Thomas–Yau stability: L almost calibrated & L cannot be decomposed into $L_1\#L_2$ with "smaller Lagrangian angles"



Main result

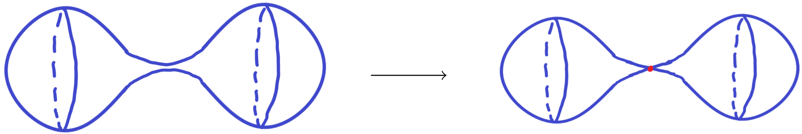
Conjecture (Thomas–Yau)

L stable $\Rightarrow$ LMCF starting at L exists for all time and converges

Theorem (L.–Schulze–Székelyhidi)

$L_t^2 \subseteq \mathbb{C}^2$ *exact almost calibrated LMCF with singularity at* (p, T),
one tangent flow = union of two transverse planes $\rightsquigarrow$

- *neck pinch modelled on Lawlor neck occurs;*
- $L_t \to L_T$ *C^1-immersed Lagrangian;*
- *can continue LMCF for* $t > T$;
- $L_T \setminus \{p\}$ *disconnected* $\Rightarrow$ L_0 *unstable*



Proof: key points

Uniqueness of tangent flow: same pair of transverse planes

- first uniqueness theorem for singular tangent flow
- moduli space: planes with same Lagrangian angle
- "non-integrable" deformations: planes with different angles

Finding Lawlor necks

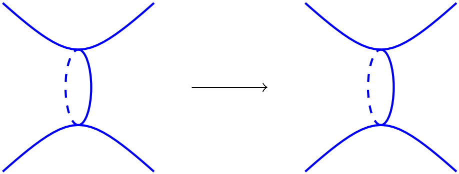
- moduli space: $\mathbb{R} \setminus \{0\} \rightsquigarrow$ choose correct scale
- Lawlor necks for $r > 0$ and $r < 0$ not Hamiltonian isotopic

Key tools

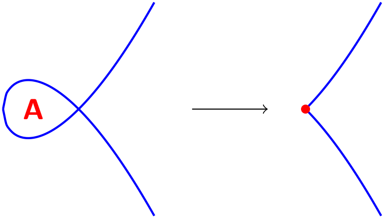
- excess: (weak) closeness to some pair of planes with same angle $\rightsquigarrow$ monotone
- distance: (strong) closeness to fixed pair of planes (uses hyperkähler rotation) $\rightsquigarrow$ "three annulus lemma"

Questions

- **mechanism for Lagrangian neck pinches?**



- **role of J-holomorphic curves?**



- **generic LMCF singularities?**