A multiscale model of complex endothelial cell dynamics in early angiogenesis

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Variable	Indexes	Description
$E_i = 1$	$i \in \mathcal{I}_{init}$	Initial distribution of cell nuclei.
$E_i = 0$	$i \in \mathcal{I} \setminus \mathcal{I}_{init}$	
$N_i = \operatorname{Unif}[(1-\xi)b_N, (1+\xi)b_N]$		
$D_i = \text{DUnif}[(1-\xi)b_D, (1+\xi)b_D]$	$i \in \mathcal{I}_{init}$	Cells are initialised with ligand/receptor numbers corresponding to their baseline gene expression with a correction for random fluctuations included via the parameter ξ . At the voxels where there is no cell nucleus, subcellular variables are initialised with the value zero.
$I_i = \text{DUnif}[(1 - \xi)I_0, (1 + \xi)I_0]$		
$R2_i = \text{DUnif}[(1 - \xi)b_{R2}, (1 + \xi)b_{R2}]$		
$R2_i^* = \text{DUnif}[(1-\xi)R2_0^*, (1+\xi)R2_0^*]$		
$N_i = D_i = I_i = R2_i = R2_i^* = 0$	$i \in \mathcal{I} \setminus \mathcal{I}_{init}$	
$l_i^{s_{init}} = 2\Delta_{init}$	$i \in \mathcal{I}_{init}$	The alignment of ECM fibrils for voxels where cells were initially placed in the direction $s_{init} \in S$.
$l_i^s = \text{Unif}[0, \Delta_{init}]$	$\begin{array}{l} i \in \mathcal{I} \setminus \mathcal{I}_{init}, \\ \forall s \in \mathcal{S} \end{array}$	The alignment of ECM fibrils for the rest of the voxels is initialised with a small random value in a given range, $[0, \Delta_{init}]$, imitating random orientation of fibrils prior to their realignment due to cell migration.
$l_i^s = \text{Unif}[0, \Delta_{init}]$	$i \in \mathcal{I}_{init}, \\ \forall s \neq s_{init} \in \mathcal{S}$	
$c_i = c_{init}$	$i \in \mathcal{I}_{init}$	The ECM concentration at the voxels with cells is equal to $c_{init} \in [0, c_{max}]$ (specified for each numerical experiment). For other voxels, the ECM is assumed to be unchanged, thus equal to the maximum ECM concentration
$c_i = c_{max}$	$i \in \mathcal{I} \setminus \mathcal{I}_{init}$	
		the maximum EOM concentration, c_{max} .
$m_i = m_{init}$	$i \in \mathcal{I}_{init}$	The concentration of BM components at the voxels with cells is equal to $m_{init} \in [0, 1]$ (specified for each numerical experiment). For other voxels, no BM components have been deposited, thus the concentration is set to zero.
$m_i = 0$	$i \in \mathcal{I} \setminus \mathcal{I}_{init}$	

S3 Table. Initial conditions for numerical simulations. Here \mathcal{I} is the set of all voxels; \mathcal{S} is the set of all possible migration directions. DUnif[a, b] is a discrete uniform distribution over all integer numbers lying within the interval [a, b]; Unif[a, b] is the uniform distribution on the interval [a, b]. Baseline gene expression parameters for the VEGF-Delta-Notch signalling are listed in S1 Table. $\Delta_{init} = 1.0$ for all numerical simulations (this value, as, in general, for the value of the OL variable, is non-dimensional). The fluctuation parameter, ξ , is set to 0.1 in all numerical simulations. The exact values for c_{init} and m_{init} are given for each numerical experiment in S4 Table, as well as the set of initial cell positions, \mathcal{I}_{init} . For the description of model variables see Table 1 in the main text.