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Ranjit Kumar Upadhyay

Professor, Department of Mathematics and Computing, Indian Institute of Technology (Indian School of Mines), Dhanbad, 826004, Jharkhand, India

Satteluri R. K. Iyengar

Professor of Mathematics (Retired), Indian Institute of Technology, New Delhi, India



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Foreword

Mathematical modelling is playing an increasingly important role in ecology, epidemiology, biology, and medicine. As I write this foreword, mathematical models are being used to inform governments across the world on the impact of different strategies for dealing with the COVID-19 pandemic. This book is therefore very timely as it provides an introduction to the deterministic (and some stochastic) modelling of spatiotemporal phenomena in ecology, epidemiology, and neural systems, surveying classical models in the field with up-to-date applications.

It begins with a detailed description of spatial dynamics, showing how movement phenomena from many different areas can be modelled within the same general mathematical framework. The methods of analysis of the resultant partial differential equations are presented. This is built upon by adding kinetics, leading to systems of reaction– diffusion equations. The classical models in this area are motivated and their analyses presented. The next three chapters discuss modelling virus dynamics in space and time, with applications to the recent Ebola and Zika virus pandemics. The final chapter presents an introduction to the biology of the brain, together with the classical models for neural dynamics.

This book is written in a very accessible way, carefully and clearly explaining all steps. It describes in detail the scientific problem, development of the appropriate mathematical models, subsequent analysis (including techniques such as linear stability analysis, weakly nonlinear analysis, control theory, and numerical simulation) and resultant insights gained into the scientific problem. It is an ideal introduction to modelling spatiotemporal dynamics for anyone wishing to enter the field of mathematical biology.

> Philip K. Maini, FRS, FMedSci, FNA Oxford, UK June 2020