2016 Q1B
An irregular hexagon with all sides of equal length is placed inside a square of side length 1, as shown below (not to scale). What is the length of one of the hexagon’s sides?

(a) $\sqrt{2} - 1$, (b) $2 - \sqrt{2}$, (c) 1, (d) $\frac{\sqrt{2}}{2}$, (e) $2 + \sqrt{2}$.

2015 Q1J
Which is the largest of the following numbers?

(a) $\frac{\sqrt{7}}{2}$, (b) $\frac{5}{4}$, (c) $\frac{\sqrt{10!}}{3(6!)}$, (d) $\frac{\log_2(30)}{\log_3(85)}$, (e) $\frac{1 + \sqrt{6}}{3}$.

2018 Q1G
The parabolas with equations $y = x^2 + c$ and $y^2 = x$ touch (that is, meet tangentially) at a single point. It follows that $c$ equals

(a) $\frac{1}{2\sqrt{3}}$, (b) $\frac{3}{4\sqrt{4}}$, (c) $-\frac{1}{2}$, (d) $\sqrt{5} - \sqrt{3}$, (e) $\sqrt{\frac{2}{3}}$.

For solutions see www.maths.ox.ac.uk/r/mat
2017 Q2
There is a unique real number $\alpha$ that satisfies the equation

$$\alpha^3 + \alpha^2 = 1.$$  

[You are not asked to prove this.]

(i) Show that $0 < \alpha < 1$.

(ii) Show that

$$\alpha^4 = -1 + \alpha + \alpha^2.$$  

(iii) Four functions of $\alpha$ are given in (a) to (d) below. In a similar manner to part (ii), each is equal to a quadratic expression

$$A + B\alpha + C\alpha^2$$

in $\alpha$, where $A, B, C$ are integers. (So in (ii) we found $A = -1, B = 1, C = 1$.) You may assume in each case that the quadratic expression is unique.

In each case below find the quadratic expression in $\alpha$.

(a) $\alpha^{-1}$.

(b) The infinite sum

$$1 - \alpha + \alpha^2 - \alpha^3 + \alpha^4 - \alpha^5 + \cdots.$$  

(c) $(1 - \alpha)^{-1}$.

(d) The infinite product

$$(1 + \alpha)(1 + \alpha^2)(1 + \alpha^4)(1 + \alpha^8)(1 + \alpha^{16}) \cdots.$$  

For solutions see \url{www.maths.ox.ac.uk/r/mat}