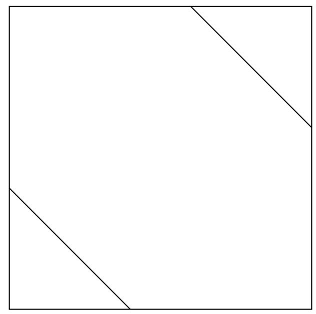
2016 Q1B

An irregular hexagon with all sides of equal length is placed inside a square of side length 1, as shown below (not to scale). What is the length of one of the hexagon's sides?



(a) $\sqrt{2}-1$, (b) $2-\sqrt{2}$, (c) 1, (d) $\frac{\sqrt{2}}{2}$, (e) $2+\sqrt{2}$.

2015 Q1J

Which is the largest of the following numbers?

(a) $\frac{\sqrt{7}}{2}$, (b) $\frac{5}{4}$, (c) $\frac{\sqrt{10!}}{3(6!)}$, (d) $\frac{\log_2(30)}{\log_3(85)}$, (e) $\frac{1+\sqrt{6}}{3}$.

2018 Q1G

The parabolas with equations $y = x^2 + c$ and $y^2 = x$ touch (that is, meet tangentially) at a single point. It follows that c equals

(a) $\frac{1}{2\sqrt{3}}$, (b) $\frac{3}{4\sqrt[3]{4}}$, (c) $\frac{-1}{2}$, (d) $\sqrt{5}-\sqrt{3}$, (e) $\sqrt{\frac{2}{3}}$.

2017 Q2

There is a unique real number α that satisfies the equation

$$\alpha^3 + \alpha^2 = 1.$$

[You are not asked to prove this.]

(i) Show that $0 < \alpha < 1$.

(ii) Show that

$$\alpha^4 = -1 + \alpha + \alpha^2.$$

(iii) Four functions of α are given in (a) to (d) below. In a similar manner to part (ii), each is equal to a quadratic expression

$$A + B\alpha + C\alpha^2$$

in α, where A, B, C are integers. (So in (ii) we found $A = -1$, $B = 1$, $C = 1$.) You may assume in each case that the quadratic expression is unique.

In each case below find the quadratic expression in α.

(a) α^{-1}.

(b) The infinite sum

$$1 - \alpha + \alpha^2 - \alpha^3 + \alpha^4 - \alpha^5 + \cdots .$$

(c) $(1 - \alpha)^{-1}$.

(d) The infinite product

$$(1 + \alpha)(1 + \alpha^2)(1 + \alpha^4)(1 + \alpha^8)(1 + \alpha^{16}) \cdots .$$