Computational Algebraic Topology Topic B: Exercise Sheet 1

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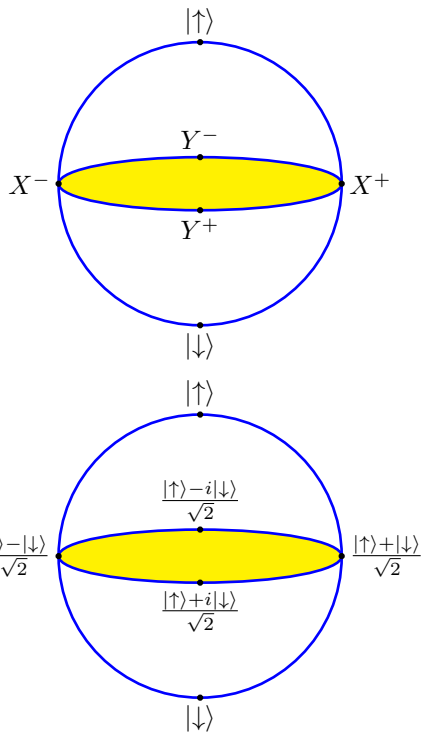
Question 1

Consider the 3-qubit GHZ state:

$$\frac{|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle}{\sqrt{2}}.$$

We can think of this state as shared by 3 parties, each of whom has two one-qubit observables, corresponding to the X and Y directions in the Bloch Sphere.



This specifies an empirical model, which can be displayed as an 8×8 table, with 8 rows corresponding to the choices of X or Y by each party, *i.e.* the measurement contexts, and 8 columns corresponding to the possible outcomes of measuring the variables in each context, 0 or 1.

(a) Use the Born rule to compute the following 4 rows of this table:

$$XXX, \quad XYY, \quad YXY, \quad YYX.$$

Characterize the supports of these rows in terms of the *parities* of the outcomes, *i.e.* the result of xor-ing the 0/1 outcomes.

(b) Now consider the generalization to n parties.

Question 2

(a) Let $h : R \longrightarrow S$ be a homomorphism of commutative semirings. Show that this induces a natural transformation $\mathcal{D}_R \Longrightarrow \mathcal{D}_S$, and hence a mapping of empirical models over the semiring R to empirical models over the semiring S. Show that this mapping preserves compatibility, *i.e.* no-signalling.

(b) Show that there is a unique semiring homomorphism from $\mathbb{R}_{\geq 0}$, the semiring of non-negative reals, to $\mathbb{B}$, the semiring of booleans. Show that there is *no* semiring homomorphism from the reals $\mathbb{R}$ to $\mathbb{B}$.

(c) Use the previous parts to show that every no-signalling probabilistic model can be mapped to a no-signalling possibilistic model.

(d) Is this mapping surjective? That is, does every possibilistic no-signalling model arise as the collapse of a probabilistic no-signalling model?