Robustness issues on regulatory risk measures

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Based on some joint work with

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Haiyan Liu (Michigan State)
Alex Schied (Waterloo)
Bin Wang (CAS Beijing)

- Embrechts-Liu-W., Quantile-based risk sharing. Operations Research, 2018
- Embrechts-Schied-W., Robustness in the optimization of risk measures. Working paper, 2018
Agenda

1. Background
2. Classic statistical robustness
3. Some other perspectives of robustness
4. Robustness in optimization
5. Conclusion
A risk measure $\rho : \mathcal{X} \to \mathbb{R} = (-\infty, \infty]$

- Risks are modelled by random losses in a specified period
  - e.g. 10d in Basel III & IV market risk
- $\mathcal{X}$ is a convex cone of rvs in some probability space $(\Omega, \mathcal{F}, \mathbb{P})$

Roles of risk measures

- regulatory capital calculation ← our main interpretation
- management, optimization and decision making
- performance analysis and capital allocation
- pricing
General Question

Question

What is a “good” risk measure for regulatory capital calculation?

- Regulator’s and firm manager’s perspectives can be different or even conflicting
  - well-being of the society versus interest of the shareholders
  - systemic risk in an economy versus risk of a single firm
Value-at-Risk (VaR) at level $p \in (0, 1)$

$$\text{VaR}_p : L^0 \to \mathbb{R},$$

$$\text{VaR}_p(X) = F_X^{-1}(p) = \inf\{x \in \mathbb{R} : \mathbb{P}(X \leq x) \geq p\}.$$  

Expected Shortfall (ES/TVaR/CVaR/AVaR) at level $p \in (0, 1)$

$$\text{ES}_p : L^0 \to \overline{\mathbb{R}},$$

$$\text{ES}_p(X) = \frac{1}{1 - p} \int_p^1 \text{VaR}_q(X) dq = (F_X \text{ cont.}) \mathbb{E}[X|X > \text{VaR}_p(X)].$$

$F_X$ above is the distribution function of $X$. 

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Value-at-Risk and Expected Shortfall

density function or data histogram of $X$

$\text{VaR}_{0.99}(X)$

$\text{ES}_{0.99}(X)$
Value-at-Risk and Expected Shortfall

The ongoing **co-existence** of VaR and ES:

- Basel IV - **both**
- Solvency II - **VaR**
- Swiss Solvency Test - **ES**
ES is generally advocated by academia for desirable properties in the past two decades; in particular,

- subadditivity or coherence (Artzner-Delbaen-Eber-Heath’99)
- convex optimization properties (Rockafellar-Uryasev’00)

Some other examples of impact from academic research

- Gneiting’11: backtesting ES is unclear, whereas backtesting VaR is straightforward
- Cont-Deguest-Scandolo’10: ES is not robust, whereas VaR is
BCBS Consultative Document, May 2012, Page 41, Question 8:

“What are the likely constraints with moving from VaR to ES, including any challenges in delivering robust backtesting, and how might these be best overcome?”
## VaR versus ES

<table>
<thead>
<tr>
<th>Features/Risk measure</th>
<th>VaR</th>
<th>Tail-VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency captured?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Severity captured?</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Sub-additive?</td>
<td>Not always</td>
<td>Always</td>
</tr>
<tr>
<td>Diversification captured?</td>
<td>Issues</td>
<td>Yes</td>
</tr>
<tr>
<td>Back-testing?</td>
<td>Straight-forward</td>
<td>Issues</td>
</tr>
<tr>
<td>Estimation?</td>
<td>Feasible</td>
<td>Issues with data limitation</td>
</tr>
<tr>
<td>Model uncertainty?</td>
<td>Sensitive to aggregation</td>
<td>Sensitive to tail modelling</td>
</tr>
<tr>
<td>Robustness I (with respect to “Lévy metric”)</td>
<td>Almost, only minor issues</td>
<td>No</td>
</tr>
<tr>
<td>Robustness II (with respect to “Wasserstein metric”)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table copied from IAIS Consultation Document Dec 2014, page 42
Progress

1. Background

2. Classic statistical robustness

3. Some other perspectives of robustness

4. Robustness in optimization

5. Conclusion
VaR and ES are law-based (thus statistical risk functionals):
\[ \rho(X) = \rho(Y) \text{ if } X \overset{d}{=} P Y \text{ (equal in distribution under } P) \]

- The calculation requires knowledge of the distribution of a risk
- This may never be the exact case: model uncertainty
  - statistical error
  - computational error
  - modeling error
  - conceptual error
- Models are at most “approximately correct” \( \Rightarrow \) robustness!
Robust Statistics

Statistical robustness addresses the question of “what if the data is compromised with small error?” (e.g. outlier)

- Originally robustness is defined on estimators (estimation procedures)
- Would the estimation be ruined if the underlying model is compromised?
  - e.g. an outlier is added to the sample
VaR and ES Robustness

density function or data histogram of X

VaR_{0.99}(X)
ES_{0.99}(X)

VaR_{0.99}(X^*)
ES_{0.99}(X^*)

single point huge value
Non-robustness of $\text{VaR}_p$ only happens if the quantile has a gap at $p$

Is this situation relevant for risk management practice?

- one must be very unlucky to hit precisely where it has a gap ...
Robust Statistics

Classic qualitative robustness:

- **Hampel’71**: the robustness of a consistent estimator of $T$ is equivalent to the continuity of $T$ with respect to underlying distributions (both with respect to the same metric).

- When we talk about the robustness of a statistical functional, (Huber-Hampel’s) robustness typically refers to continuity with respect to some metric.

- (Pseudo-)metrics: $\pi^q = L^q$ ($q \geq 1$), $\pi^\infty = L^\infty$, $\pi^W = $ Lévy, ...

General reference: Huber-Ronchetti’07
Consider the continuity of $\rho : \mathcal{X} \rightarrow \mathbb{R}$.

- A strong sense of continuity is w.r.t. weak convergence.
  - $X_n \rightarrow X$ in distribution $\Rightarrow \rho(X_n) \rightarrow \rho(X)$.

- Quite restrictive

- Practitioners like weak convergence (e.g. estimation, simulation)
With respect to weak convergence $p \in (0, 1)$:

- $\text{VaR}_p$ is continuous at distributions whose quantile is continuous at $p$. $\text{VaR}_p$ is argued as being almost robust.
- $\text{ES}_p$ is not continuous for any $\mathcal{X} \supset L^\infty$

$\text{ES}_p$ is continuous w.r.t. some other (stronger) metric, e.g. $\pi^q$ (or the Wasserstein-$L^q$ metric)
Range-Value-at-Risk (RVaR)

A two-parameter family of risk measures, for $\alpha, \beta > 0$, $\alpha + \beta < 1$,

$$\text{RVaR}_{\alpha, \beta}(X) = \frac{1}{\beta} \int_{\alpha}^{\alpha+\beta} \text{VaR}_\gamma(X) d\gamma, \quad X \in \mathcal{X}.$$  

- RVaR bridges the gap between VaR and ES (limiting cases).
- RVaR is continuous w.r.t. weak convergence
- RVaR is not convex or coherent; it is finite on $L^0$
- Practically:

$$\text{RVaR}_{\alpha, \beta}(X) = \mathbb{E}[X | \text{VaR}_\alpha(X) < X \leq \text{VaR}_{\alpha+\beta}(X)].$$

First proposed by Cont-Deguest-Scandolo’10; name in W.-Bignozzi-Tsanakas’15.

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Classic Robustness

The general perception of robustness, from worst to best:

\[ \text{ES} \prec \text{VaR} \prec \text{RVaR} \]
A distortion risk measure is defined as, for $X \in \mathcal{X}$,

$$\rho(X) = \int_0^\infty h(\mathbb{P}(X > x))dx + \int_{-\infty}^0 (h(\mathbb{P}(X > x)) - 1)dx,$$

where $h$ is an increasing function on $[0, 1]$ with $h(0) = 0$ and $h(1) = 1$. $h$ is called a distortion function. If $h$ is continuous,

$$\rho(X) = \int_0^1 \text{VaR}_p(X)dg(p), \ X \in \mathcal{X},$$

where $g(t) = 1 - h(1 - t), t \in [0, 1]$.

- ES and VaR are special cases of distortion risk measures

Yaari’87’s dual utility
Distortion Risk Measures

Some summary.

▶ A distortion risk measure is continuous (wrt $\pi^W$) on $L^\infty \iff$ its distortion function has a derivative which vanishes at neighbourhoods of 0 and 1 (classic property of $L$-statistics).

▶ From weak to strong:
  
  • Continuity w.r.t. $\pi^\infty$: all monetary risk measures
  • Continuity w.r.t. $\pi^q$, $q \geq 1$: finite convex risk measures on $L^q$, e.g. ES$_p$
  • Continuity w.r.t. weak/a.s./P convergence: e.g. RVaR$_{\alpha,\beta}$, VaR$_p$ (almost); no convex risk measure satisfies this

Some results: Bäuerle-Müller’06, Cont-Deguest-Scandolo’10, Kou-Peng-Heyde’13;
general references: Rüschenhof’13, Föllmer-Schied’16

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Robustness of Risk Measures

Is robustness w.r.t. weak convergence necessarily a good thing?

- **Toy example.**
  - Let $X_n = n^2 I\{U \leq 1/n\}$ for some $U[0,1]$ random variable $U$ (e.g. a credit default risk). Clearly $X_n \to 0$ a.s. but $X_n$ is getting more “dangerous” in many senses. If $\rho$ preserves weak convergence, then
    $$\rho(X_n) \to \rho(0) \ (= 0 \ \text{typically}).$$
  - $\text{VaR}_{0.999}(X_{10000}) = 0$
  - $\text{ES}_{0.999}(X_{10000}) = 10^7$

- May be reasonable for internal management; not so much for regulation.
One-in-ten-thousand Event

On the other hand,

- the 1/10,000-event-type risks are very difficult to capture statistically (accuracy is impossible)

UK House of Lords/House of Commons, June 12, 2013, Output of a “stress test” exercise, from HBOS:

“We actually got an external advisor [to assess how frequently a particular event might happen] and they came out with one in 100,000 years and we said “no”, and I think we submitted one in 10,000 years. But that was a year and a half before it happened. It doesn’t mean to say it was wrong: it was just unfortunate that the 10,000th year was so near.”
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Uncertainty in Risk Aggregation

General setup

- **To calculate** $\rho(S)$ where $S = \Lambda(X_1, \ldots, X_n)$ for risk factors $X_1, \ldots, X_n \in \mathcal{X}$ and aggregation function $\Lambda : \mathbb{R}^n \to \mathbb{R}$

- **Two levels of model uncertainty:**
  - the marginal distributions $F_i$ of $X_i$, $i = 1, \ldots, n$
  - the dependence structure (copula) of $(X_1, \ldots, X_n)$

- **Both VaR$_p(S)$ and ES$_p(S)$ depend on both levels**
  - The second level of uncertainty is arguably more challenging due to data, computation and modeling limitations
  - In the Basel IV market risk formulas, the value of ES$_p(S)$ requires a calculation under the worst-case dependence

Some references on risk aggregation under dependence uncertainty:
Embrecchts-Puccetti-Rüschendorf’13, Bernard-Jiang-W.’14, Cai-Liu-W.’18
Uncertainty in Risk Aggregation

- **Uncertainty at the second level** (with first level fixed):
  - **Robustness**: is $\rho \circ \Lambda$ continuous with respect to the modeling in dependence ($\pi^W$)? ⇒ robustness in risk aggregation
  - **Uncertainty spread**: how large is the spread of $\rho \circ \Lambda$ if we do not know about the dependence?

- We focus on the natural aggregation function
  $$\Lambda(x_1 + \cdots + x_n) = \sum_{i=1}^n x_i.$$  
  - $\mathcal{X} = L^1, L^\infty, \ldots$
Some results. In the problem of risk aggregation,

- A distortion risk measure is robust on $L^\infty \iff$ its distortion function is *continuous* on $[0, 1]$.
- $ES_p$ is *robust* on $L^1$;
- $VaR_p$ is *not robust* on $L^\infty$ (but almost)
- $RVaR_{\alpha, \beta}$ is *robust* on $L^0$
- The uncertainty spread of $VaR_p$ is generally *bigger* than that of $ES_q$ for $q \leq p$
  - In Basel III & IV market risk calculation, $VaR_{0.99}$ is replaced by $ES_{0.975}$

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Embretchs-Wang-W.'15
Robustness in Risk Aggregation

On robustness in risk aggregation:

\[ \text{VaR} \prec \text{ES} \prec \text{RVaR} \]

Remark.

- The robustness of \( \text{ES}_p \) is due to uniform integrability in risk aggregation.
Robustness in Risk Sharing

Simplistic setup

- **n agents** sharing a total risk (or asset) $X \in \mathcal{X}$ (set of rvs)
- $\rho_1, \ldots, \rho_n$: underlying risk measures (objectives to minimize)
  - The risk measures are chosen as VaR, ES and RVaR.
- **Optimality**: aggregate risk $\iff$ collaborative $\iff$ competitive
- **Robustness**: small model misspecification does not lead to very different aggregate risk value
Robustness in Risk Sharing

Some results.

- There exists a $\pi^1$-robust optimal allocation of $X \iff$ no VaR is involved
- If $X$ is bounded, then there exists a $\pi^\infty$-robust optimal allocation of $X \iff$ no VaR is involved
- There exists a $\pi^W$-robust optimal allocation of $X \iff$ no VaR is involved and at least one RVaR.

On robustness in risk sharing:

$\text{VaR} \prec \prec \text{ES} \prec \text{RVaR}$

Results in Embrechts-Liu-W.’18
Progress

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The Optimization Problem

General setup

- \( \mathcal{G}_n = \{ \text{measurable functions from } \mathbb{R}^n \text{ to } \mathbb{R} \} \)
- \( X \in (L^0)^n \) is an economic vector, representing all random sources
- \( \mathcal{G} \subset \mathcal{G}_n \) is an admissible set (decision set)
- \( g(X) \) for \( g \in \mathcal{G} \) represents a risky position of an investor
- \( \rho \) is an objective functional mapping \( \{g(X) : g \in \mathcal{G}\} \) to \( \overline{\mathbb{R}} \)

"The optimization problem":

- to minimize \( \rho(g(X)) \) over \( g \in \mathcal{G} \)

(e.g. think about a classic hedging/optimal investment problem)
The Optimization Problem

Denote

\[ \rho(X; G) = \inf \{ \rho(g(X)) : g \in G \}, \]

and (possibly empty)

\[ G^*(X, \rho) = \{ g \in G : \rho(g(X)) = \rho(X; G) \}, \]

We call

- \( g^* \in G^*(X, \rho) \) an optimizing function
- \( g^*(X) \) an optimized position
Uncertainty in Optimization

- The optimization problem is often subject to severe model uncertainty resulting from the assumptions made for $X$.
- Let $\mathcal{Z}$ be a set of possible economic vectors including $X$; $\mathcal{Z}$ may be interpreted as the set of alternative models.
  - E.g. a parametric family of models (parameter uncertainty)
- The real economic vector $Z \in \mathcal{Z}$ is likely different from the perceived economic vector $X$.
  - $X$: best-of-knowledge model
  - $Z$: real model ( unknowable)
Uncertainty in Optimization

- We choose $g \in \mathcal{G}^*(X, \rho)$ to optimize our objective $\rho$ (best-of-knowledge decision).
  - real position $g(Z)$
  - perceived position $g(X)$
- If the modeling is good, $Z$ and $X$ are close to each other according to some metric $\pi$
- $\rho(g(Z))$ should be close to $\rho(g(X))$ to make sense of the optimizing function $g$
- We desire some continuity of the mapping $Z \mapsto \rho(g(Z))$ at $Z = X$
We call \((G, \mathcal{Z}, \pi_\mathcal{Z})\) an uncertainty triplet if \(G \subset \mathcal{G}_n\) and \((\mathcal{Z}, \pi_\mathcal{Z})\) is a pseudo-metric space of \(n\)-random vectors.

\(\rho\) is compatible if it maps \(G(\mathcal{Z})\) to \(\overline{\mathbb{R}}\), and \(\rho(g(Y)) = \rho(g(Z))\) for all \(g \in G\) and \(Y, Z \in \mathcal{Z}\) with \(\pi_\mathcal{Z}(Y, Z) = 0\).

**Definition 1**

A compatible objective functional \(\rho\) is robust at \(X \in \mathcal{Z}\) relative to the uncertainty triplet \((G, \mathcal{Z}, \pi_\mathcal{Z})\) if there exists \(g \in G^*(X, \rho)\) such that the function \(Z \mapsto \rho(g(Z))\) is \(\pi_\mathcal{Z}\)-continuous at \(Z = X\).
Remarks.

- Robustness is a joint property of the tuple \((\rho, X, \mathcal{G}, \mathcal{Z}, \pi_{\mathcal{Z}})\).

- Only a \(\pi_{\mathcal{Z}}\)-neighbourhood of \(X\) in \(\mathcal{Z}\) matters.

- If \(\rho\) is robust at \(X\) relative to \((\mathcal{G}, \mathcal{Z}, \pi_{\mathcal{Z}})\), then \(\rho\) is also
  - robust at \(X\) relative to \((\mathcal{G}, \mathcal{Y}, \pi_{\mathcal{Z}})\) if \(X \in \mathcal{Y} \subset \mathcal{Z}\);
  - robust at \(X\) relative to \((\mathcal{G}, \mathcal{Z}, \hat{\pi}_{\mathcal{Z}})\) if \(\hat{\pi}_{\mathcal{Z}}\) is stronger than \(\pi_{\mathcal{Z}}\).

- If \(\mathcal{G}^*(X, \rho) = \emptyset\), then \(\rho\) is not robust at \(X\).

- One can use topologies instead of metrics.
- One can use uncertainty on \(\mathbb{P}\) instead of on \(X\).
- Conceptually different from the field of robust optimization or optimizing robust preferences.
Representative Optimization Problems

Representative optimization problems.

- $n = 1$ and $X$ is a random loss
- The pricing density $\gamma = \gamma(X)$ is a measurable function of $X$
  - $\gamma > 0$, $\mathbb{E}[\gamma] = 1$ and $\mathbb{E}[\gamma X] < \infty$
- The budget constraint is $\mathbb{E}[\gamma g(X)] \geq x_0$
- Problems: to minimize $\rho(g(X))$ over $g \in \mathcal{G}$ for some $\mathcal{G} \subset \mathcal{G}_n$
in three settings $\mathcal{G} = \mathcal{G}_{cm}, \mathcal{G}_{ns}, \mathcal{G}_{bd}$
Representative Optimization Problems

(a) Complete market:

$$G_{cm} = \{ g \in G_1 : \mathbb{E}[\gamma g(X)] \geq x_0 \}.$$  

(b) No short-selling or over-hedging constraint:

$$G_{ns} = \{ g \in G_1 : \mathbb{E}[\gamma g(X)] \geq x_0, \ 0 \leq g(X) \leq X \}.$$  

Assume $0 \leq x_0 < \mathbb{E}[\gamma X]$ to avoid triviality.

(c) Bounded constraint: for some $m > 0$,

$$G_{bd} = \{ g \in G_1 : \mathbb{E}[\gamma g(X)] \geq x_0, \ 0 \leq g(X) \leq m \}.$$  

Assume $0 \leq x_0 < m$ to avoid triviality.
Remark.

- Problem (c) is not a special case of Problem (b) as $X$ in (b) is both the constraint and the source of randomness.

For (a)-(c), assume

- $X \geq 0$ and the distribution function of $X$ is continuous and strictly increasing on $(\text{ess-inf}X, \text{ess-sup}X)$.
- $X \in \mathcal{Z}$, and $(\mathcal{Z}, \pi_{\mathcal{Z}})$ is one of the classic choices $(L^q, \pi^q)$ for $q \in [1, \infty]$ and $(L^0, \pi^W)$.

Problem (c) for some distortion risk measures is studied by He-Zhou'11.
Robustness in the Optimization of $\text{VaR}$

Let

$$q = \text{VaR}_p(X; G_{ns}) = \inf \left\{ \text{VaR}_p(g(X)) : g \in G_{ns} \right\},$$

$$q' = \text{VaR}_p(X; G_{bd}) = \inf \left\{ \text{VaR}_p(g(X)) : g \in G_{bd} \right\}.$$

**Assumption 1**

$q > 0$ and $\mathbb{P}(\gamma(X - q) \leq \text{VaR}_p(\gamma(X - q))) = p$.

**Assumption 2**

$q' > 0$ and $\mathbb{P}(\gamma \leq \text{VaR}_p(\gamma)) = p$.

- $q, q' > 0$ means the optimization does not result in zero risk
- Assumptions 1-2 are very weak
Robustness in the Optimization of VaR

Theorem 2

For $p \in (0, 1)$ and $X \in \mathcal{Z}$,

(i) $\text{VaR}_p$ is not robust relative to $(\mathcal{G}_{cm}, \mathcal{Z}, \pi_{\mathcal{Z}})$;

(ii) under Assumption 1, $\text{VaR}_p$ is not robust at $X$ relative to $(\mathcal{G}_{ns}, \mathcal{Z}, \pi_{\mathcal{Z}})$;

(iii) under Assumption 2, $\text{VaR}_p$ is not robust at $X$ relative to $(\mathcal{G}_{bd}, \mathcal{Z}, \pi_{\mathcal{Z}})$.

- Robustness of VaR in optimization is very bad
### Assumption 3

\[
\text{ess-sup} \gamma \leq \frac{1}{1-p}.
\]

- Assumption 3 may be interpreted as a **no-arbitrage** condition for a market with ES participants.

### Assumption 4

*Either \(\gamma\) is a constant, or \(\gamma\) is a continuous function and \(\gamma(X)\) is continuously distributed.*

- Assumption 4 is commonly satisfied.
Robustness in the Optimization of ES

Theorem 3

For \( p \in (0, 1) \) and \( X \in \mathcal{Z} \),

(i) under Assumption 3, \( \text{ES}_p \) is \textit{robust} at \( X \) relative to \((\mathcal{G}_{cm}, \mathcal{Z}, \pi_{\mathcal{Z}})\);

(ii) under Assumption 4, \( \text{ES}_p \) is \textit{robust} at \( X \) relative to \((\mathcal{G}_{ns}, \mathcal{Z}, \pi_{\mathcal{Z}})\), where \((\mathcal{Z}, \pi_{\mathcal{Z}}) = (L^q, \pi^q)\) for \( q \in [1, \infty] \);

(iii) under Assumption 4, \( \text{ES}_p \) is \textit{robust} at \( X \) relative to \((\mathcal{G}_{bd}, \mathcal{Z}, \pi_{\mathcal{Z}})\), where \((\mathcal{Z}, \pi_{\mathcal{Z}}) = (L^q, \pi^q)\) for \( q \in [1, \infty] \).

▶ Robustness of ES in optimization is \textit{quite good}
Robustness in Optimization for VaR and ES

On robustness in optimization:

$\text{VaR} \ll \text{ES}$ (RVaR/ES not easy to compare)

Observations.

- The discontinuity in $Z \mapsto g^*(Z)$ comes from the fact that optimizing VaR is “too greedy”: always ignores tail risk, and hoping the probability of the tail risk is correctly modelled.

- None of the two values $\text{VaR}_p(g^*(X))$ and $\text{VaR}_p(g^*(Z))$

  is a rational measure of the “optimized” risk.
Robustness in Optimization for VaR and ES

Is risk positions of type $g^*$ realistic?

“Starting in 2006, the CDO group at UBS noticed that their risk-management systems treated AAA securities as essentially riskless even though they yielded a premium (the proverbial free lunch). So they decided to hold onto them rather than sell them.”

- From Feb 06 to Sep 07, UBS increased investment in AAA-rated CDOs by more than 10 times; many large banks did the same.
  - Take a risk of big loss with small probability, $X_i = X_{I_A}$
  - Treat it as free money - profit
  - Model uncertainty?

quoted from Acharya-Cooley-Richardson-Walter’10
Other Questions

Other questions

- other risk measures
- other optimization problems
- utility maximization problems
- risk measures as constraints instead of objectives
- robust preferences
Progress

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Some conclusions on robustness

- **Classic notion**
  - $ES \prec VaR \prec RVaR$
  - However this robustness may not be desirable

- **Novel perspectives**
  - $VaR \prec ES \prec RVaR$ in risk aggregation
  - $VaR \iff ES \prec RVaR$ in risk sharing
  - $VaR \iff ES$ in optimization
  - The rationality of optimizing $VaR$ under model uncertainty is questionable
CEO of AIG Financial Products, August 2007:

“It is **hard** for us, without being flippant, to even see a **scenario** within any kind of **realm of reason** that would see us **losing one dollar** in any of those transactions.”

- AIGFP sold protection on super-senior tranches of CDOs
- $180 billion bailout from the federal government in September 2008
Thank You
Industry Perspectives

From the **International Association of Insurance Supervisors**:  

- Document (version June 2015)  
  Compiled Responses to ICS Consultation 17 Dec 2014 - 16 Feb 2015

**In summary**

- Responses from insurance organizations and companies in the world.
- 49 responses are public
- 34 commented on Q42: VaR versus ES (TVaR)
Industry Perspectives

- 5 responses are supportive about ES:
  - Canadian Institute of Actuaries, CA
  - Liberty Mutual Insurance Group, US
  - National Association of Insurance Commissioners, US
  - Nematrian Limited, UK
  - Swiss Reinsurance Company, CH

- Some are indecisive; most favour VaR.

  Regulator and firms may have different views
Discussion

Major reasons to favour VaR from the insurance industry (IAIS report June 2015)

- Implementation of ES is expensive (staff, software, capital)
- ES does not exist for certain heavy-tailed risks
- ES is more costly on distributional information, data and simulation
- ES has trouble with a change of currency