Robust pricing–hedging duality for American options in discrete time financial markets

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based on joint work with
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Robust pricing-hedging duality for American option
Toy example
Two period model, stock price $S$
$S_0 = S_1 = 0$, $S_2 \in \{2, 1, -1, -2\}$
Two period model, stock $S$, American option $\Phi$

$\Phi_1 = 1$, $\Phi_2(S_2 \in \{2, -2\}) = 0$, $\Phi_2(S_2 \in \{1, -1\}) = 2$
Stock $S$, American option $\Phi$

Superhedging cost is the minimal $x \in \mathbb{R}$ such that there exists $\gamma \in \mathbb{R}$ satisfying $x \geq 1$ and $x + \gamma S_2 \geq \Phi_2$. 
Stock $S$, American option $\Phi$

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So, superhedging cost of $\Phi$ is 2.
Stock $S$, American option $\Phi$

Model price is $\sup_{Q \in \mathcal{M}} \sup_{\tau} \mathbb{E}^{Q}[\Phi_{\tau}]$. 
Stock $S$, American option $\Phi$

Model price is $\sup_{\mathcal{Q} \in \mathcal{M}} \sup_{\tau} \mathbb{E}^\mathcal{Q}[\Phi_{\tau}]$.

$\mathcal{Q}$ is a martingale probability measure $\mathcal{Q}(S_2 = i) = q_i$ satisfies $q_2 + q_1 + q_{-1} + q_{-2} = 1$ and $2q_2 + q_1 - q_{-1} - 2q_{-2} = 0$. 
Stock $S$, American option $\Phi$

Model price is $\sup_{Q \in \mathcal{M}} \sup_{\tau} \mathbb{E}^Q[\Phi_{\tau}]$.

Since $\Phi \leq 2$ and for $\tilde{Q}$ given by $\tilde{q}_1 = \tilde{q}_{-1} = 1/2$, model price of $\Phi$ equals $\mathbb{E}^{\tilde{Q}}[\Phi_2] = 2$. 
Stock $S$, American option $\Phi$

Model price is $\sup_{Q \in \mathcal{M}} \sup_{\tau} E^Q[\Phi_\tau]$.

Since $\Phi \leq 2$ and for $\tilde{Q}$ given by $\tilde{q}_1 = \tilde{q}_{-1} = 1/2$, model price of $\Phi$ equals $E^{\tilde{Q}}[\Phi_2] = 2$. So, superhedging cost and model price are equal.
Stock $S$, European option $g = \mathbb{1}_{\{|S|\leq 1\}} - 1/2$ with price 0, American option $\Phi$
Stock $S$, European option $g$, American option $\Phi$

Superhedging cost is the minimal $x \in \mathbb{R}$ such that there exists $\gamma \in \mathbb{R}$ and $\alpha \in \mathbb{R}$ satisfying $x + \alpha g \geq 1$ and $x + \gamma S_2 + \alpha g \geq \Phi_2$. 
Stock $S$, European option $g$, American option $\Phi$

Superhedging cost is the minimal $x \in \mathbb{R}$ such that there exists $\gamma \in \mathbb{R}$ and $\alpha \in \mathbb{R}$ satisfying $x + \alpha g \geq 1$ and $x + \gamma S_2 + \alpha g \geq \Phi_2$, equivalently $x \geq (1 + 1/2\alpha) \vee (2 - 1/2\alpha)$. 
Stock $S$, European option $g$, American option $\Phi$

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So, superhedging cost of $\Phi$ is $3/2$ and superhedging portfolio consists of keeping $3/2$ in cash and buying one option $g$. 
Stock $S$, European option $g$, American option $\Phi$

Model price is $\sup_{Q \in \mathcal{M}_g} \sup_{\tau} \mathbb{E}^Q[\Phi_\tau]$. 
Stock $S$, European option $g$, American option $\Phi$

Model price is $\sup_{Q \in \mathcal{M}_g} \sup_{\tau} \mathbb{E}^Q[\Phi_\tau]$. $\mathcal{M}_g$ is a set of calibrated martingale measures, i.e., additionally satisfying that $\mathbb{E}^Q[g] = 0$. 
Stock $S$, European option $g$, American option $\Phi$

<table>
<thead>
<tr>
<th></th>
<th>0, 0</th>
<th>0, 1</th>
<th>2, -1/2, 0</th>
<th>1, 1/2, 2</th>
<th>-1, 1/2, 2</th>
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Stock $S$, European option $g$, American option $\Phi$

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**Problem:** $g$ is traded only at time 0.

**Solution:** Allow dynamic trading in $g$. 

```
0, 0   0, 1
      / \
    2, -1/2, 0
  /     \n1, 1/2, 2
/       \n-1, 1/2, 2
       \n-2, -1/2, 0
```
**Problem:** $g$ is traded only at time 0.

**Solution:** Allow dynamic trading in $g$. We introduce process $Y = (Y_t : t = 0, 1, 2)$ such that $Y_2 = g$, $Y_0 = 0$ and $Y_1$ can take any value. In particular:

- $0, 0$
- $0, -1/2, 1$
- $0, 1/2, 1$
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- $Y_0 = 0$, $Y_1 = 0, 1/2, 1$ with probability $1/2$.
- $Y_0 = 0$, $Y_1 = 0, -1/2, 1$ with probability $1/2$.
- $Y_2 = 1, 1/2, 2$.
- $Y_2 = -1, 1/2, 2$.
- $Y_2 = 2, -1/2, 0$.
- $Y_2 = -2, -1/2, 0$.

Unique probability measure $\hat{Q}$ under which $S$ and $Y$ are martingales.
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$$
\begin{array}{c}
0, 0 \\
\text{1/2} \downarrow 0, -1/2, 1 \\
\text{1/2} \downarrow 0, 1/2, 1 \\
\text{1/2} \\
\hline
1/2 \\
\end{array}
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$$

Unique probability measure $\hat{Q}$ under which $S$ and $Y$ are martingales. New stopping time $\tau^* = 1_{\{Y_1=-1/2\}} + 2 1_{\{Y_1=1/2\}}$. 
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\end{array}

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\end{array}

\begin{array}{c}
1/2 \quad -1, 1/2, 2
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Unique probability measure $\widehat{Q}$ under which $S$ and $Y$ are martingales. New stopping time $\tau^* = \mathbb{1}_{\{Y_1=-1/2\}} + 2\mathbb{1}_{\{Y_1=1/2\}}$. So, model price equals $\mathbb{E}^{\widehat{Q}}[\Phi_{\tau^*}] = 3/2$ and duality is restored.
Set up

Set up

- \((\Omega, \mathbb{F}, \mathcal{F})\) is a filtered space where \(\mathbb{F} := (\mathcal{F}_t)_{t=0,1,...,T}\)
- \(\mathcal{P}\) is a set of probability measures on \((\Omega, \mathcal{F})\)
- \(S\) is an \(\mathbb{F}\)-adapted \(\mathbb{R}^d\)-valued process
- \(g = (g^i)_{i \in \{1,...,k\}}\) is a vector of \(\mathcal{F}\)-measurable \(\mathbb{R}\)-valued r.v.
- \(\mathcal{H}\) is the set of all \(\mathbb{F}\)-predictable \(\mathbb{R}^d\)-valued processes

Final payoff of semi–static trading strategy \((H, \alpha) \in (\mathcal{H}, \mathbb{R}^k)\)

\[
(H \circ S)_T + \alpha g = \sum_{j=1}^{d} \sum_{t=1}^{T} H_t^j \left( S_t^j - S_{t-1}^j \right) + \sum_{i=1}^{k} \alpha^i g^i
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\(\mathcal{M} = \{\mathbb{Q} \ll \mathcal{P} \text{ and } S \text{ is an } (\mathbb{Q}, \mathcal{F})\text{-martingale}\}\)

\(\mathcal{M}_g = \{\mathbb{Q} \in \mathcal{M} : \mathbb{E}^\mathbb{Q}[g^i] = 0, \ \forall i \in \{1, \ldots, k\}\}.\)
Superhedging of American options

- American option has a payoff function $\Phi = (\Phi_t)_{1 \leq t \leq T}$ may be exercised at any time $t \in T := \{1, \cdots, T\}$

The superhedging cost of the American option $\Phi$ using semi–static strategies is given by

$$\pi^A_g(\Phi) = \inf \left\{ x : \exists (H^1, \cdots, H^T) \in \mathcal{H}^T \text{ s.t. } H^t_i = H^n_i \quad \forall i \leq t \leq n \right\}$$

and $\alpha \in \mathbb{R}^k$ satisfying $x + (H^t \circ S)_T + \alpha g \geq \Phi_t \quad \forall t \in T \quad \mathcal{P}\text{-q.s.}$

- Dynamic trading strategy $H^t$ might be adjusted after disclosure of whether the exercise of American option took place or not
- Consistency: $H^t_i = H^t_n$ whenever $i \leq n \leq t$, $H^t$ is $\mathbb{F}$-predictable
- Asymmetry: there is no way to adjust the static trading strategy due to its nature
Superhedging of American options

The superhedging cost of the American option \( \Phi \) using semi–static strategies is given by

\[
\pi_g^A (\Phi) = \inf \left\{ x : \exists (H^1, ..., H^T) \in \mathcal{H}^T \text{ s.t. } H^t_i = H^n_i \ \forall i \leq t \leq n \right. \\
\text{and } \alpha \in \mathbb{R}^k \text{ satisfying } x + (H^t \circ S)_T + \alpha g \geq \Phi_t \ \forall t \in \mathbb{T} \ \mathcal{P}\text{-q.s.} \left. \right\}
\]

\[
> \sup_{Q \in \mathcal{M}_g} \sup_{\tau \in \mathcal{T}(\mathbb{F})} \mathbb{E}^Q[\Phi_{\tau}]
\]

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- Asymmetry: there is no way to adjust the static trading strategy due to its nature
Enlarged space $\overline{\Omega} := \Omega \times \{1, \ldots, T\}$

- Let $\overline{\Omega} := \Omega \times \mathbb{T}$ and $\mathbb{T} := \{1, \ldots, T\}$ with $\overline{\omega} := (\omega, \theta)$

- Natural embedding of $\Omega$ into $\overline{\Omega}$ extends $S$ and $g^i$ as $S(\overline{\omega}) = S(\omega)$ and $g^i(\overline{\omega}) = g^i(\omega)$

- The canonical time $\Theta : \overline{\Omega} \rightarrow \mathbb{T}$ is given by $\Theta(\overline{\omega}) := \theta$

- The filtration $\overline{\mathbb{F}} := (\overline{\mathbb{F}}_t)_{t=0,1,\ldots,T}$ is the smallest filtration containing $\mathbb{F}$ and making $\Theta$ a stopping time, i.e., $\overline{\mathbb{F}}_t = \mathbb{F}_t \otimes \mathbb{V}_t$ and $\mathbb{V}_t = \sigma(\Theta \wedge (t + 1))$, and the $\sigma$-field $\overline{\mathbb{F}} = \mathbb{F} \otimes \mathbb{V}_T$

- $\Theta$ is an $\overline{\mathbb{F}}$-stopping time

- Sets of probability measures:

$$\overline{\mathbb{P}} := \{\overline{\mathbb{P}} : \overline{\mathbb{P}}|_{\Omega} \in \mathbb{P}\},$$

$$\overline{\mathbb{M}} := \{\overline{Q} \ll \overline{\mathbb{P}} \text{ and } S \text{ is an } (\overline{Q}, \overline{\mathbb{F}})\text{-martingale}\},$$

$$\overline{\mathbb{M}}_g := \{\overline{Q} \in \overline{\mathbb{M}} : \mathbb{E}^\overline{Q}[g^i] = 0 \ \forall i \in \{1, \ldots, k\}\}$$
Reformulation of a superhedging of an American option

We identify an American option $\Phi$ on $\Omega$ with a European option on $\overline{\Omega}$ via

$$\Phi(\omega) = \Phi_{\theta}(\omega)$$

The superhedging cost of the option $\Phi$ on $\overline{\Omega}$

$$\pi_g^E(\Phi) := \inf \{ x : \exists (H, \alpha) \in \overline{H} \times \mathbb{R}^k \text{ s.t. } x + (H \circ S)_T + \alpha g \geq \Phi \ \mathcal{P}-\text{q.s.} \}$$

where $\overline{H}$ is the class of $\overline{\mathcal{F}}$-predictable processes

**Theorem**

We have that $\pi_{g}^{A}(\Phi) = \pi_{g}^{E}(\Phi)$ and, in particular, if the European pricing–hedging duality on $\overline{\Omega}$ holds for $\Phi$ then

$$\pi_{g}^{A}(\Phi) = \pi_{g}^{E}(\Phi) = \sup_{\overline{Q} \in \mathcal{M}_g} \mathbb{E}^{\overline{Q}}[\Phi].$$
What models are in $\overline{M}_g$?

• Instead of stopping times relative to $\mathbb{F}$, it allows us to consider any random time which can be made into a stopping time under some calibrated martingale measure.

• Comparing with formulation on $\Omega$:

$$\sup_{\tau: \text{random time}} \sup_{Q \in \mathcal{M}_g(\mathbb{F}^\tau)} E^Q[\Phi_\tau] = \sup_{Q \in \overline{M}_g} E^Q[\Phi]$$

• $\overline{M}_g$ is equivalent to weak formulation.

• Is there a minimal way of enlarging a space which is equivalent to $\overline{M}_g$?
Dynamic perspective: case $k = 0$:

- Let $\mathcal{E}(\xi) := \sup_{Q \in \mathcal{M}} \mathbb{E}^Q[\xi]$.

- Suppose that there is a family of operators $(\mathcal{E}_t)$ such that $\mathcal{E}_t(\xi)$ is $\mathcal{F}_t$-measurable for all $\xi$.

- We say that the family $(\mathcal{E}_t)$ provides a dynamic programming representation of $\mathcal{E}$ if

$$\mathcal{E}(\xi) = \mathcal{E}_0 \circ \mathcal{E}_1 \circ \ldots \circ \mathcal{E}_{T-1}(\xi), \quad \forall \xi \in \Upsilon$$

- The family $(\mathcal{E}_t)$ extends to the family $(\overline{\mathcal{E}}_t)$ as

$$\overline{\mathcal{E}}_0(\Phi)(\overline{\omega}) := \mathcal{E}_0(\Phi(\cdot, 1))(\omega), \quad \text{for all } \overline{\omega} = (\omega, \theta),$$

$$\overline{\mathcal{E}}_t(\Phi)(\overline{\omega}) := \begin{cases} \mathcal{E}_t(\Phi(\cdot, \theta))(\omega) & \text{if } \theta < t \\ \mathcal{E}_t(\Phi(\cdot, t))(\omega) \lor \mathcal{E}_t(\Phi(\cdot, t + 1))(\omega) & \text{if } \theta \geq t \end{cases}, \quad \text{for } t \neq 0.$$
Duality for an American option: case $k = 0$

**Theorem**

Suppose $(\mathcal{E}_t)$ satisfies DPP. Then,

$$\sup_{Q \in \mathcal{M}} \mathbb{E}^Q[\Phi] = \sup_{Q \in \mathcal{M}} \sup_{\tau \in \mathcal{T}(\mathbb{F})} \mathbb{E}^Q[\Phi_\tau].$$

If, further, the European pricing–hedging duality holds on $\bar{\Omega}$, then

$$\pi^A(\Phi) = \sup_{Q \in \mathcal{M}} \sup_{\tau \in \mathcal{T}(\mathbb{F})} \mathbb{E}^Q[\Phi_\tau].$$

There exists an $\mathbb{F}$-stopping time $\tau^*$, defined via $(\mathcal{E}_t)$ and $(\bar{\mathcal{E}}_t)$ as

$$\tau^* := \inf \{ t \geq 1 : \mathcal{E}_t \circ \cdots \circ \mathcal{E}_{T-1} (\Phi) = \bar{\mathcal{E}}_t \circ \cdots \circ \bar{\mathcal{E}}_{T-1}(\Phi) \}$$

which provides the optimal exercise policy for $\Phi \in \bar{\Upsilon}$:

$$\sup_{Q \in \mathcal{M}} \mathbb{E}^Q[\Phi] = \sup_{Q \in \mathcal{M}} \mathbb{E}^Q[\Phi_{\tau^*}] = \sup_{Q \in \mathcal{M}} \sup_{\tau \in \mathcal{T}(\mathbb{F})} \mathbb{E}^Q[\Phi_\tau].$$
Embedding into a larger space $\hat{\Omega}$: case $k \geq 1$

- Presence of statically traded options unables/breaks dynamic programming principle
- Embed the market into a fictitious larger one where both $S$ and all the options $(g^1, ..., g^k)$ are traded dynamically
- Let us denote by $\hat{S} := (S, Y)$ which will now correspond to dynamically traded assets
Recovering a dynamic perspective

- Let $\hat{\Omega} = \Omega \times \mathbb{R}^{(T-1) \times k}$, an element $\hat{\omega}$ in $\hat{\Omega}$ can be written as $\hat{\omega} = (\omega, y)$ where $y = (y^1, ..., y^k) \in \mathbb{R}^{(T-1) \times k}$ with $y^i = (y^i_1, ..., y^i_{T-1})$

- Let $Y$ be a process given by

$$Y_t(\hat{\omega}) := \begin{cases} 0 & \text{for } t = 0 \\ y_t & \text{for } t \in \{1, ..., T - 1\} \\ g(\omega) & \text{for } t = T \end{cases}$$

- Let $\hat{\mathbb{F}} := (\hat{\mathcal{F}})_{t=0,1,...,T}$ be a filtration given by $\hat{\mathcal{F}}_t = \mathcal{F}_t \otimes \mathcal{Y}_t$, $\mathcal{Y}_t = \sigma(Y_t : t \leq T)$

- Let $\hat{\mathbb{M}} := \{\hat{\mathbb{Q}} \ll \hat{\mathbb{P}} \text{ and } \hat{\mathbb{S}} := (S, Y) \text{ is an } (\hat{\mathbb{Q}}, \hat{\mathbb{F}})\text{-martingale}\}$ where $\hat{\mathbb{P}} := \{\hat{\mathbb{P}} : \hat{\mathbb{P}}|_\Omega \in \mathbb{P}\}$

- For each $\mathbb{Q} \in \mathbb{M}_g$, there exists $\hat{\mathbb{Q}} \in \hat{\mathbb{M}}$ such that $\hat{\mathbb{Q}}|_\Omega = \mathbb{Q}$ and $\mathcal{L}_{\hat{\mathbb{Q}}}(Y) = \mathcal{L}_\mathbb{Q}(Y^\mathbb{Q})$ where $Y^\mathbb{Q} := (E[\mathbb{Q}[g^i|\mathcal{F}_t]])_{t \leq T}$
Corollary

Suppose that \((\hat{\mathcal{E}}_t)\) satisfies DPP. Assume that the European pricing–hedging duality holds on \(\hat{\Omega}\). Then,

\[
\pi^A_g(\Phi) = \hat{\pi}^A(\Phi) = \sup_{\hat{\mathcal{Q}} \in \hat{\mathcal{M}}} \sup_{\tau \in \mathcal{T}(\hat{\mathcal{F}})} \mathbb{E}^{\hat{\mathcal{Q}}}[\Phi_\tau].
\]

Follows by

\[
\bar{\pi}^E_g(\Phi) = \pi^A_g(\Phi) \geq \hat{\pi}^A(\Phi) = \bar{\pi}^E(\Phi) \sup_{\mathcal{Q} \in \mathcal{M}} \mathbb{E}^{\mathcal{Q}}[\Phi] \geq \sup_{\mathcal{Q} \in \mathcal{M}_g} \mathbb{E}^{\mathcal{Q}}[\Phi],
\]

note that \(\pi^A_g \geq \hat{\pi}^A\) since a buy–and–hold strategy is a special case of a dynamic trading strategy and \(\mathcal{P} = \hat{\mathcal{P}}|_\Omega\).
Conclusions

Recovering duality for American options:

• Solution 1: American option rendered European option
  \( \Omega = \Omega \times \{1, \ldots, T\} \) and \( \Phi(\omega, t) = \Phi_t(\omega) \)

• Solution 2: Presence of statically traded options breaks dynamic programming principle. We allow dynamic trading in these options by enlarging the probability space to \( \hat{\Omega} \).

Applications to:


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Applications to:


THANK YOU!