Robust pricing–hedging duality for American options in discrete time financial markets

Anna Aksamit

The University of Sydney

based on joint work with

Shuoqing Deng, Jan Obłój and Xiaolu Tan

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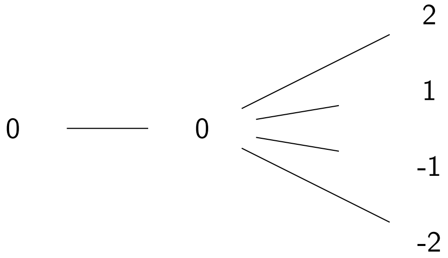
Robust pricing-hedging duality for American option

Toy example

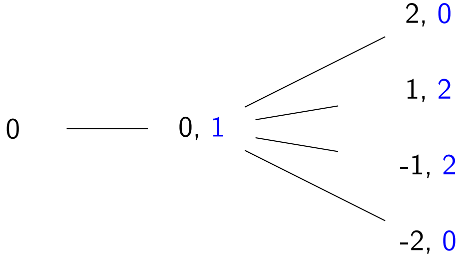
Two period model, stock price S

$S_0 = S_1 = 0$, $S_2 \in \{2, 1, -1, -2\}$

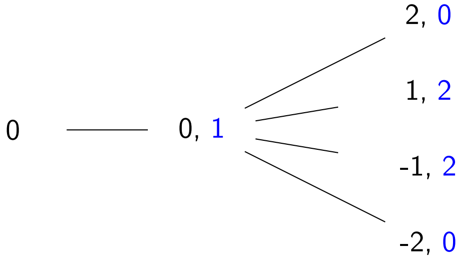


Two period model, stock S, American option Φ

$\Phi_1 = 1$, $\Phi_2(S_2 \in \{2, -2\}) = 0$, $\Phi_2(S_2 \in \{1, -1\}) = 2$

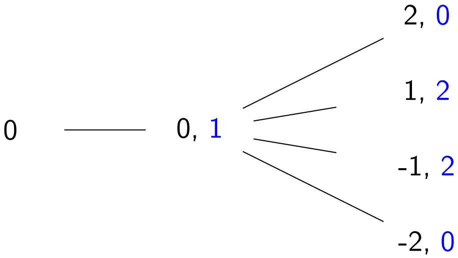


Stock S, American option Φ



Superhedging cost is the minimal $x \in \mathbb{R}$ such that there exists $\gamma \in \mathbb{R}$ satisfying $x \geq 1$ and $x + \gamma S_2 \geq \Phi_2$.

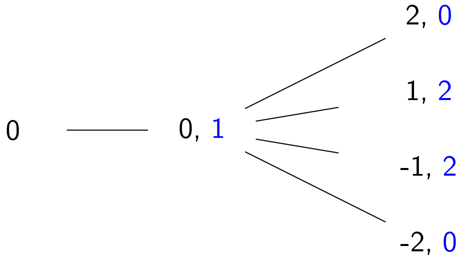
Stock S, American option Φ



Superhedging cost is the minimal $x \in \mathbb{R}$ such that there exists $\gamma \in \mathbb{R}$ satisfying $x \geq 1$ and $x + \gamma S_2 \geq \Phi_2$.

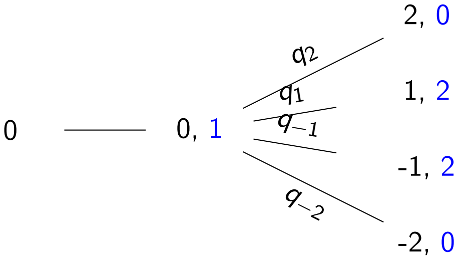
So, superhedging cost of Φ is 2.

Stock S, American option Φ



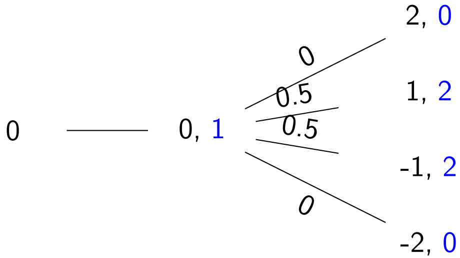
Model price is $\sup_{\mathbb{Q}\in\mathcal{M}} \sup_{\tau} \mathbb{E}^{\mathbb{Q}}[\Phi_\tau]$.

Stock S, American option Φ



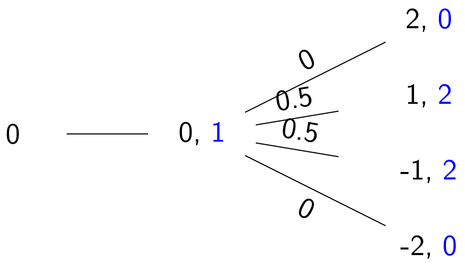
Model price is $\sup_{\mathbb{Q}\in\mathcal{M}} \sup_{\tau} \mathbb{E}^{\mathbb{Q}}[\Phi_{\tau}]$.
$\mathbb{Q}$ is a martingale probability measure $\mathbb{Q}(S_2 = i) = q_i$ satisfies $q_2 + q_1 + q_{-1} + q_{-2} = 1$ and $2q_2 + q_1 - q_{-1} - 2q_{-2} = 0$.

Stock S, American option Φ



Model price is $\sup_{\mathbb{Q}\in\mathcal{M}} \sup_{\tau} \mathbb{E}^{\mathbb{Q}}[\Phi_\tau]$.
Since $\Phi \leq 2$ and for $\widetilde{\mathbb{Q}}$ given by $\widetilde{q}_1 = \widetilde{q}_{-1} = 1/2$, model price of Φ equals $\mathbb{E}^{\widetilde{\mathbb{Q}}}[\Phi_2] = 2$.

Stock S, American option Φ



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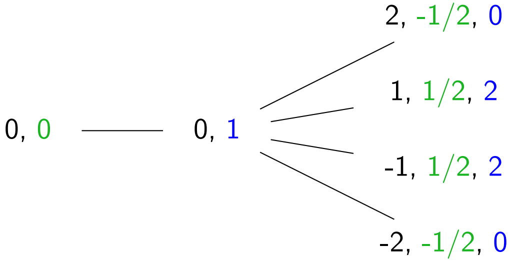
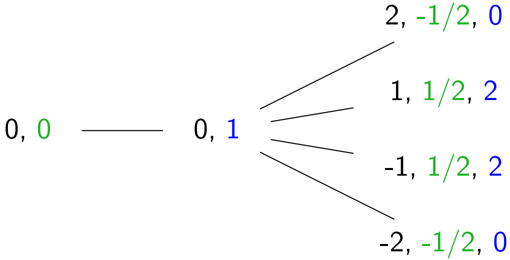
Since $\Phi \leq 2$ and for $\widetilde{\mathbb{Q}}$ given by $\widetilde{q}_1 = \widetilde{q}_{-1} = 1/2$, model price of Φ equals $\mathbb{E}^{\widetilde{\mathbb{Q}}}[\Phi_2] = 2$. **So, superhedging cost and model price are equal.**

Stock S, European option $g = \mathbb{1}_{\{|S_2|=1\}} - 1/2$ with price 0, American option Φ

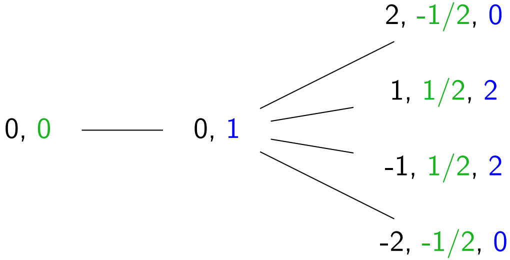


Stock S, European option g, American option Φ



Superhedging cost is the minimal $x \in \mathbb{R}$ such that there exists $\gamma \in \mathbb{R}$ and $\alpha \in \mathbb{R}$ satisfying $x + \alpha g \geq 1$ and $x + \gamma S_2 + \alpha g \geq \Phi_2$.

Stock S, European option g, American option Φ



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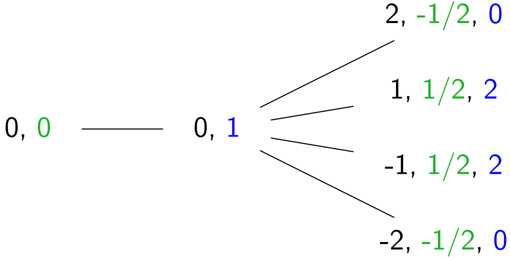
Stock S, European option g, American option Φ



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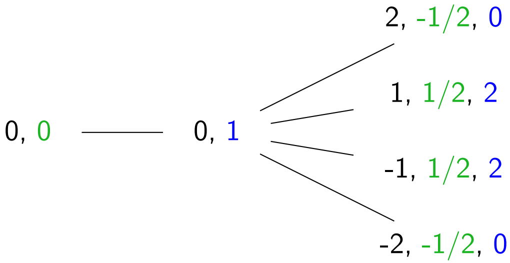
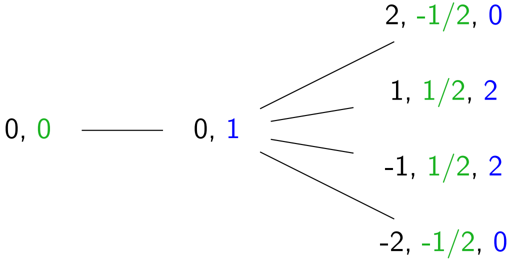
So, superhedging cost of Φ is 3/2 and superhedging portfolio consists of keeping 3/2 in cash and buying one option g.

Stock S, European option g, American option Φ



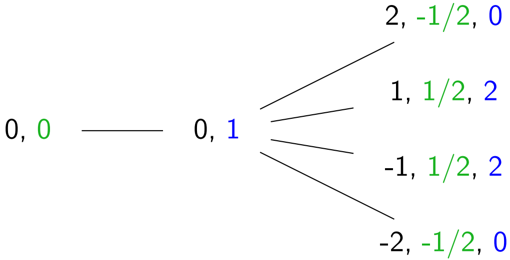
Model price is $\sup_{\mathbb{Q}\in\mathcal{M}_g} \sup_\tau \mathbb{E}^{\mathbb{Q}}[\Phi_\tau]$.

Stock S, European option g, American option Φ



Model price is $\sup_{\mathbb{Q}\in\mathcal{M}_g} \sup_\tau \mathbb{E}^{\mathbb{Q}}[\Phi_\tau]$. $\mathcal{M}_g$ is a set of calibrated martingale measures, i.e., additionally satisfying that $\mathbb{E}^{\mathbb{Q}}[g] = 0$.

Stock S, European option g, American option Φ



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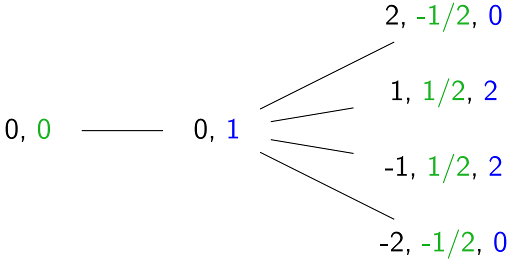

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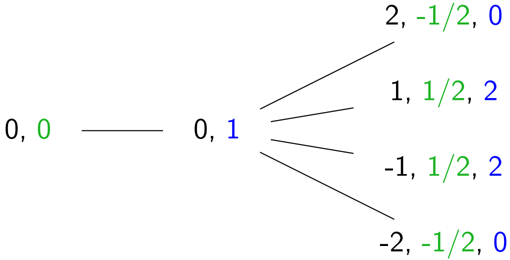
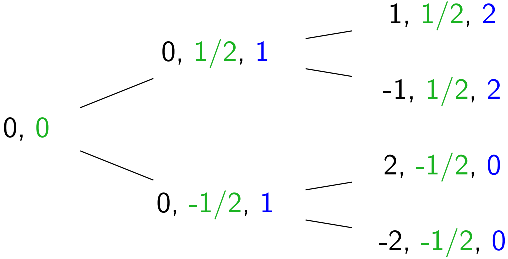
PROBLEM: g is traded only at time 0.
SOLUTION: Allow dynamic trading in g.



PROBLEM: g is traded only at time 0.
SOLUTION: Allow dynamic trading in g. We introduce process $Y = (Y_t : t = 0, 1, 2)$ such that $Y_2 = g$, $Y_0 = 0$ and Y_1 can take any value. In particular:



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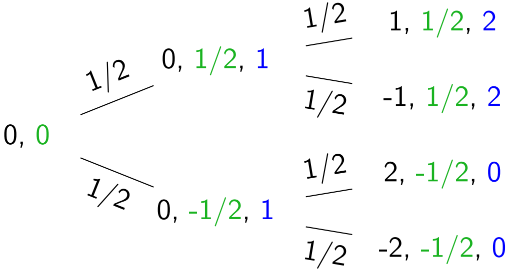

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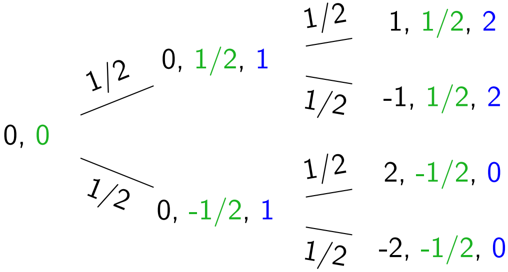

Unique probability measure $\widehat{\mathbb{Q}}$ under which S and Y are martingales. New stopping time $\tau^* = \mathbb{1}_{\{Y_1=-1/2\}} + 2\mathbb{1}_{\{Y_1=1/2\}}$.

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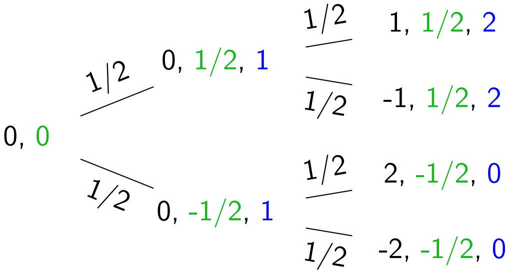

Unique probability measure $\widehat{\mathbb{Q}}$ under which S and Y are martingales. New stopping time $\tau^* = \mathbb{1}_{\{Y_1=-1/2\}} + 2\mathbb{1}_{\{Y_1=1/2\}}$.
So, model price equals $\mathbb{E}^{\widehat{\mathbb{Q}}}[\Phi_{\tau^*}] = 3/2$ and duality is restored.

Set up

- $(\Omega, \mathbb{F}, \mathcal{F})$ is a filtered space where $\mathbb{F} := (\mathcal{F}_t)_{t=0,1,\dots,T}$
- $\mathcal{P}$ is a set of probability measures on $(\Omega, \mathcal{F})$
- S is an $\mathbb{F}$-adapted $\mathbb{R}^d$-valued process
- $g = (g^i)_{i\in\{1,\dots,k\}}$ is a vector of $\mathcal{F}$-measurable $\mathbb{R}$-valued r.v.
- $\mathcal{H}$ is the set of all $\mathbb{F}$-predictable $\mathbb{R}^d$-valued processes

Final payoff of semi–static trading strategy $(H, \alpha) \in (\mathcal{H}, \mathbb{R}^k)$

$$(H \circ S)_T + \alpha g = \sum_{j=1}^{d} \sum_{t=1}^{T} H_t^j \left(S_t^j - S_{t-1}^j \right) + \sum_{i=1}^{k} \alpha^i g^i$$

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$$\mathcal{M} = \{\mathbb{Q} \lll \mathcal{P} \text{ and } S \text{ is an } (\mathbb{Q}, \mathbb{F})\text{-martingale}\}$$
$$\mathcal{M}_g = \{\mathbb{Q} \in \mathcal{M} : \mathbb{E}^{\mathbb{Q}}[g^i] = 0,\ \forall i \in \{1, \dots, k\}\}.$$

Superhedging of American options

- American option has a payoff function $\Phi = (\Phi_t)_{1\leq t\leq T}$ may be exercised at any time $t \in \mathbb{T} := \{1, \cdots, T\}$

The superhedging cost of the American option Φ using semi–static strategies is given by

$$\pi_g^A(\Phi) = \inf \big\{ x : \exists (H^1, ..., H^T) \in \mathcal{H}^T \text{ s.t. } H_i^t = H_i^n \ \forall i \leq t \leq n$$
$$\text{and } \alpha \in \mathbb{R}^k \ \text{ satisfying } \ x + (H^t \circ S)_T + \alpha g \geq \Phi_t \ \forall t \in \mathbb{T} \ \mathcal{P}\text{-q.s.} \big\}$$

- Dynamic trading strategy H^t might be adjusted after disclosure of whether the exercise of American option took place or not
- Consistency: $H_i^t = H_i^n$ whenever $i \leq n \leq t$, H^t is $\mathbb{F}$-predictable
- Asymmetry: there is no way to adjust the static trading strategy due to its nature

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Enlarged space $\overline{\Omega} := \Omega \times \{1, ..., T\}$

- Let $\overline{\Omega} := \Omega \times \mathbb{T}$ and $\mathbb{T} := \{1, ..., T\}$ with $\overline{\omega} := (\omega, \theta)$
- Natural embedding of Ω into $\overline{\Omega}$ extends S and g^i as $S(\overline{\omega}) = S(\omega)$ and $g^i(\overline{\omega}) = g^i(\omega)$
- The canonical time $\Theta : \overline{\Omega} \to \mathbb{T}$ is given by $\Theta(\overline{\omega}) := \theta$
- The filtration $\overline{\mathbb{F}} := (\overline{\mathcal{F}}_t)_{t=0,1,...,T}$ is the smallest filtration containing $\mathbb{F}$ and making Θ a stopping time, i.e., $\overline{\mathcal{F}}_t = \mathcal{F}_t \otimes \vartheta_t$ and $\vartheta_t = \sigma(\Theta \wedge (t+1))$, and the σ-field $\overline{\mathcal{F}} = \mathcal{F} \otimes \vartheta_T$
- Θ is an $\overline{\mathbb{F}}$-stopping time
- Sets of probability measures:

$$\begin{aligned}
\overline{\mathcal{P}} &:= \{\overline{\mathbb{P}} : \overline{\mathbb{P}}_{|\Omega} \in \mathcal{P}\}, \\
\overline{\mathcal{M}} &:= \{\overline{\mathbb{Q}} \lll \overline{\mathcal{P}} \text{ and } S \text{ is an } (\overline{\mathbb{Q}}, \overline{\mathbb{F}})\text{-martingale}\}, \\
\overline{\mathcal{M}}_g &:= \{\overline{\mathbb{Q}} \in \overline{\mathcal{M}} : \mathbb{E}^{\overline{\mathbb{Q}}}[g^i] = 0 \ \forall i \in \{1, ..., k\}\}
\end{aligned}$$

Reformulation of a superhedging of an American option

We identify an American option Φ on Ω with a European option on $\overline{\Omega}$ via

$$\Phi(\overline{\omega}) = \Phi_\theta(\omega)$$

The superhedging cost of the option Φ on $\overline{\Omega}$

$$\overline{\pi}_g^E(\Phi) := \inf\left\{x : \exists\ (\overline{H}, \alpha) \in \overline{\mathcal{H}} \times \mathbb{R}^k \ \text{ s.t. } \ x + (\overline{H} \circ S)_T + \alpha g \geq \Phi \ \ \overline{\mathcal{P}}\text{-q.s.}\right\}$$

where $\overline{\mathcal{H}}$ is the class of $\overline{\mathbb{F}}$-predictable processes

Theorem

We have that $\pi_g^A(\Phi) = \overline{\pi}_g^E(\Phi)$ and, in particular, if the European pricing–hedging duality on $\overline{\Omega}$ holds for Φ then

$$\pi_g^A(\Phi) = \overline{\pi}_g^E(\Phi) = \sup_{\overline{\mathbb{Q}} \in \overline{\mathcal{M}}_g} \mathbb{E}^{\overline{\mathbb{Q}}}[\Phi].$$

What models are in $\overline{\mathcal{M}}_g$?

- Instead of stopping times relative to $\mathbb{F}$, it allows us to consider any random time which can be made into a stopping time under some calibrated martingale measure
- Comparing with formulation on Ω:

$$\sup_{\tau:\text{random time}} \sup_{\mathbb{Q}\in\mathcal{M}_g(\mathbb{F}^\tau)} \mathbb{E}^{\mathbb{Q}}[\Phi_\tau] = \sup_{\overline{\mathbb{Q}}\in\overline{\mathcal{M}}_g} \mathbb{E}^{\overline{\mathbb{Q}}}[\Phi]$$

- $\overline{\mathcal{M}}_g$ is equivalent to *weak formulation*
- Is there a minimal way of enlarging a space which is equivalent to $\overline{\mathcal{M}}_g$?

Dynamic perspective: case $k = 0$:

- Let $\mathcal{E}(\xi) := \sup_{\mathbb{Q} \in \mathcal{M}} \mathbb{E}^{\mathbb{Q}}[\xi]$
- Suppose that there is a family of operators $(\mathcal{E}_t)$ such that $\mathcal{E}_t(\xi)$ is $\mathcal{F}_t$-measurable for all ξ
- We say that the family $(\mathcal{E}_t)$ provides a dynamic programming representation of $\mathcal{E}$ if

$$\mathcal{E}(\xi) = \mathcal{E}_0 \circ \mathcal{E}_1 \circ \ldots \circ \mathcal{E}_{T-1}(\xi), \quad \forall \xi \in \Upsilon$$

- The family $(\mathcal{E}_t)$ extends to the family $(\overline{\mathcal{E}}_t)$ as

$$\overline{\mathcal{E}}_0(\Phi)(\overline{\omega}) := \mathcal{E}_0(\Phi(\cdot, 1))(\omega), \qquad \text{for all } \overline{\omega} = (\omega, \theta),$$

$$\overline{\mathcal{E}}_t(\Phi)(\overline{\omega}) := \begin{cases} \mathcal{E}_t(\Phi(\cdot, \theta))(\omega) & \text{if } \theta < t \\ \mathcal{E}_t(\Phi(\cdot, t))(\omega) \vee \mathcal{E}_t(\Phi(\cdot, t+1))(\omega) & \text{if } \theta \geq t \end{cases}, \text{ for } t \neq 0.$$

Duality for an American option: case $k = 0$

Theorem

Suppose $(\mathcal{E}_t)$ satisfies DPP. Then,

$$\sup_{\overline{\mathbb{Q}} \in \overline{\mathcal{M}}} \mathbb{E}^{\overline{\mathbb{Q}}}[\Phi] = \sup_{\mathbb{Q} \in \mathcal{M}} \sup_{\tau \in \mathcal{T}(\mathbb{F})} \mathbb{E}^{\mathbb{Q}}[\Phi_\tau].$$

If, further, the European pricing–hedging duality holds on $\overline{\Omega}$, then

$$\pi^A(\Phi) = \sup_{\mathbb{Q} \in \mathcal{M}} \sup_{\tau \in \mathcal{T}(\mathbb{F})} \mathbb{E}^{\mathbb{Q}}[\Phi_\tau].$$

There exists an $\mathbb{F}$-stopping time τ^*, defined via $(\mathcal{E}_t)$ and $(\overline{\mathcal{E}}_t)$ as

$$\tau^* := \inf\left\{ t \geq 1 : \mathcal{E}_t \circ \ldots \circ \mathcal{E}_{T-1}(\Phi) = \overline{\mathcal{E}}_t \circ \ldots \circ \overline{\mathcal{E}}_{T-1}(\Phi) \right\}$$

which provides the optimal exercise policy for $\Phi \in \overline{\Upsilon}$:

$$\sup_{\overline{\mathbb{Q}} \in \overline{\mathcal{M}}} \mathbb{E}^{\overline{\mathbb{Q}}}[\Phi] = \sup_{\mathbb{Q} \in \mathcal{M}} \mathbb{E}^{\mathbb{Q}}[\Phi_{\tau^*}] = \sup_{\mathbb{Q} \in \mathcal{M}} \sup_{\tau \in \mathcal{T}(\mathbb{F})} \mathbb{E}^{\mathbb{Q}}[\Phi_\tau]$$

Embedding into a larger space $\widehat{\Omega}$: case $k \geq 1$

- Presence of statically traded options unables/breaks dynamic programming principle
- Embed the market into a fictitious larger one where both S and all the options $(g^1, ..., g^k)$ are traded dynamically
- Let us denote by $\widehat{S} := (S, Y)$ which will now correspond to dynamically traded assets

Recovering a dynamic perspective

- Let $\widehat{\Omega} = \Omega \times \mathbb{R}^{(T-1)\times k}$, an element $\widehat{\omega}$ in $\widehat{\Omega}$ can be written as $\widehat{\omega} = (\omega, y)$ where $y = (y^1, ..., y^k) \in \mathbb{R}^{(T-1)\times k}$ with $y^i = (y^i_1, ..., y^i_{T-1})$
- Let Y be a process given by

$$Y_t(\widehat{\omega}) := \begin{cases} 0 & \text{for} \quad t = 0 \\ y_t & \text{for} \quad t \in \{1, ..., T-1\} \\ g(\omega) & \text{for} \quad t = T \end{cases}$$

- Let $\widehat{\mathbb{F}} := (\widehat{\mathcal{F}})_{t=0,1,\dots,T}$ be a filtration given by $\widehat{\mathcal{F}}_t = \mathcal{F}_t \otimes \mathcal{Y}_t$, $\mathcal{Y}_t = \sigma(Y_t : t \leq T)$
- Let $\widehat{\mathcal{M}} := \left\{ \widehat{\mathbb{Q}} \lll \widehat{\mathcal{P}} \text{ and } \widehat{S} := (S, Y) \text{ is an } (\widehat{\mathbb{Q}}, \widehat{\mathbb{F}})\text{-martingale} \right\}$ where $\widehat{\mathcal{P}} := \{\widehat{\mathbb{P}} : \widehat{\mathbb{P}}_{|\Omega} \in \mathcal{P}\}$
- For each $\mathbb{Q} \in \mathcal{M}_g$, there exists $\widehat{\mathbb{Q}} \in \widehat{\mathcal{M}}$ such that $\widehat{\mathbb{Q}}_{|\Omega} = \mathbb{Q}$ and $\mathcal{L}_{\widehat{\mathbb{Q}}}(Y) = \mathcal{L}_{\mathbb{Q}}(Y^{\mathbb{Q}})$ where $Y^{\mathbb{Q}} := (E^{\mathbb{Q}}[g^i|\mathcal{F}_t])_{t\leq T}$

Duality for an American option on $\widehat{\Omega}$

Corollary

Suppose that $(\widehat{\mathcal{E}}_t)$ satisfies DPP. Assume that the European pricing–hedging duality holds on $\overline{\Omega}$. Then,

$$\pi_g^A(\Phi) = \widehat{\pi}^A(\Phi) = \sup_{\widehat{\mathbb{Q}} \in \widehat{\mathcal{M}}} \sup_{\tau \in \mathcal{T}(\widehat{\mathbb{F}})} \mathbb{E}^{\widehat{\mathbb{Q}}}\left[\Phi_\tau\right].$$

Follows by

$$\overline{\pi}_g^E(\Phi) = \pi_g^A(\Phi) \geq \widehat{\pi}^A(\Phi) = \overline{\widehat{\pi}}^E(\Phi) \geq \sup_{\mathbb{Q} \in \overline{\widehat{\mathcal{M}}}} \mathbb{E}^{\mathbb{Q}}\left[\Phi\right] \geq \sup_{\mathbb{Q} \in \overline{\mathcal{M}}_g} \mathbb{E}^{\mathbb{Q}}\left[\Phi\right],$$

note that $\pi_g^A \geq \widehat{\pi}^A$ since a buy–and–hold strategy is a special case of a dynamic trading strategy and $\mathcal{P} = \widehat{\mathcal{P}}_{|\Omega}$

Conclusions

Recovering duality for American options:

- Solution 1: American option rendered European option $\overline{\Omega} = \Omega \times \{1, ..., T\}$ and $\overline{\Phi}(\omega, t) = \Phi_t(\omega)$
- Solution 2: Presence of statically traded options breaks dynamic programming principle. We allow dynamic trading in these options by enlarging the probability space to $\widehat{\Omega}$.

Applications to:

B. Bouchard and M. Nutz, *Arbitrage and duality in nondominated discrete-time models*, The Annals of Applied Probability, 25(2), 823-859, 2015.

A. Cox and S. Källblad, *Model-independent bounds for Asian options: a dynamic programming approach* SIAM Journal on Control and Optimization, 55(6), 3409-3436, 2017.

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THANK YOU!