Martingale Optimal Transport (and Friends)

18th – 19th September 2017

Mathematical Institute, University of Oxford

held at the

Oxford-Man Institute, Eagle House, Walton Well Rd, Oxford OX2 6ED

Organised by:

Gaoyue Guo *and* Jan Obłój

University of Oxford

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Mathematical Institute
University of Oxford
AWB, ROQ, Woodstock Road
OX2 6GG, Oxford, UK

http://people.maths.ox.ac.uk/obloj/MOT2017.html

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AGENDA – *the long talks are 45+15 and the short talks are 20+10*

Monday 18th September – All talks held in AHL Lecture Theatre	
9:30-10:30	*Coffee and Registration*
10:30-11:00	Welcome and Opening statements
11:00-12:00	**Nicolas Juillet** "Markovinification of the quantile process"
12:10-12:40	**Christian Leonard** "From entropic to deterministic optimal transport"
12:45-13:45	*Lunch - conservatory*
13:45-14:45	**Martin Huesmann** "A Benamou-Brenier approach to martingale optimal transport"
14:45-15:15	**Julio Backhoff Veraguas** "Martingale Benamou-Brenier: a probabilistic perspective (I)"
15:15-15:45	**Mathias Beiglboeck** "Martingale Benamou-Brenier: a probabilistic perspective (II)"
15:45-16:15	*Coffee - conservatory*
16:15-16:45	**Pierre Henry-Labordere** "Quantum optimal transport"
16:45-17:15	**Halil Mete Soner** "Robust duality – a survey"
17:20-17:50	**David Hobson** "Robust hedging of American puts"
17:50-18:20	**Florian Stebegg** "Robust pricing and hedging of American and Asian options in continuous time"
19:00for19:30	*Dinner at St John's College*

Tuesday 19th September – All talks held in AHL Lecture Theatre	
9:15-9:45	**Bruno Levy** "Geometric algorithms for computational optimal transport"
9:45-10:15	**Gaoyue Guo** "Numerical computation of martingale optimal transportation"
10:15-10:45	**Xiaolu Tan** "Numerical approximation of the MOT problem and the optimal SEP"
10:45-11:15	*Coffee - conservatory*
11:15-12:15	**Alexander Cox** "Discretisation and duality of optimal Skorokhod embedding problems"
12:15-12:45	**David Promel** "On Skorokhod embeddings and Poisson equations"
12:45-13:45	*Lunch - conservatory*
13:45-14:15	**Sigrid Kallblad** *"Measure-valued martingales and optimality of the Skorokhod embedding problem"*
14:15-14:45	**Hadrien De March** "Some results about multi-dimensional martingale optimal transport"
14:50-15:20	**Pietro Siorpaes** "Structure of martingale transports in finite dimensions"
15:20-15:50	**Tongseok Lim** "Dual attainability for the martingale transport problem"
15:50-16:30	*Coffee - conservatory*
16:30-18:15	***Open Problems session and Discussion***

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Abstracts

Nicolas Juillet (University of Strasbourg) *"Markovinification of the quantile process"*

Kellerer's theorem (1972) states that, for measures (μ_t)_t in (increasing) convex order, there exists a Markovian martingale, respectively a Markovian submartingale, with marginals μ_t . In view of the Doob–Meyer decomposition theorem it may appear surprising that the corresponding statement for the stochastic order and Markovian increasing processes has never been established. Note that, due to the atomic part of the measures μ_t , the problem can not trivially be solved by using cumulative distribution functions and also Kellerer's proof can not be readily adapted. That the statement is yet true without further assumption is the result of a joint work with Charles Boubel, that I will present in this talk. In particular, I will explain how Kellerer's proof can be modified as less as possible to obtain a common proof for all three statements on Markovian martingales, submartingales and increasing processes. In a second part, I will distinguish a special process, that we called Markov-quantile process, that coincides with the quantile process when all the measures are continuous. In the context of (non-martingale) optimal transport and the transport equation, it opens the way to proving uniqueness statements for optimal Markovian processes attached to a curve (μ_t)_t, completing previous works by Ambrosio–Gigli–Savare and Lisini.

Christian Leonard (Paris Nanterre University) *"From entropic to deterministic optimal transport"*

In the early 2000, T. Mikami proved the convergence of entropic transport to deterministic quadratic transport, when a fluctuation parameter vanishes, by means of h−processes. Inspired by some features of Mikami's method and applying elementary large deviation technics to the Schrodinger problem, we extend Mikami's result to any continuous cost function and recover the standard results about optimal transport without appealing to c−monotonicity.

Martin Huesmann (University of Bonn) *"A Benamou-Brenier approach to martingale optimal transport"*

We introduce and analyze a continuous time martingale optimal transport problem which can be seen as the "Benamou-Brenier" formulation of MOT. It is naturally linked to the discrete MOT problem via a weak length relaxation. Based on a duality result we derive the "geodesic equation" which allows us to explicitly construct optimizers (based on joint work with Dario Trevisan).

Julio Backhoff Veraguas & Mathias Beiglboeck (TU Vienna) *"Martingale Benamou-Brenier: a probabilistic perspective (I) & (II)"*

In classical optimal transport, the contributions by Benamou, Brenier and McCann (among others) regarding the time-dependent version of the problem, have had a lasting impact in the field and led to many applications. It is remarkable that this is achieved even if in continuoustime classical optimal transport mass/particles only travel in straight lines. Of course this fails to happen when we consider (continuous-time) martingale optimal transport. In this talk we discuss the existence of a martingale analogue to McCann's interpolation and the BenamouBrenier formula from a probabilistic - as opposed to analytic - point of view. This remarkable martingale is characterized by very natural optimality and geometric properties, leading us to say that it provides a canonical martingale way to connect two measures in convex order. This is joint work with M. Huesmann and S. Kallblad.

Pierre Henry-Labordere (Societe Generale) *"Quantum Optimal Transport"*

We study quantum entanglement through a non-commutative generalization of the optimal transport.

Halil Mete Soner (ETH Zurich) *"Robust duality - a survey"*

In the past decade many interesting convex duality results between the super-hedging functional and the pricing functionals have been established in the contexts of martingale optimal transport, robust hedging and in model-independent finance. All these problems have in common is that the stock price process is a martingale under all pricing functionals and there is no measure that dominates them. Then, these problems are set either in discrete or continuous time and are constrained through other information on prices. A variety of techniques have been used as well. In this talk, I concentrate on the duality theorems and provide a survey of the recent result and the different techniques.

David Hobson (University of Warwick) *"Robust hedging of American puts"*

We consider the highest possible price of an American put given the prices of European puts in the robust pricing, model-independent framework. The aim is to use the specific put structure of the American option to derive the optimal model and the optimal superhedge, and to do so wherever possible by drawing pictures.

Florian Stebegg (Columbia University) *"Robust Pricing and Hedging of American and Asian Options in Continuous Time"*

We consider the Robust Pricing Problem for a class of options encompassing American, Asian, Bermudan and European Options in a martingale optimal transport setting for c`adl`ag processes. We prove strong duality of the pricing and hedging problem and the existence of an optimal pathwise hedge. Our approach provides insight into the structure of primal and dual optimizers and provides a remarkable parity of the price bounds in this setting. For finitely supported marginal laws we are able to reduce the problem to a semi-infinite linear program and in the case of piecewise linear payoffs (risk reversal, butterfly spread, etc.) a finite linear program, making it very amenable to numerical optimization methods.

Bruno Levy (Inria Nancy) *"Geometric algorithms for computational optimal transport"*

In this talk, I will present recent advances in computational optimal transport. I fill focus on the semi-discrete case (probability density transported to sum of Dirac masses). It is well known that the Kantorovich dual is a concave function [Aleksandrov, Aurenhammer, Mc Cann] that can be optimized by a Newton algorithm [Kitagawa Merigot]. I will present efficient algorithms to compute the Hessian matrix of the Kantorovich dual in 2D and 3D, as well as numerical methods to solve for the Newton step. I will also present some ideas to adapt this type of numerical algorithms to other problems, such as Martingale Optimal Transport.

Gaoyue Guo (University of Oxford) *"Numerical computation of martingale optimal transportation"*

We provide a numerical method for solving the martingale optimal transport problem. The scheme considers the approximation of marginal distributions, through which the primal problem could be approximated by a LP problem with the relaxation of martingale constraint.

Xiaolu Tan (Paris-Dauphine University) *"Numerical approximation of the MOT problem and the optimal SEP"*

We consider the numerical approximation problems for the continuous time MOT problem and the optimal SEP. Based on the numerical methods for PDEs and the gradient projection algorithm, we obtain a convergent numerical scheme to approximate the optimal value of the original continuous time problems.

Alexander Cox (University of Bath) *"Discretisation and Duality of Optimal Skorokhod Embedding Problems"*

We prove a strong duality result for a linear programming problem which has the interpretation of being a discretised optimal Skorokhod embedding problem, and we recover this continuous time problem as a limit of the discrete problems. With the discrete setup we show that for a suitably chosen objective function, the optimiser takes the form of a hitting time for a random walk. In the limiting problem we then reprove the existence of the Root, Rost, and cave embedding solutions of the Skorokhod embedding problem. The main strength of this approach is that we can derive properties of the discrete problem more easily than in continuous time, and then prove that these properties hold in the limit. For example, the strong duality result gives dual optimisers, and our limiting arguments can be used to derive properties of the continuous time dual functions, known to represent a superhedging portfolio.

David Promel (University of Oxford) *"On Skorokhod Embeddings and Poisson Equations"*

We consider the Skorokhod embedding problem for a Levy process L with initial distribution μ_0 and for a target probability measure μ_1, i.e. the task is to find a non-randomized stopping time τ such that L_τ has the law μ_1. Assuming μ_0 and μ_1 have positive densities, we propose necessary and sufficient conditions for the existence of an embedding in terms of the Poisson equation associated to the adjoint operator of the Levy process. Furthermore, we give a fairly explicit construction of the stopping time using the solution of the Poisson equation. The talk is based on a joint work with Leif Doring, Lukas Gonon and Oleg Reichmann.

Sigrid Kallblad (TU Vienna) *"Measure-valued martingales and optimality of the Skorokhod embedding problem"*

We consider (probability-)measure valued processes, which we call MVMs, which have a natural martingale structure. Following previous work of Eldan and Cox-Kallblad, these processes are known to have a close connection to the solutions to the Skorokhod Embedding Problem. Here, we consider properties of these processes, and in particular, we are able to show that the MVMs connected to the Bass and Root embeddings have natural measure-valued analogues which also possess natural optimality properties. Based on joint work with M. Beiglboeck, A.M.G. Cox and M. Huesmann.

Hadrien De March (Ecole Polytechnique) *"Some results about multi-dimensional martingale optimal transport"*

Martingale optimal transport is an extension of classical optimal transport that includes a martingale constraint on the considered coupling. New geometric constraints arise from this martingaleness. The dual problem may not provide duality if it is taken pointwise as it does in classical optimal transport. There arises a decomposition of the space in "irreducible components". We study the existence and the properties of these irreducible components and show how they allow to have componentwise duality and existence of the dual optimizer. We also provide the local structure of the optimal probability, and show that its degeneracy bring numerical instabilities.

Pietro Siorpaes (Imperial College London) *"Structure of martingale transports in finite dimensions"*

Martingale optimal transport is a variant of the classical optimal transport problem where a martingale constraint is imposed on the coupling. In a recent paper, Beiglboeck, Nutz and Touzi show that in dimension one there is no duality gap and that the dual problem admits an optimizer. A key step towards this achievement is the characterization of the polar sets of the family of all martingale couplings. Here we aim to extend this characterization to arbitrary finite dimension through a deeper study of the convex order.

Tongseok Lim (University of Oxford) *"Dual attainability for the martingale transport problem"*

On the real line, Beiglboeck-Nutz-Touzi showed that the martingale optimal transport problem attains dual optimizers in q.s. sense. We show that under certain assumptions on the cost, the dual optimizers exist in the classical sense. Furthermore, these dual optimizers can have some regularity under further assumption on the cost. These assumptions are met, for example, if the cost is C^2 and marginals are compactly supported, but not necessarily irreducible.

List of Participants	
Beatrice Acciaio	b.acciaio@lse.ac.uk
Anna Aksamit	anna.aksamit@maths.ox.ac.uk
Julio Backhoff Veraguas	jubackhoff@gmail.com
Mathias Beiglboeck	mathias.beiglboeck@gmail.com
Luciano Campi	L.Campi@lse.ac.uk
Alexander Cox	A.M.G.Cox@bath.ac.uk
Hadrien De March	hadrien.de-march@polytechnique.edu
Shuoqing Deng	deng@ceremade.dauphine.fr
Gaoyue Guo	gaoyue.guo@maths.ox.ac.uk
Pierre Henry-Labordere	pierre.henry-labordere@sgcib.com
David Hobson	D.Hobson@warwick.ac.uk
Martin Huesmann	huesmann@iam.uni-bonn.de
Nicolas Juillet	nicolas.juillet@math.unistra.fr
Sigrid Kallblad	sigrid.kaellblad@tuwien.ac.at
Matti Kiiski	matti.kiiski@gmail.com
Christian Leonard	christian.leonard@parisnanterre.fr
Bruno Levy	Bruno.Levy@inria.fr
Tongseok Lim	tongseok.lim@maths.ox.ac.uk
Michael Monoyios	Michael.monoyios@maths.ox.ac.uk
Ariel Neufeld	ariel.neufeld@math.ethz.ch
Dominykas Norgilas	D.Norgilas@warwick.ac.uk
Harald Oberhauser	harald.oberhauser@maths.ox.ac.uk
Jan Obloj	jan.obloj@maths.ox.ac.uk
David Promel	david.proemel@math.ethz.ch
Benjamin Robinson	B.A.Robinson@bath.ac.uk
Halil Mete Soner	mete.soner@math.ethz.ch
Piero Siorpaes	p.siorpaes@imperial.ac.uk
Florian Stebegg	florian.stebegg@columbia.edu
Xiaolu Tan	xiaolu.tan@gmail.com
Johannes Wiesel	johannes.wiesel@sjc.ox.ac.uk

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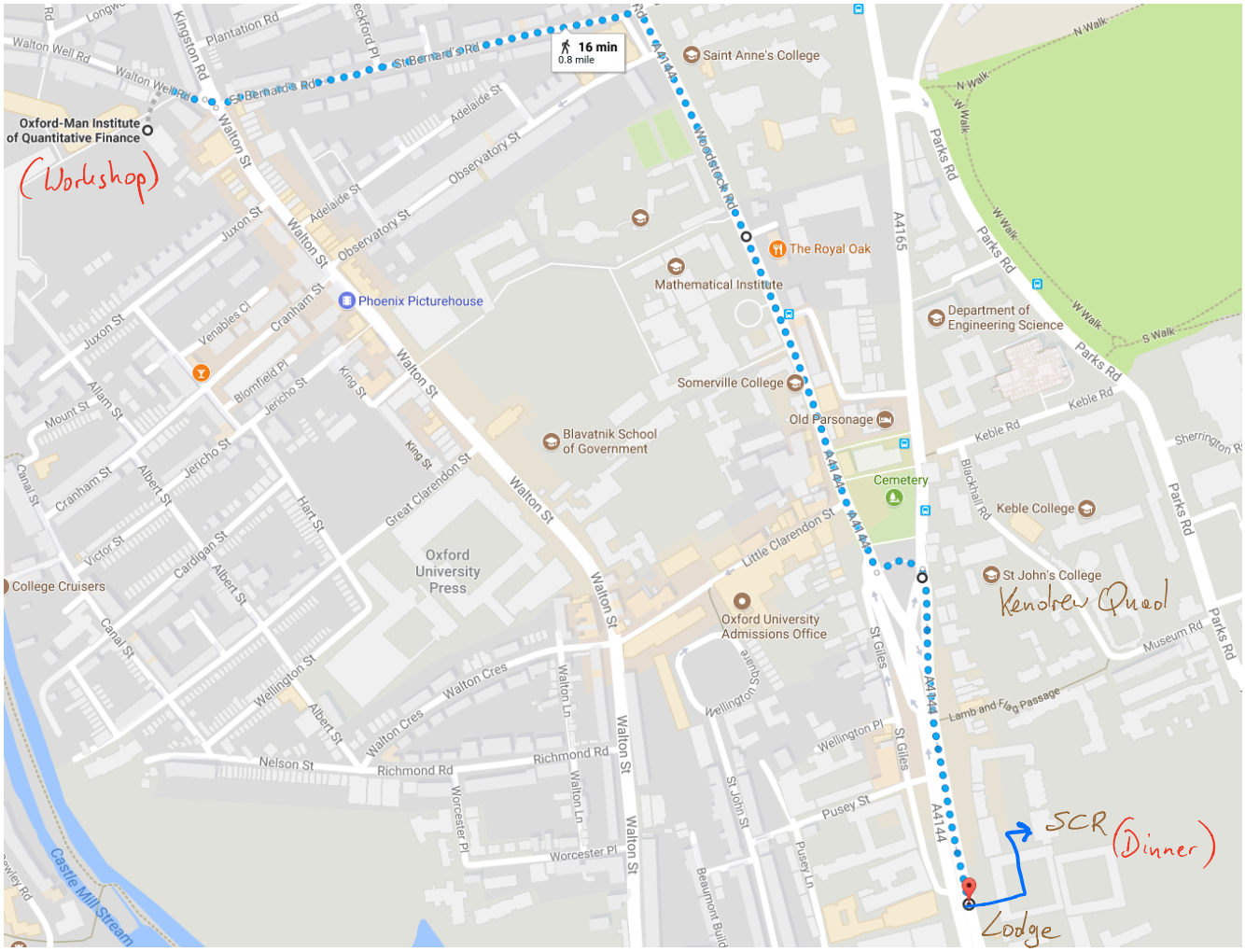
WORKSHOP DINNER

The Workshop dinner will be on Monday evening at St John's College on St Giles'. We will start with drinks at 19:00 in the SCR and follow with dinner at 19:30 in the Hall. Please be sure to arrive by 19:15 at the very latest.



To reach the SCR you enter through the main Lodge, head left into a second quad and then the SCR doors are on your right (big black door).

Porters (in the main Lodge) will be able to assist you if needed.

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