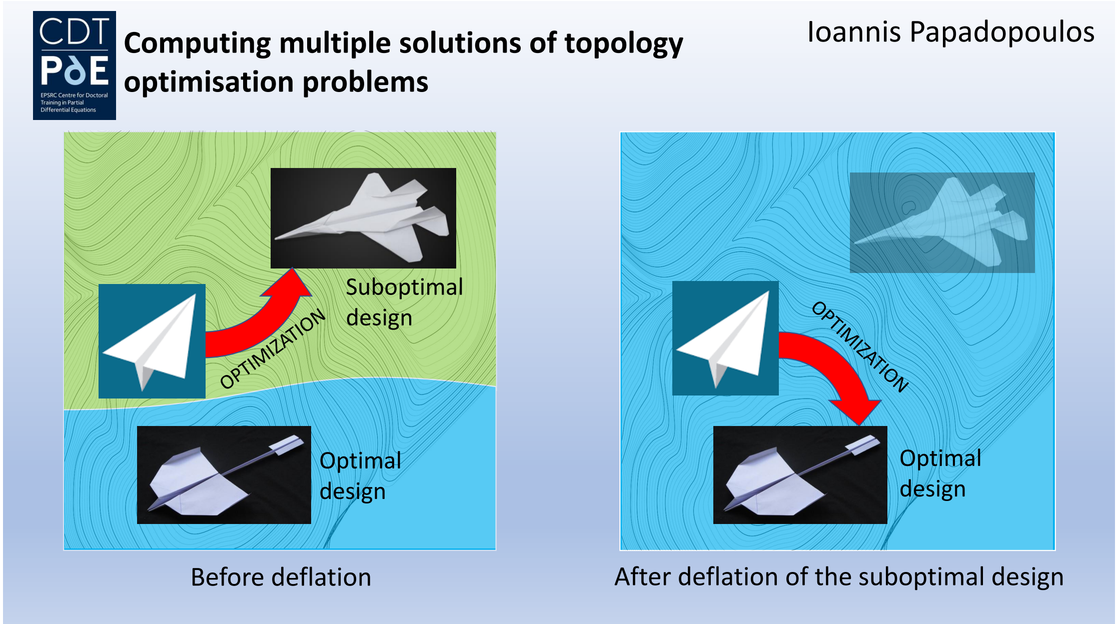

If I gave you a piece of paper, could you design *the* paper airplane that flies the furthest distance? What if you could also cut holes and rearrange the pieces? This is an example of a topology optimization problem. We can control the shape and topology of our paper airplane and we want to find the design that flies the furthest distance. The paper airplane is subject to physics described by partial differential equations. For example, the fluid flow of air over the wings of the plane could be described by the Navier–Stokes equations and the deformation of the wings of the plane could be described by the linear elasticity equations [1].

It may not be surprising that mathematically formulating this problem is quite tricky. You need to solve a problem with many constraints. Often, this means that the model supports multiple solutions; there are many paper airplanes of suboptimal design! Suppose you made an educated guess at a good initial design and over time you made small modifications to perfect it, gradually converging to an "optimal" solution. You are satisfied with the flight distance but, soon after, your friend shows you a completely different design which vastly outperforms your plane. They have discovered a more optimal solution! Iterative algorithms used in optimization encounter the same problem.

Engineers frequently use topology optimization to design lightweight airplane wings, car designs, bridges, buildings and medical devices. Finding the best design is important and plugging in the wrong initial guess into an iterative algorithm can converge to a suboptimal solution.

During my PhD, I am designing an algorithm that discovers multiple optimal designs with the same initial guess. Implementing a solver to find a single solution is an extremely interesting problem by itself and was a large part of my initial work. However, the core of the algorithm is a technique called deflation [2]. I pick an initial guess and optimize to find a solution. I then deflate this discovered solution away and with the same initial guess, run the iterative algorithm again. It can no longer converge to the deflated solution, so it will hopefully converge to a different one. This process can be repeated finding many solutions.

I have successfully applied the algorithm to pipes, bridges and levers [3]. Looking forward, I hope to apply the algorithm to several large-scale problems and have an impact in the engineering design world.

References

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[3] I. P. A. Papadopoulos, P. E. Farrell, and T. M. Surowiec, "Computing multiple solutions of topology optimization problems," *arXiv e-prints*, 2020. arXiv: `2004.11797 [math.NA]`.