Morse Homology (L24)

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Given a manifold, and a generically chosen function on it, how many critical points must the function have? Are the critical points related to the topology of the manifold? Can you tell two manifolds apart just from looking at one generic function on each manifold?

Morse homology is the analytic construction of the homology of a manifold, by studying critical points and gradient flowlines. It is based on Morse theory, which dates back to the 1930s, but its modern formulation was only developed in the 1980s. Morse homology is the model upon which many modern homology theories in geometry are based, such as: Floer homology, Lagrangian Floer homology, Seiberg-Witten theory, etc.

Morse homology is also a tool to study classical operations in algebraic topology. For example, cup product corresponds to counting gradient flowlines along a Y-shaped graph.

Topics covered include: basic differential topology, construction of the Morse chain complex, transversality and compactness for the moduli spaces of gradient trajectories, isomorphism with ordinary homology, topological quantum field theory structure.

Further topics may include: Novikov homology, infinite dimensional Morse theory and existence of closed geodesics, Floer homology theories.

Pre-requisite Mathematics

This course is an interplay of three subjects: differential geometry, algebraic topology and functional analysis. The differential geometry course is essential, but we only assume some working knowledge with the other two subjects. We will introduce most of the necessary analysis within the course (Sobolev spaces, Banach bundles, Fredholm operators, the Sard-Smale theorem).

Literature

1. A. Banyaga, D. Hurtubise, *Lectures on Morse Homology*, Kluwer, 2004.
2. R. Bott, *Morse Theory Indomitable*, Publ. Math. IHES 68, 99–114, 1988.
3. M. Guest, *Morse theory in the 1990s*, Invitations to geometry and topology, 2002, 146–207.
4. J. Milnor, *Morse Theory*, Princeton University Press, 1963.
5. M. Schwarz, *Morse Homology*, Birkhäuser, 1993.