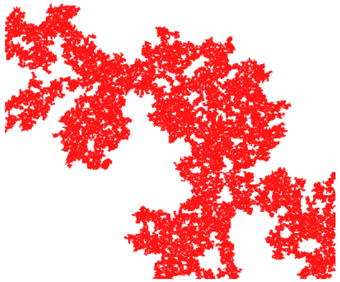
Crossing probabilities for planar percolation



Vincent Tassion

ETH zürich

Oxford Discrete Mathematics and Probability Seminar

Mai 25, 2021

Percolation: how does a fluid propagate in a random medium?

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How does water flow in rocks?

How do fires propagate in forests?


Bernoulli percolation [Broadbent and Hammersley, 1957]

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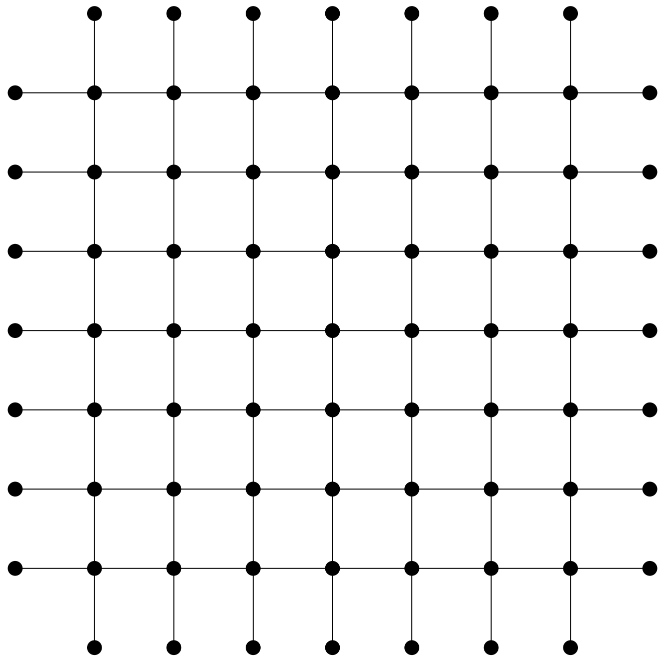

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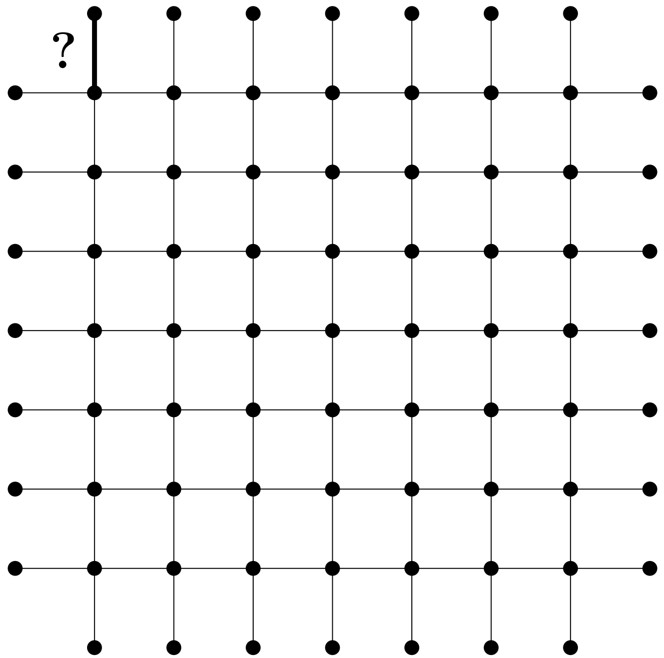

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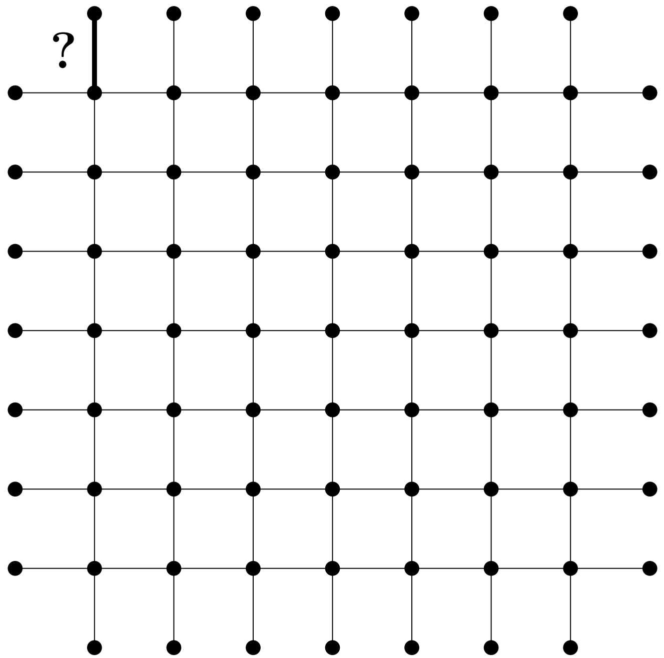

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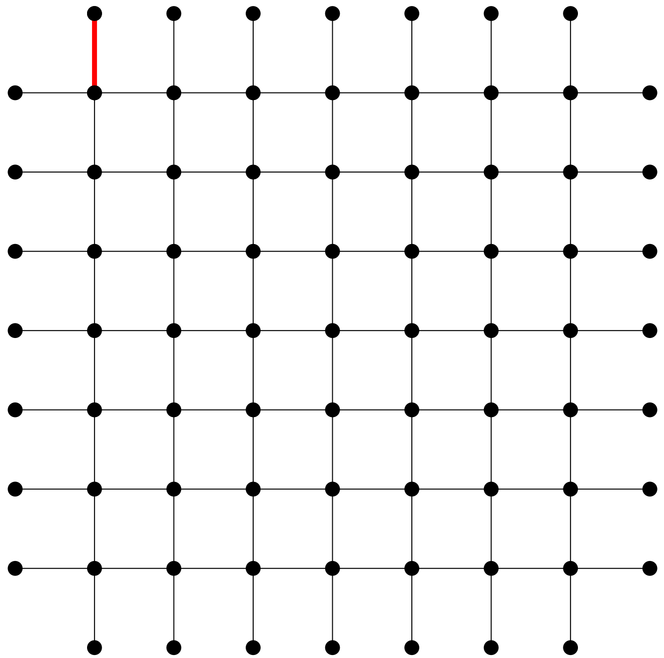

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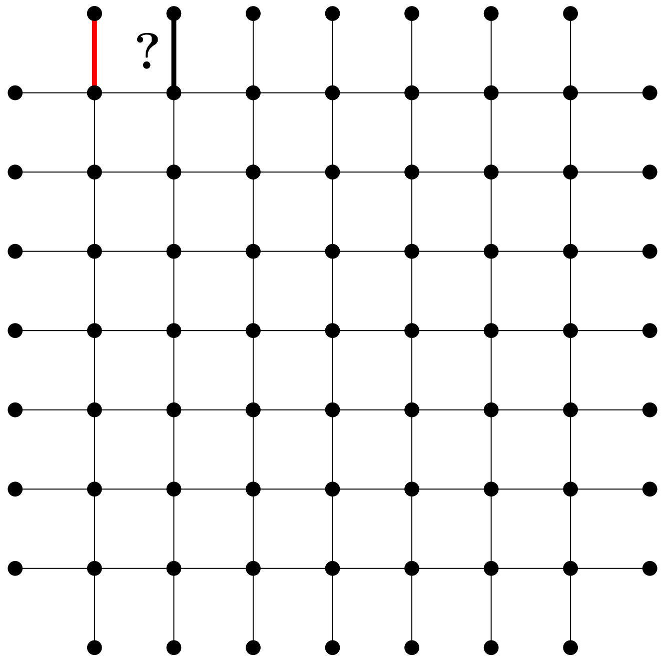

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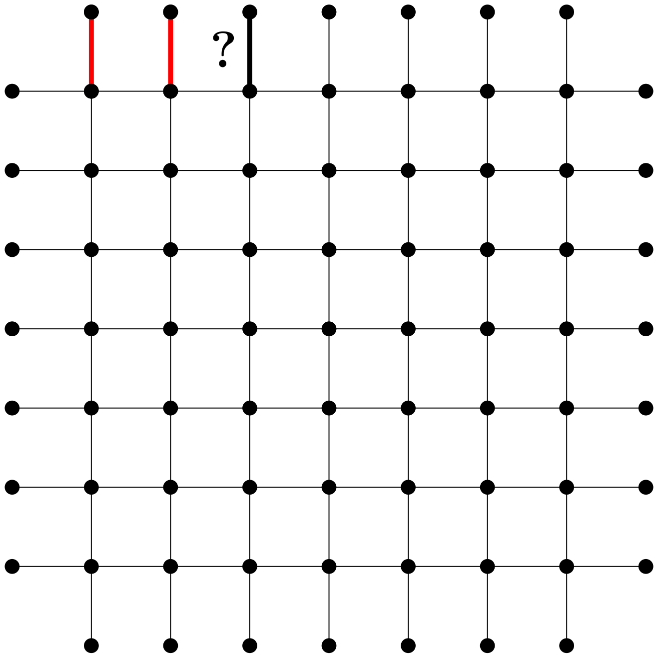

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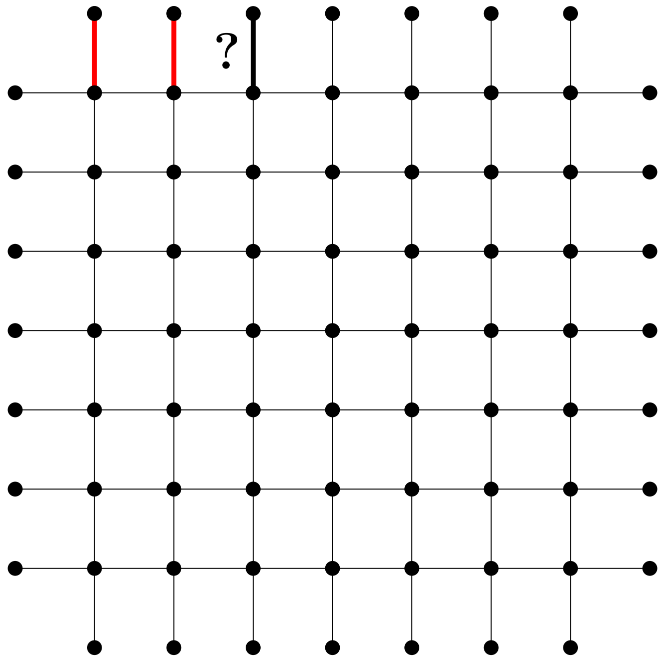

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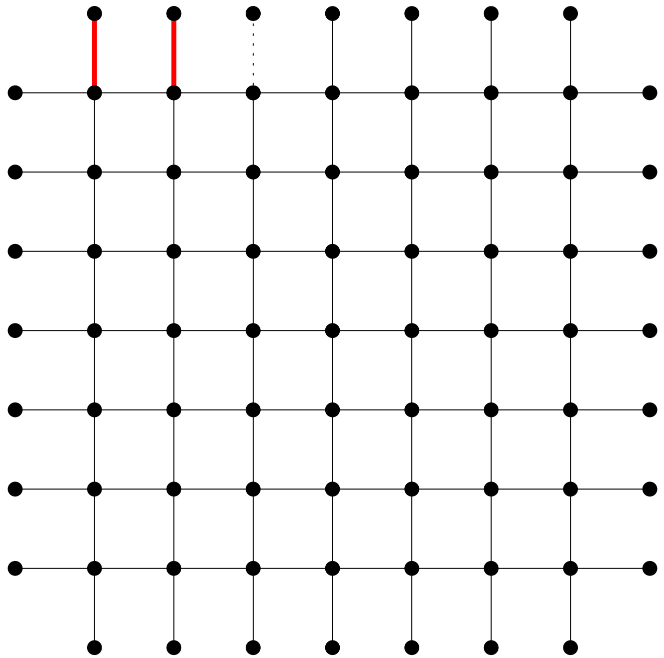

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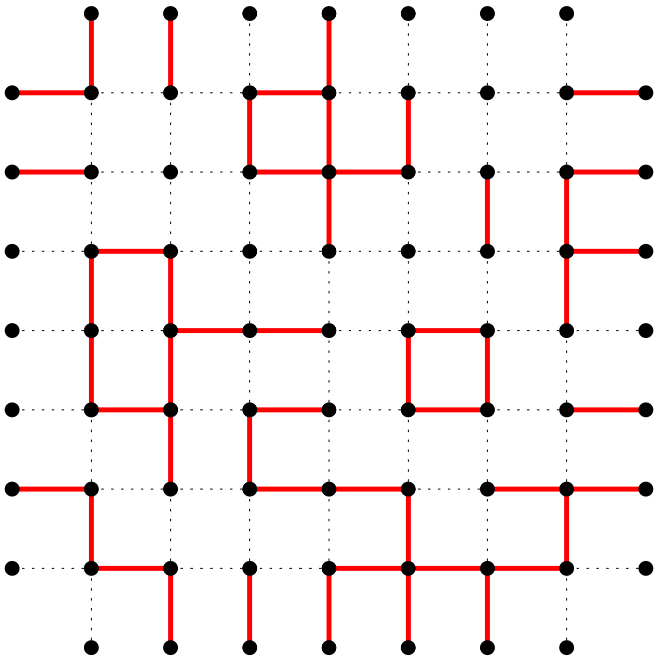

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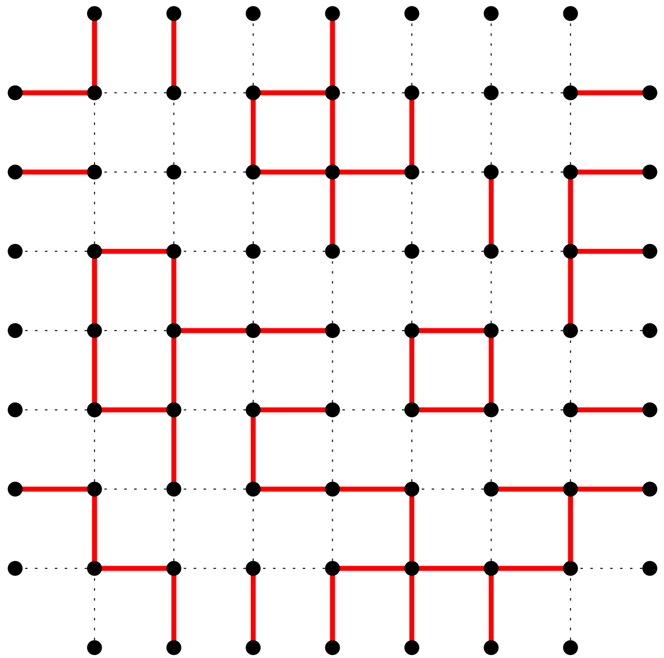

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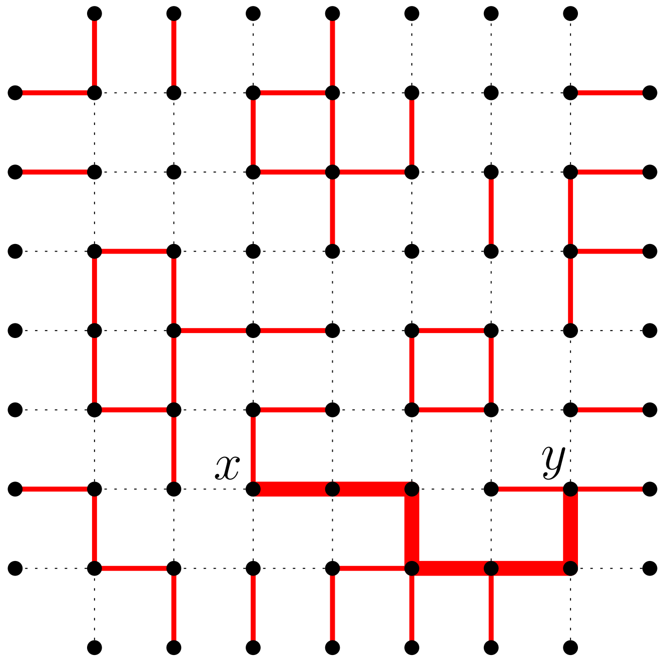

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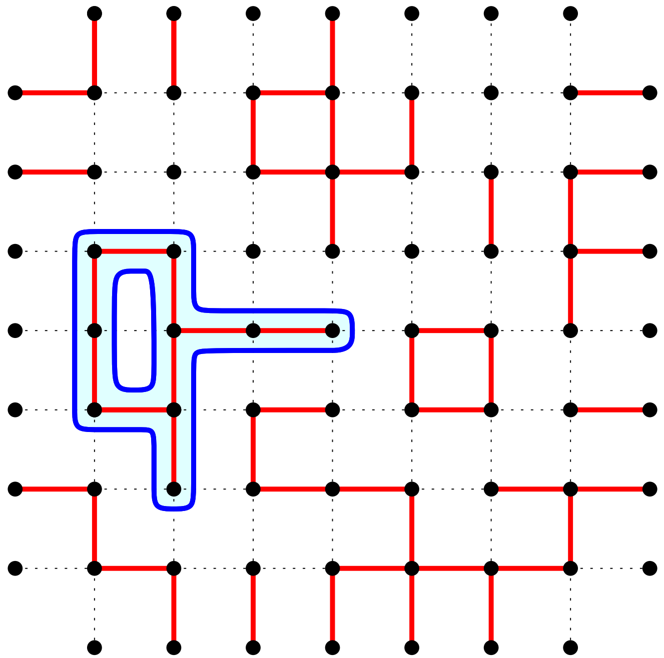

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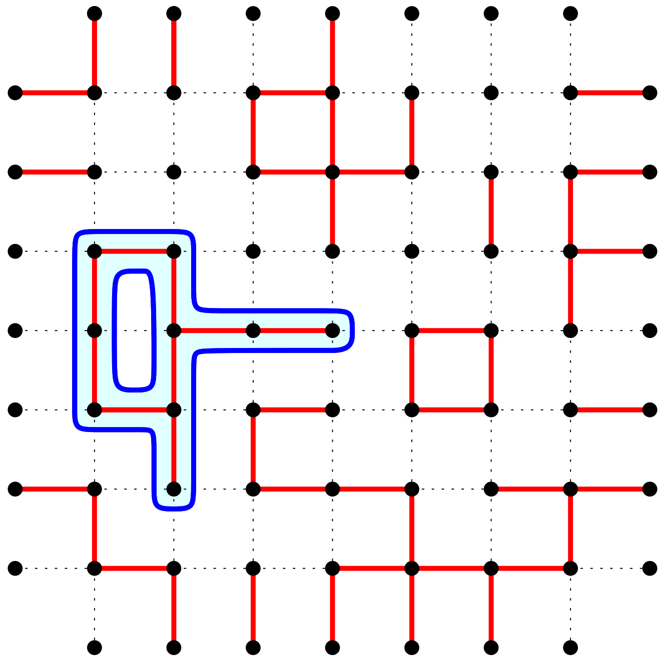

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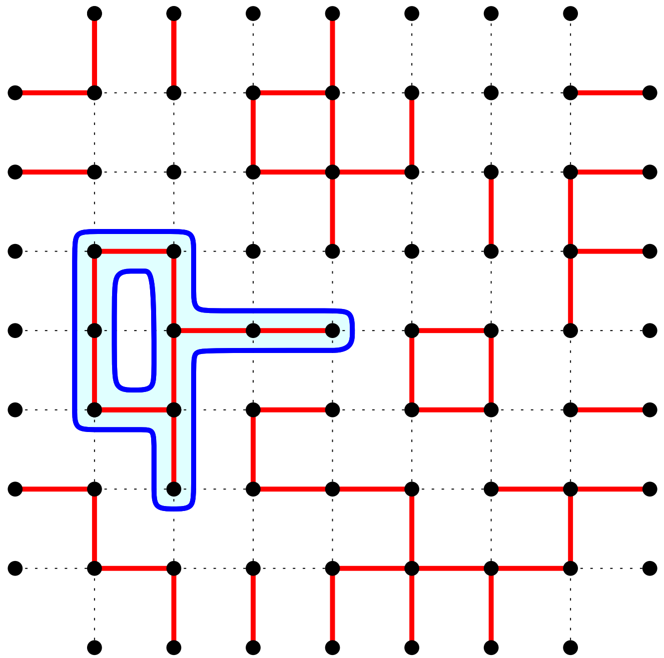

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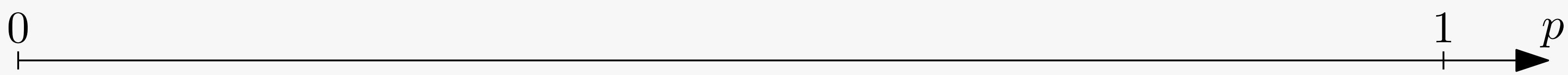

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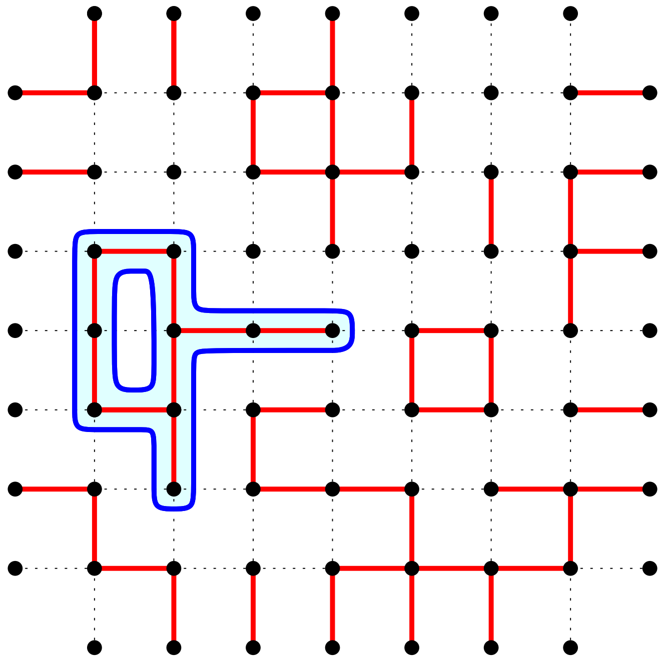

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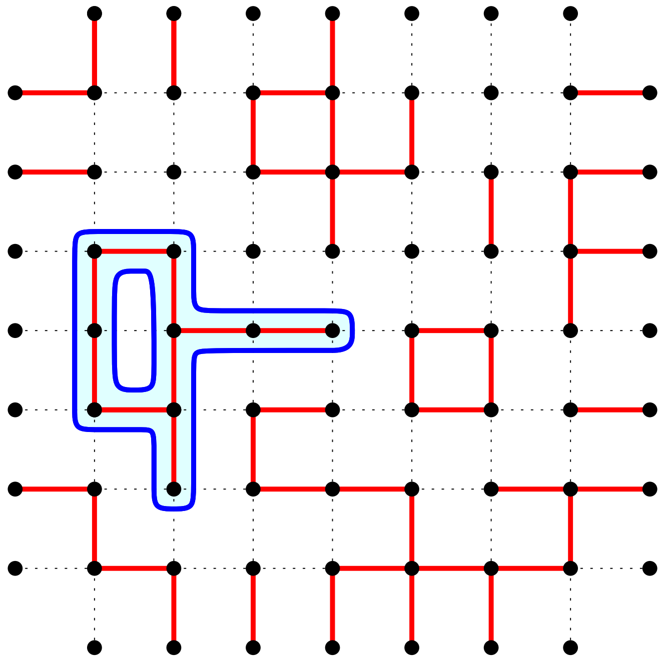

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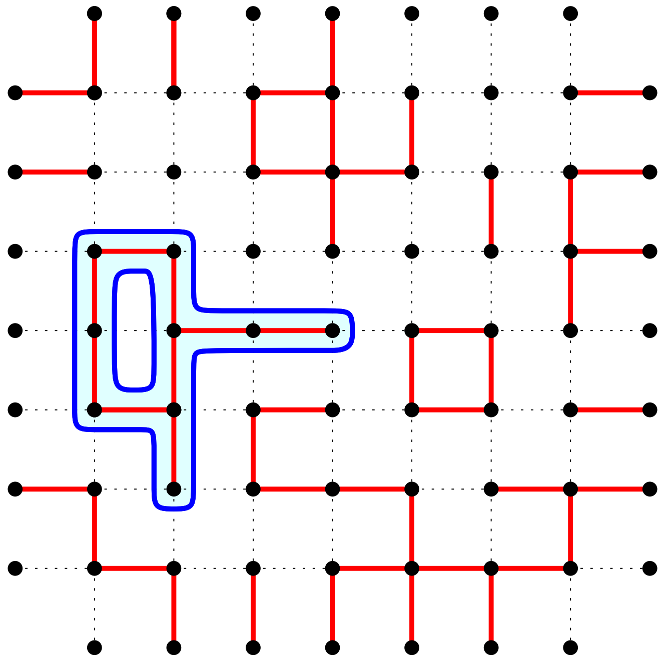

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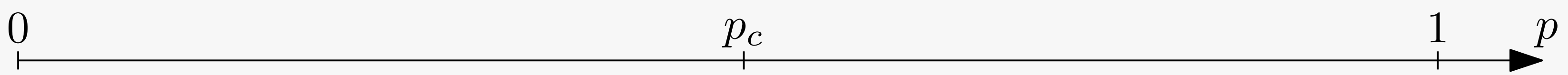

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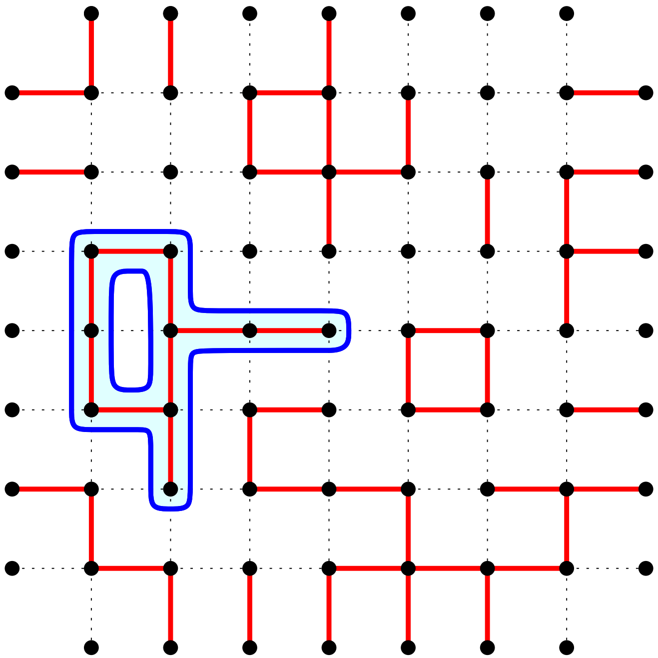

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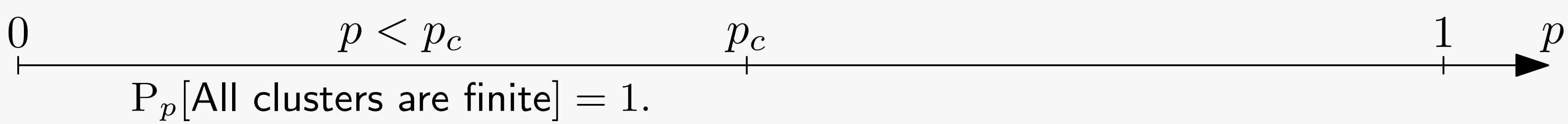

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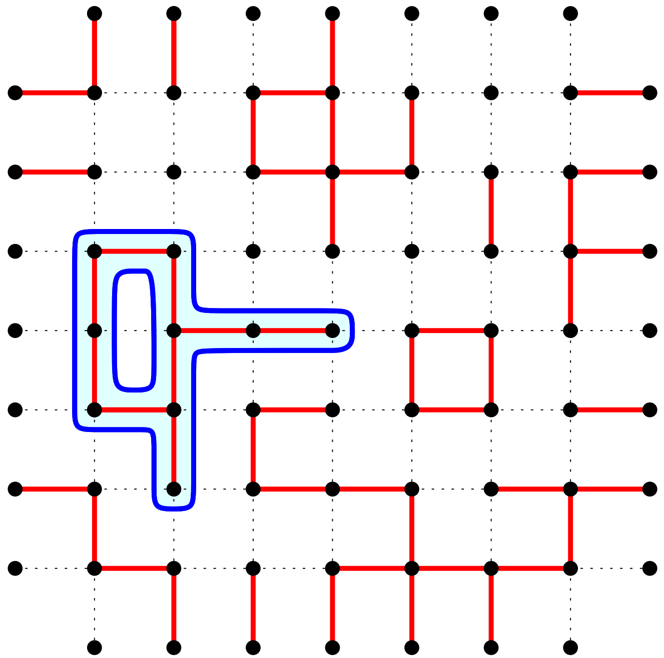

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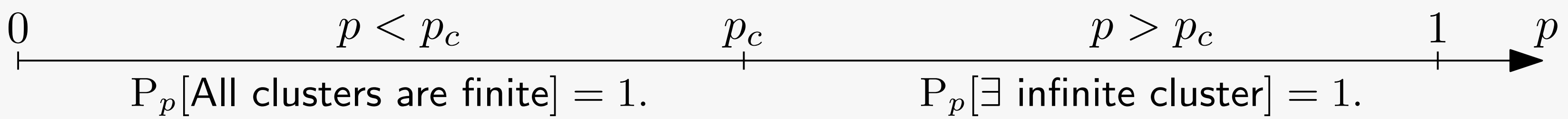
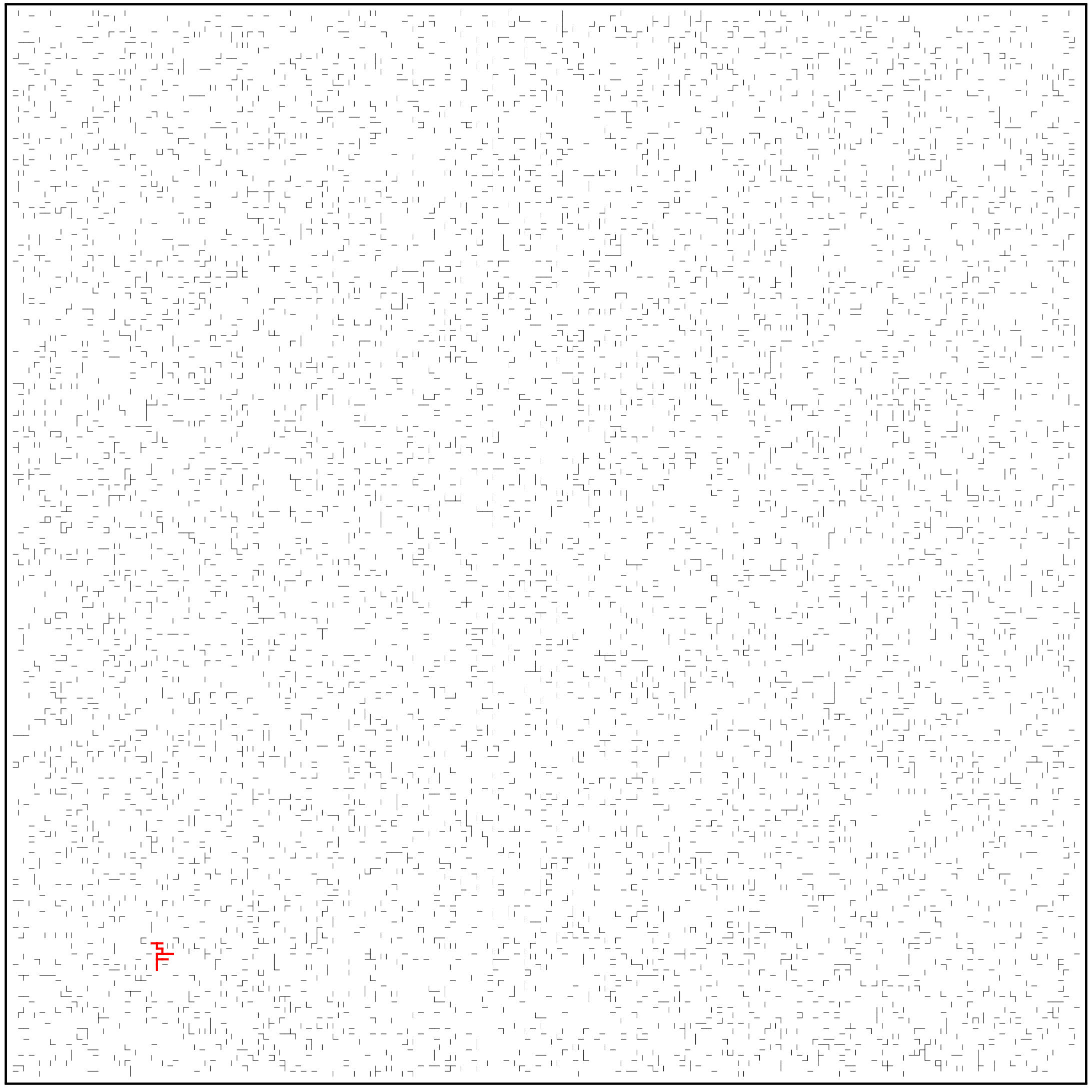
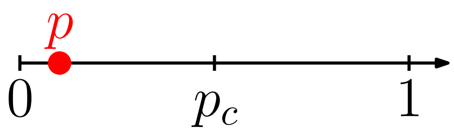


Simulations of the largest cluster in a box





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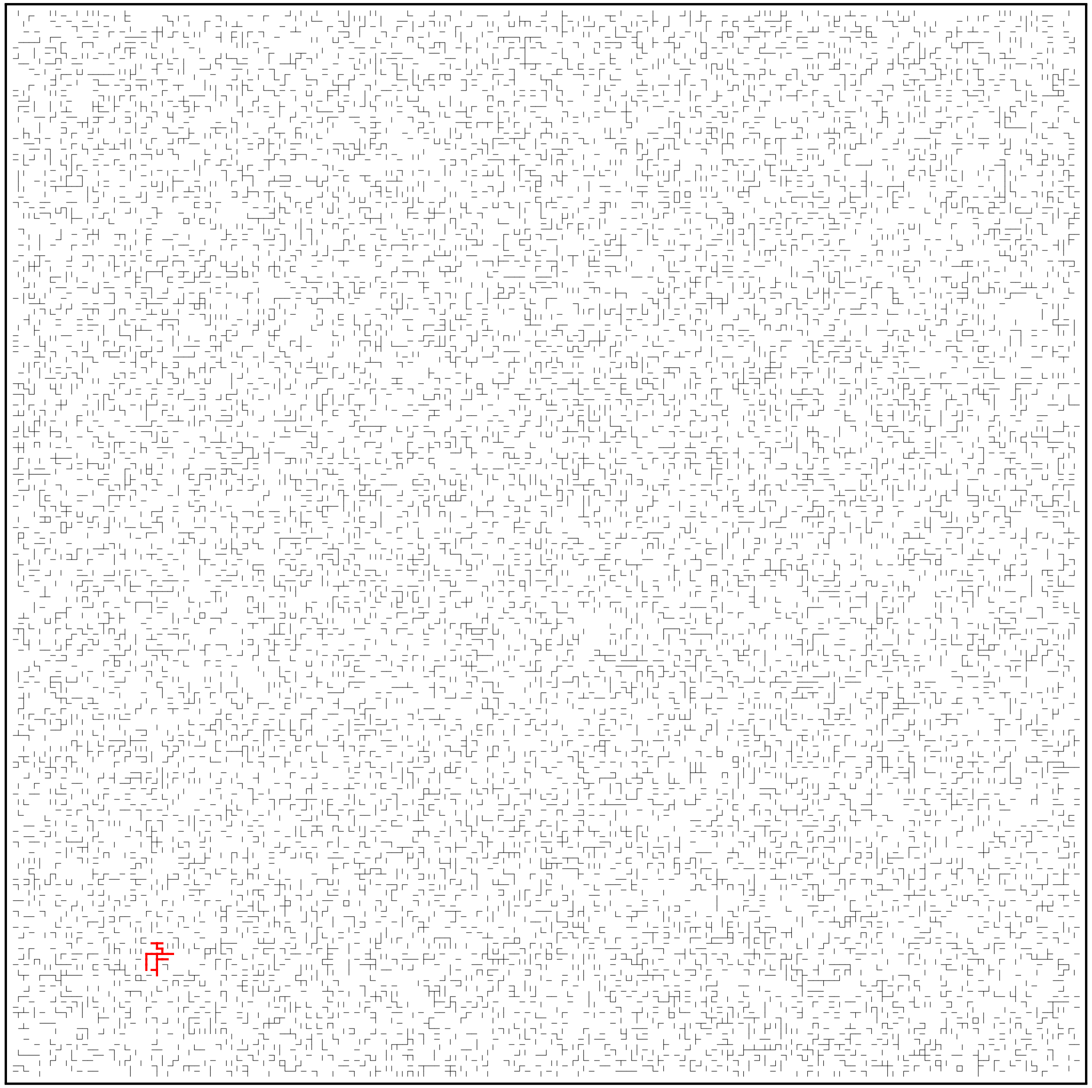

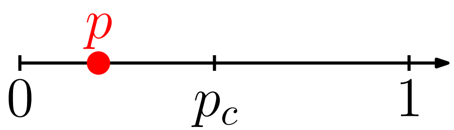

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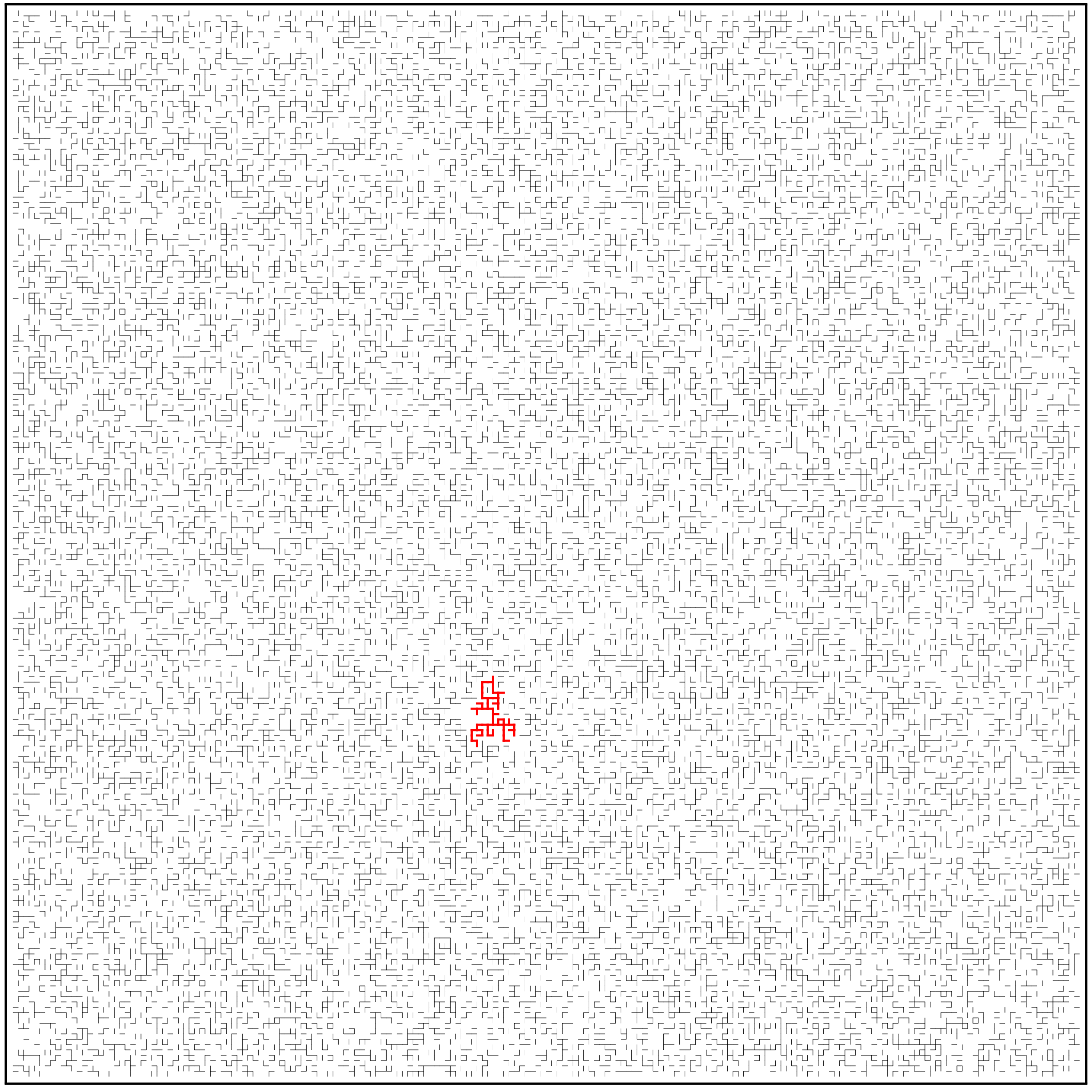

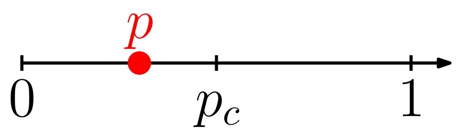

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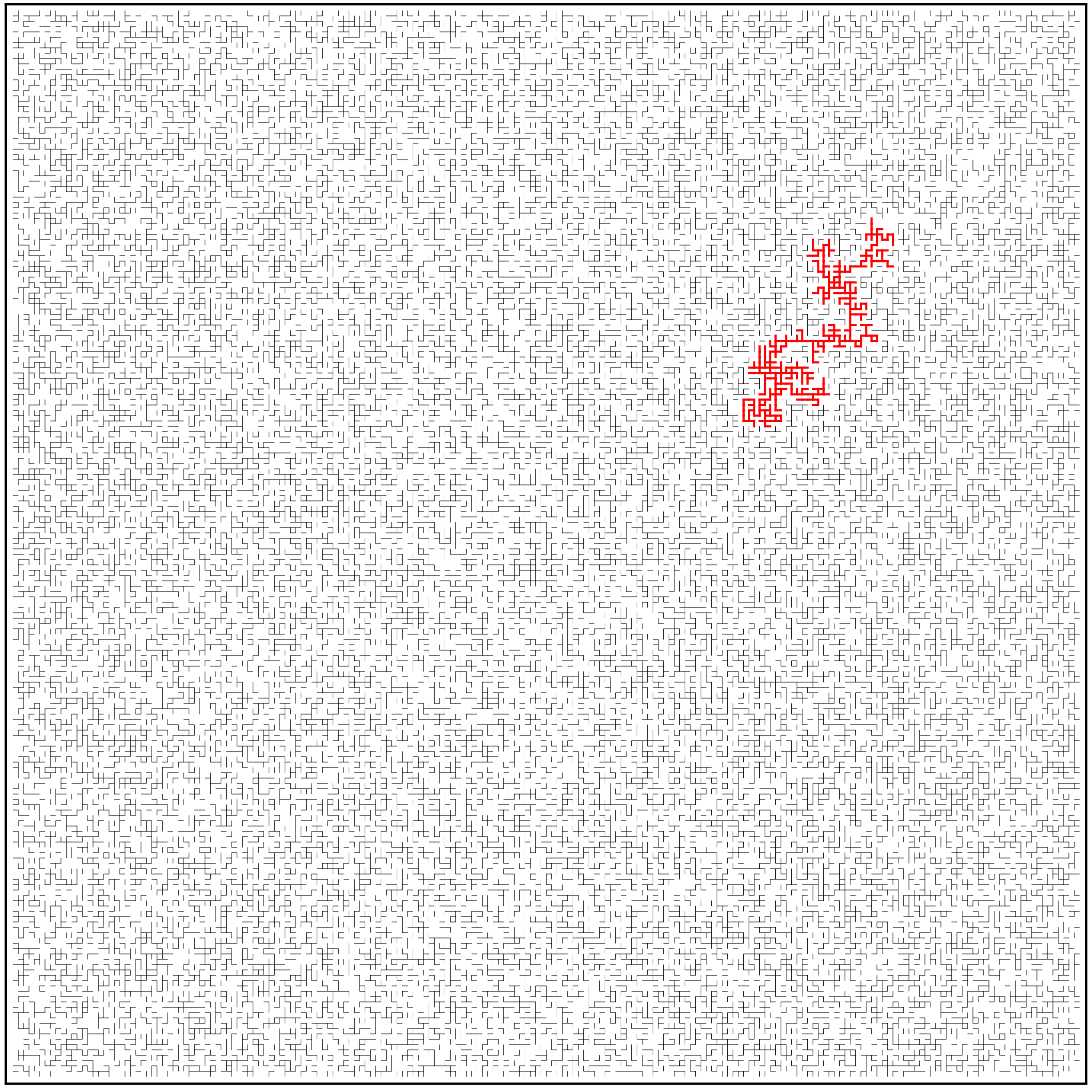

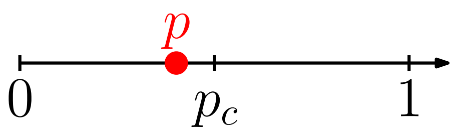

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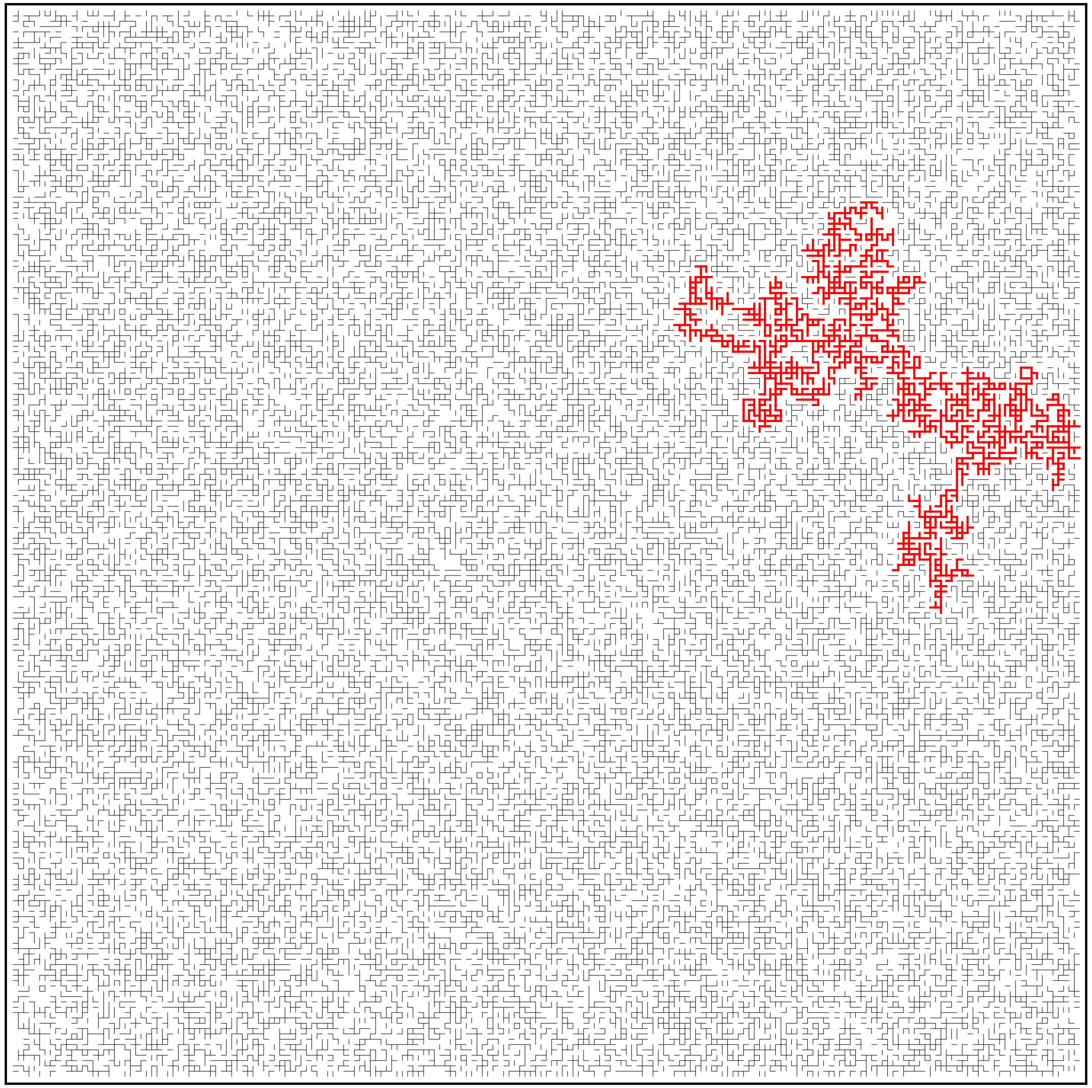

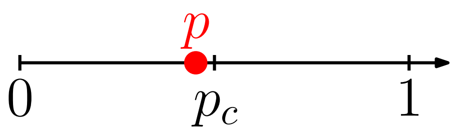

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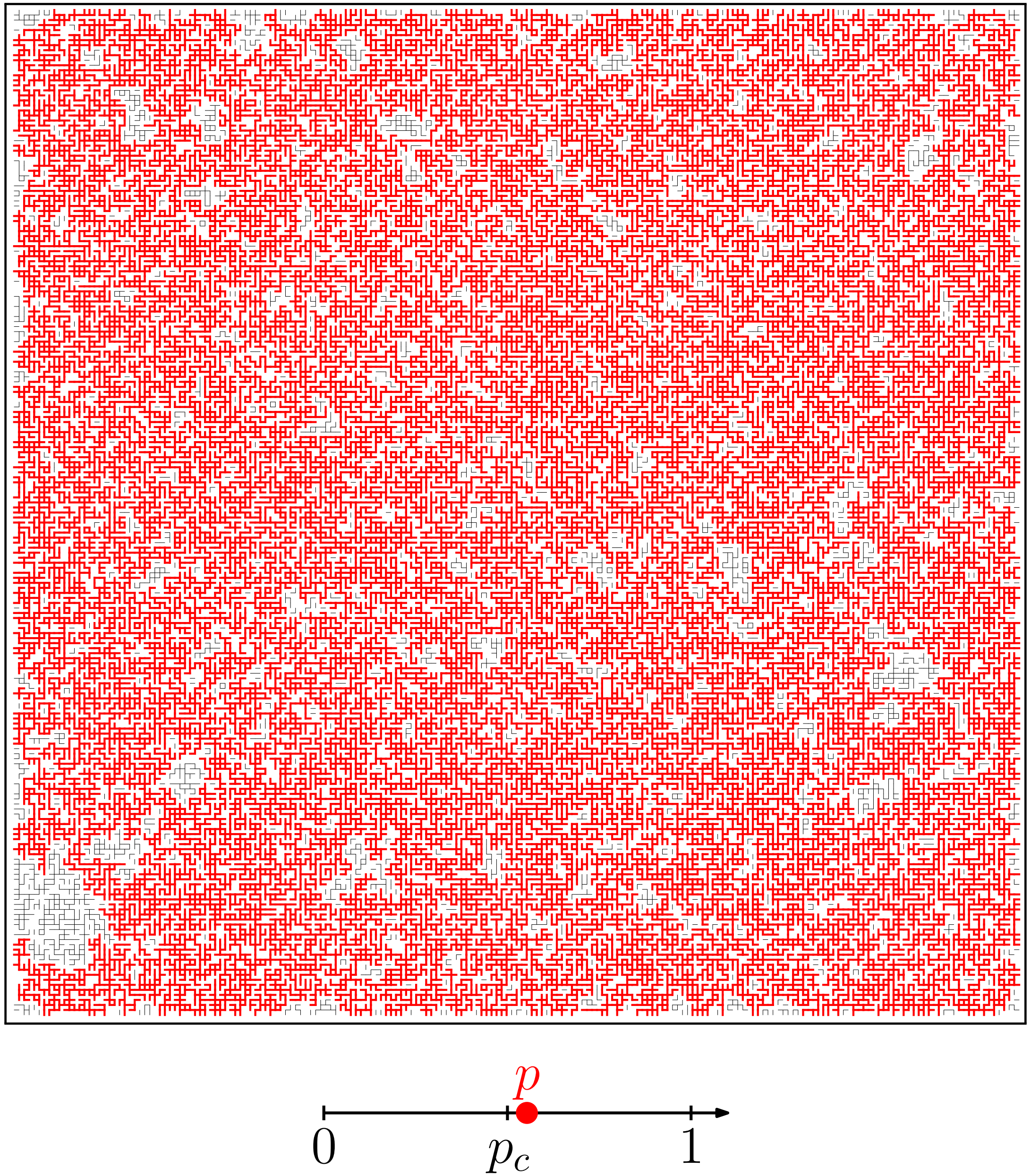

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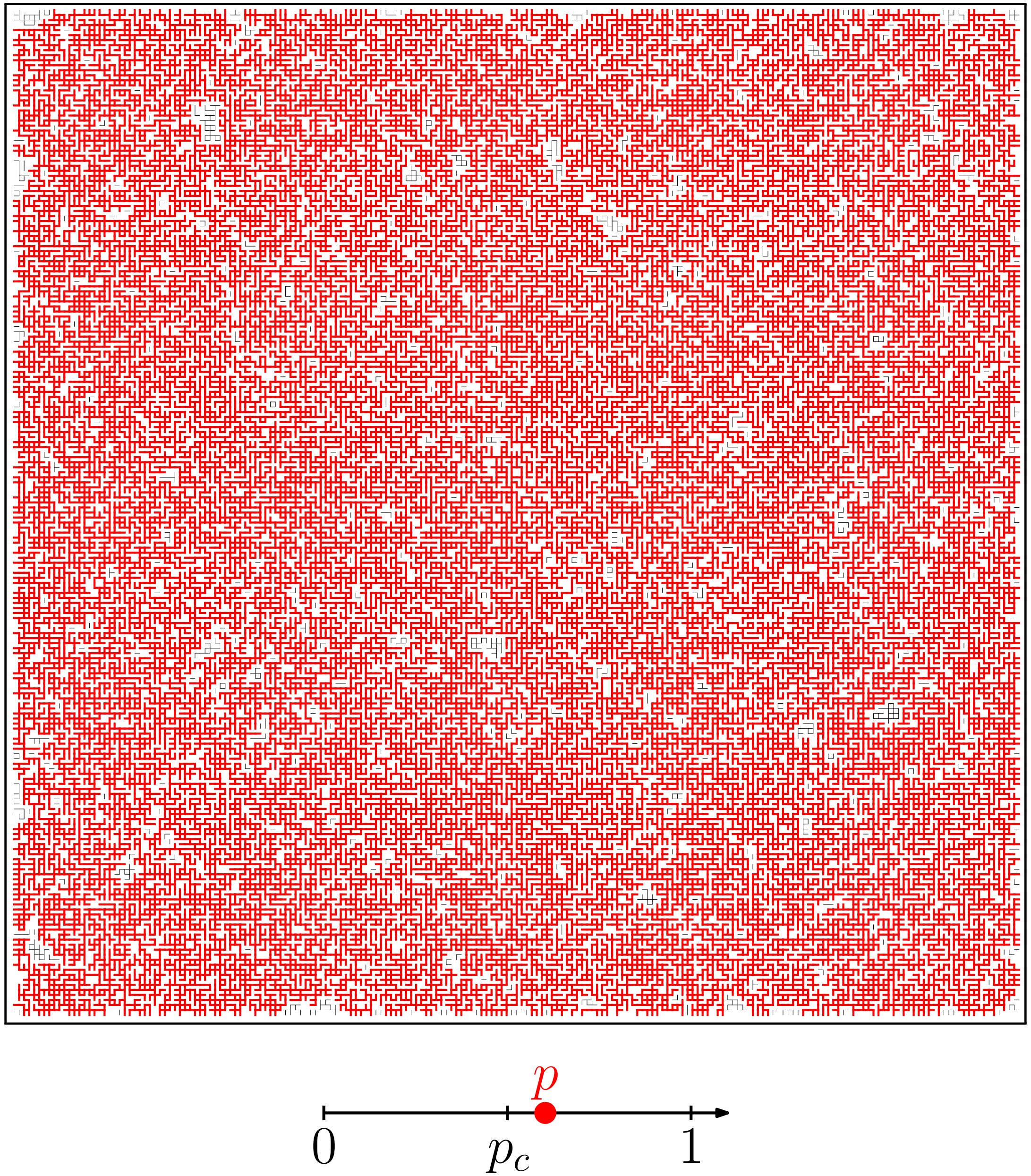

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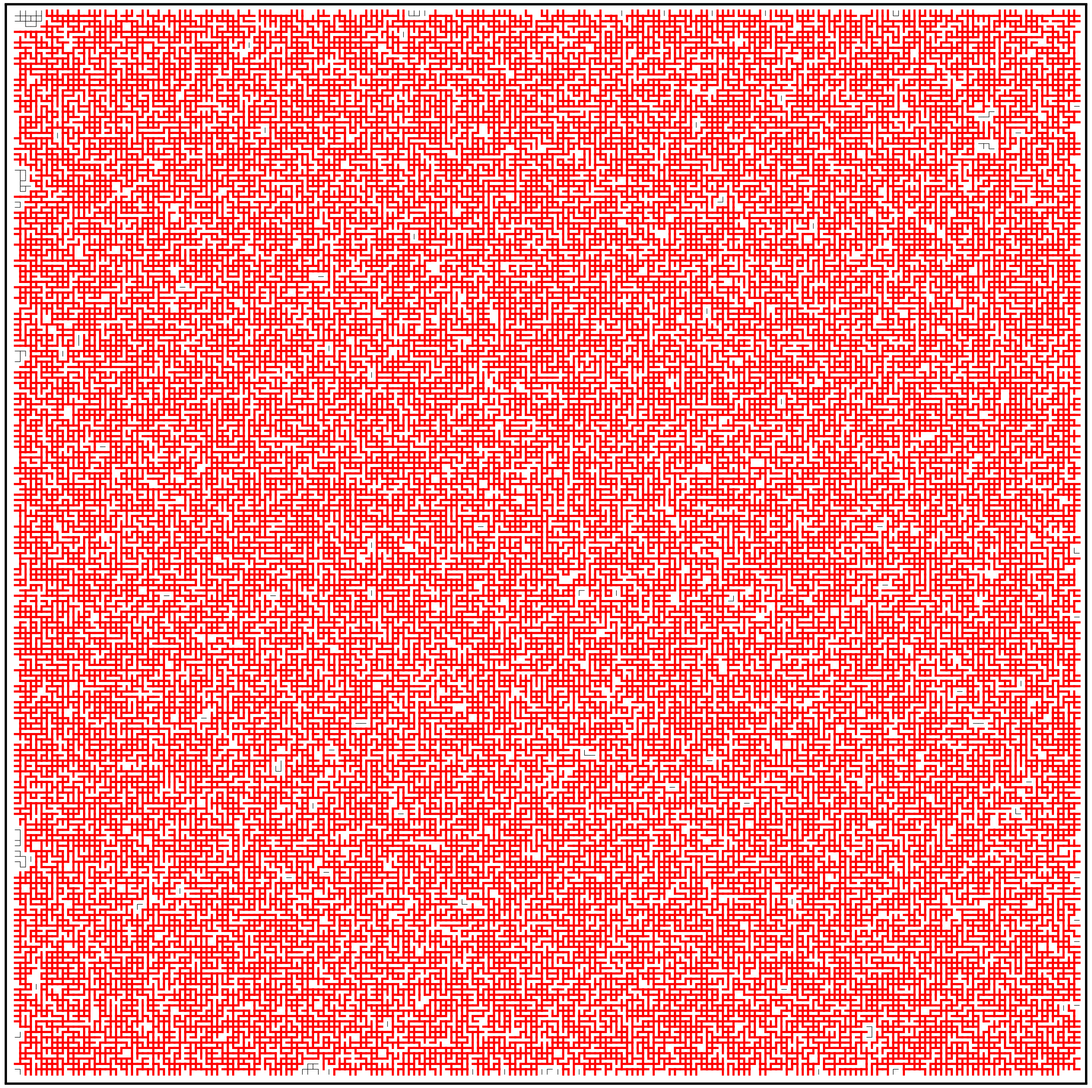

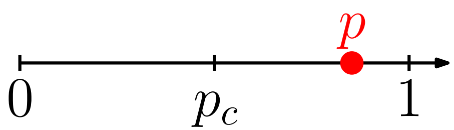

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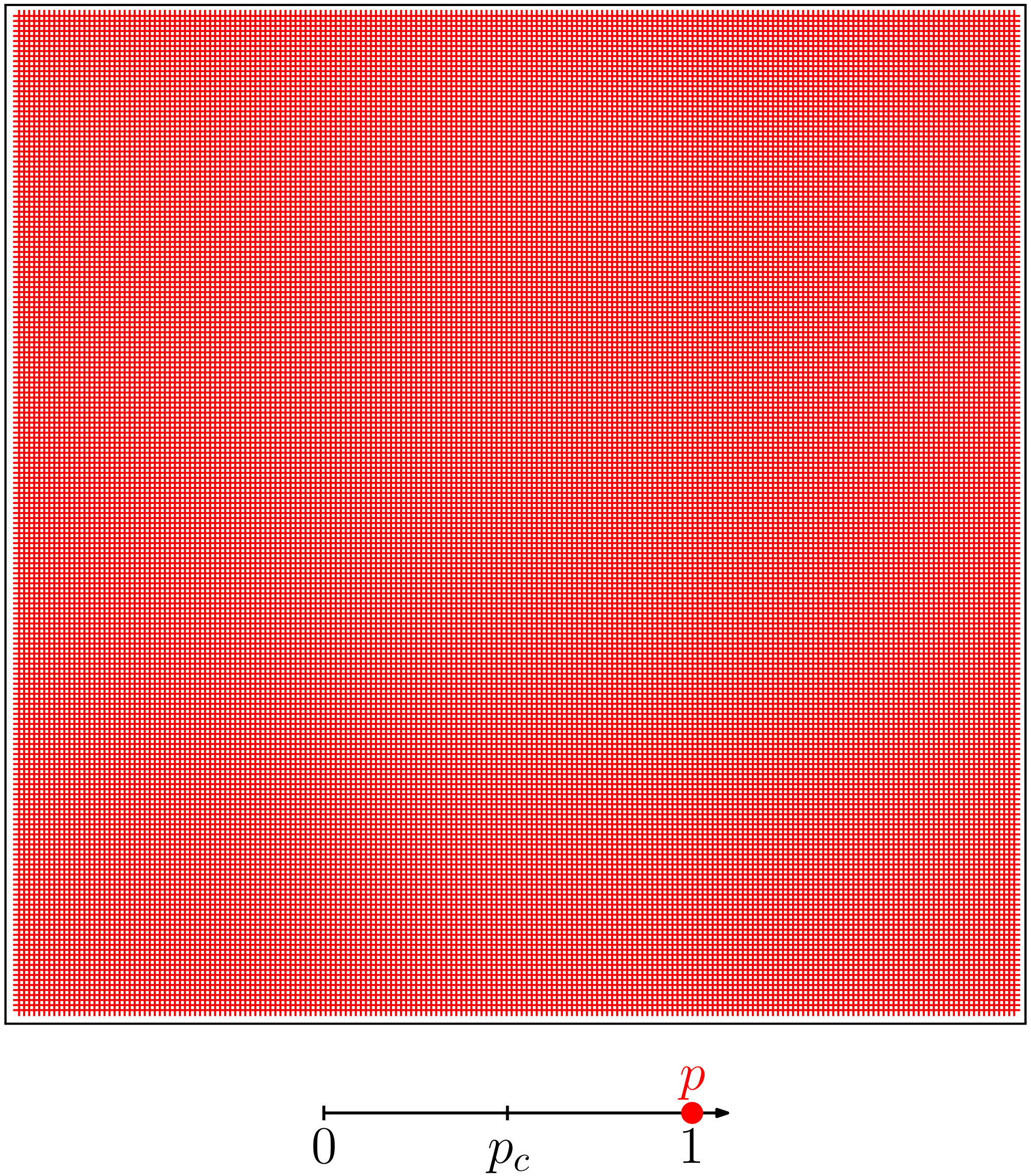

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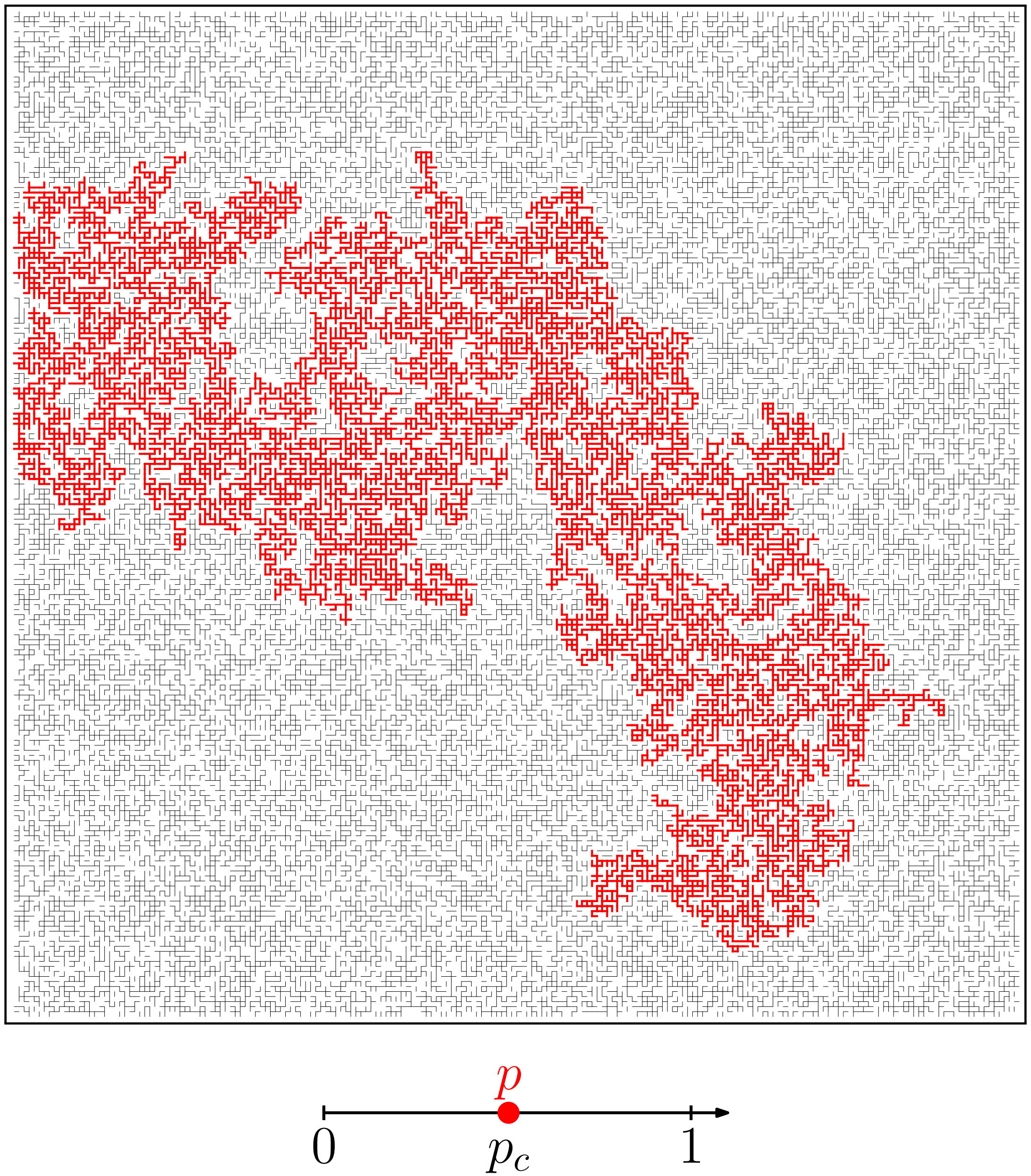

Interactions with other fields

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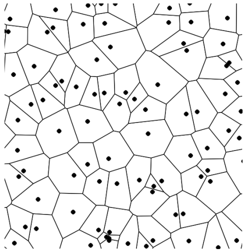
Stochastic geometry



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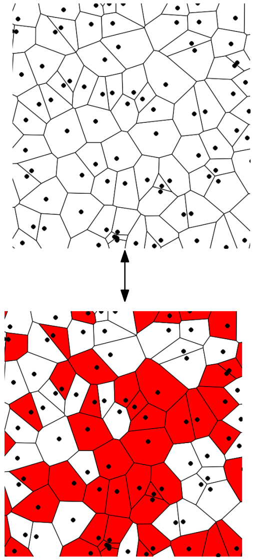

Voronoi percolation

[Vahidi-Asl Wierman 90]

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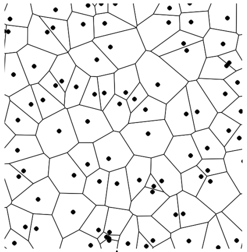

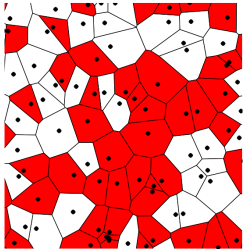

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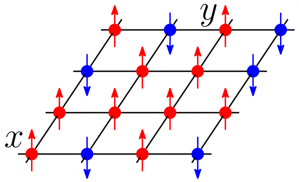
Spin systems



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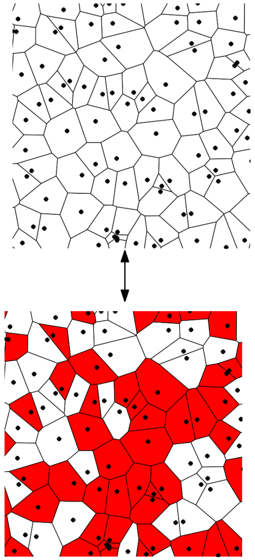

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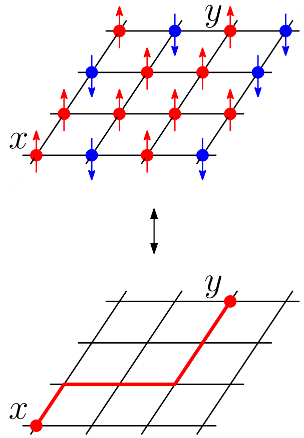

FK percolation

[Fortuin Kasteleyn 74]

Interactions with other fields

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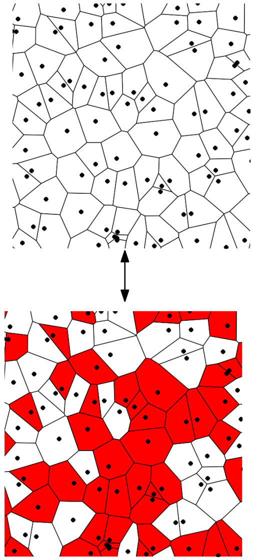

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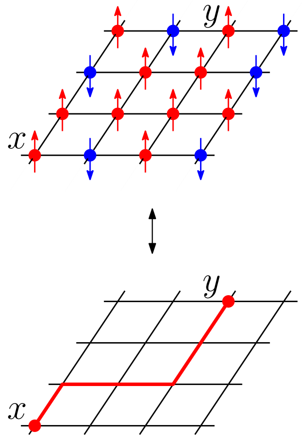

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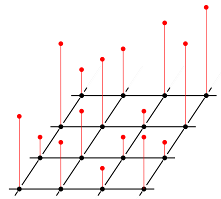

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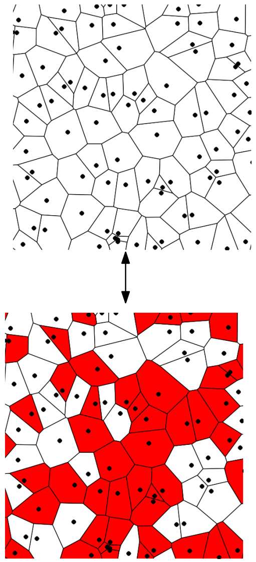

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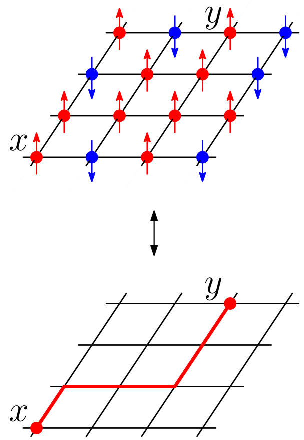

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[Fortuin Kasteleyn 74]

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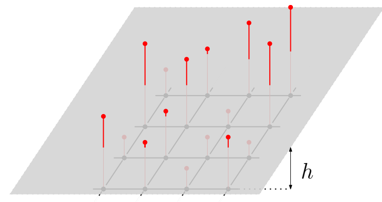
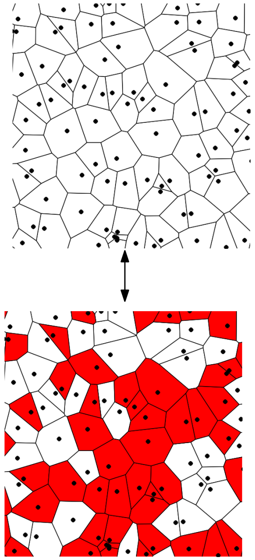


Interactions with other fields

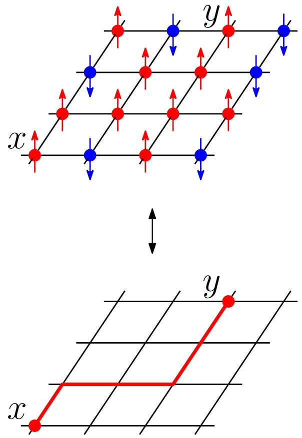
Stochastic geometry



Voronoi percolation

[Vahidi-Asl Wierman 90]

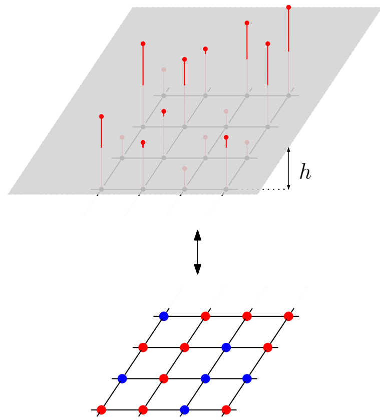
Spin systems



FK percolation

[Fortuin Kasteleyn 74]

Random functions



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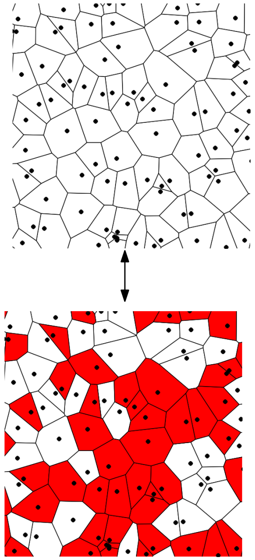

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[Vahidi-Asl Wierman 90]

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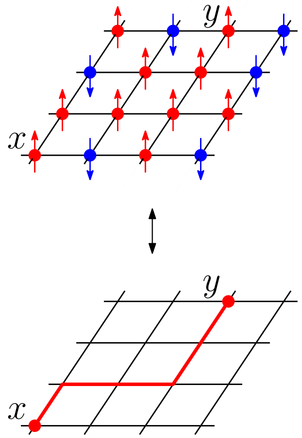
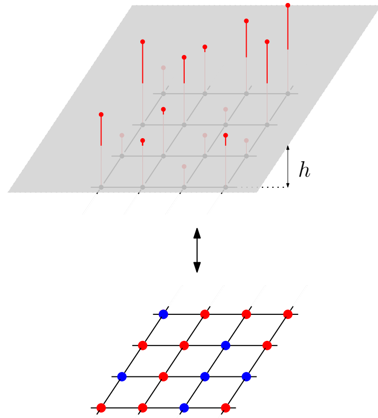


FK percolation
[Fortuin Kasteleyn 74]

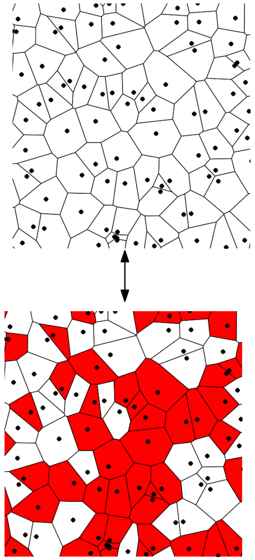
Random functions



GFF percolation
Nodal lines

Interactions with other fields

Stochastic geometry



Voronoi percolation
[Vahidi-Asl Wierman 90]

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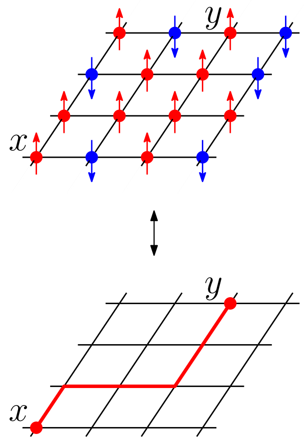
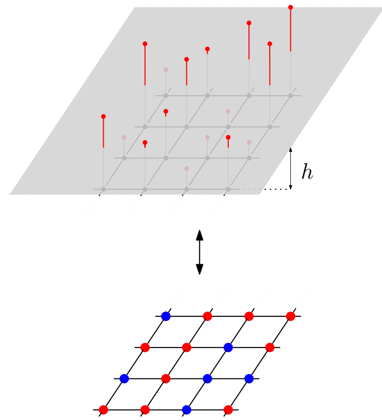


FK percolation
[Fortuin Kasteleyn 74]

Random functions

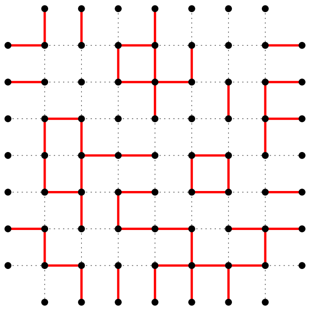


GFF percolation
Nodal lines

Motivations for robust results:

- ➜ New results in other fields.
- ➜ New methods for Bernoulli percolation.

Robust theory of crossing probabilities in dimension 2.



RSW theory for Bernoulli percolation on $\mathbb{Z}^2$

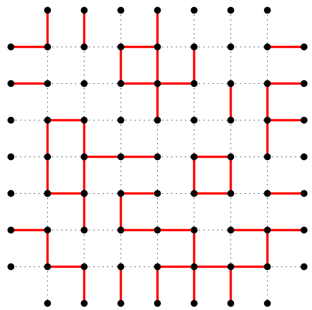
[Russo 78] [Seymour Welsh 78]

$\downarrow$

RSW theory for dependent planar models

[T. 16] [Köhler-Schindler T. 20]

1. RSW theory for Bernoulli percolation on $\mathbb{Z}^2$.



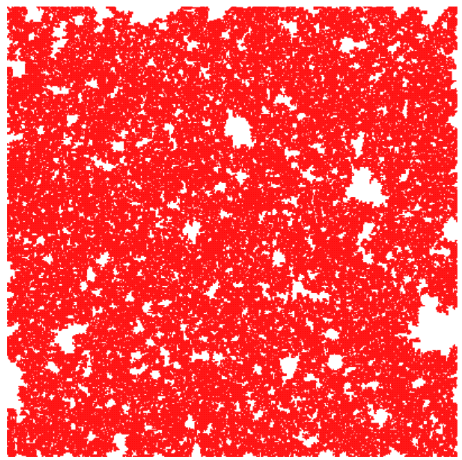
Phase transition in dimension 2

Theorem [Kesten 80]

For Bernoulli percolation on $\mathbb{Z}^2$, we have

$$p_c = \frac{1}{2}.$$

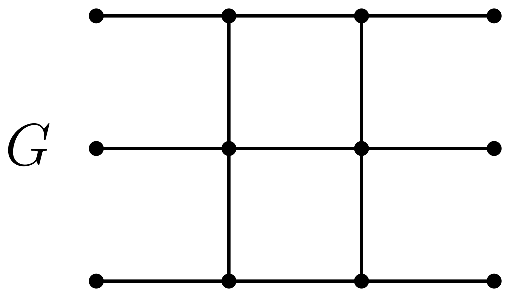
$p < \frac{1}{2}$



$p > \frac{1}{2}$

What is special about $p = 1/2$?

Planar duality:



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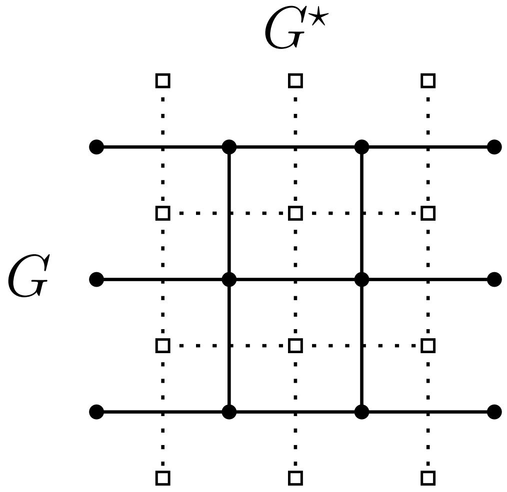

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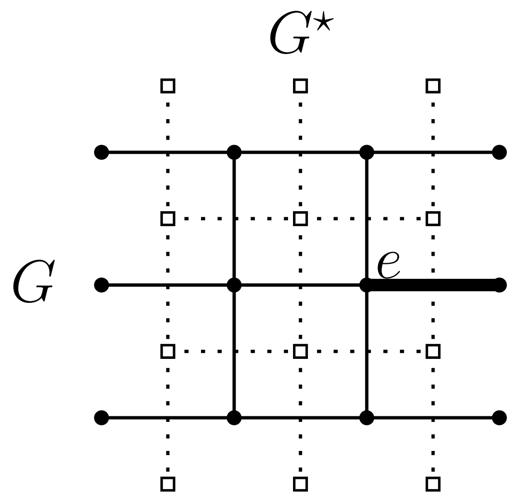

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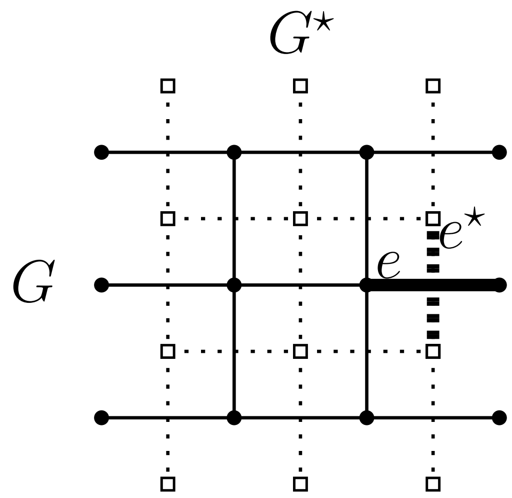

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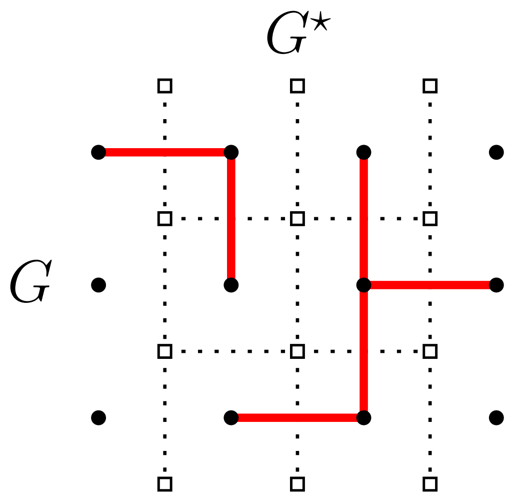

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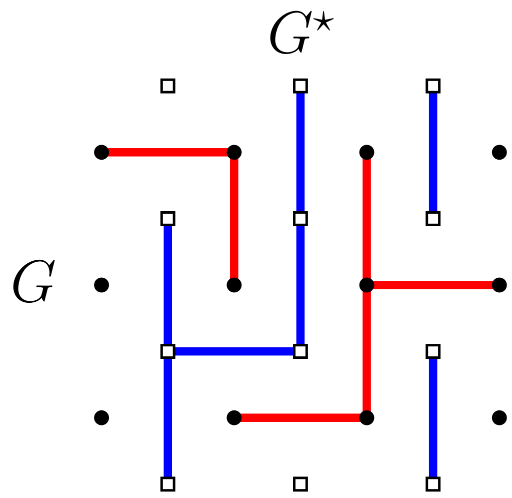

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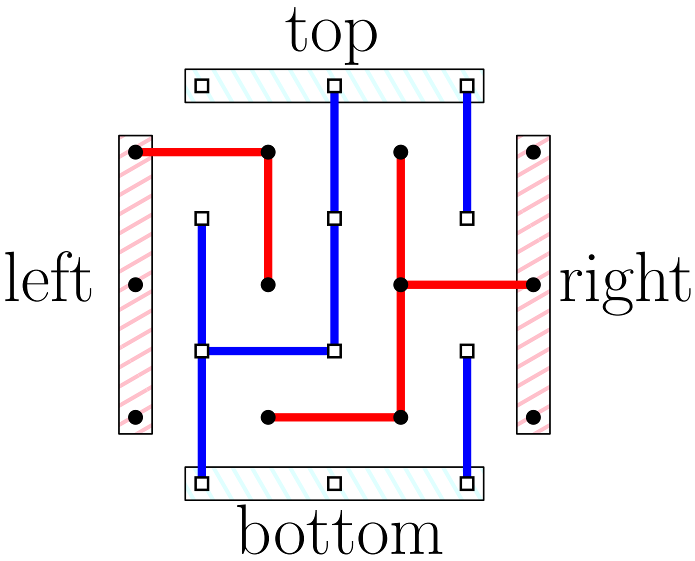

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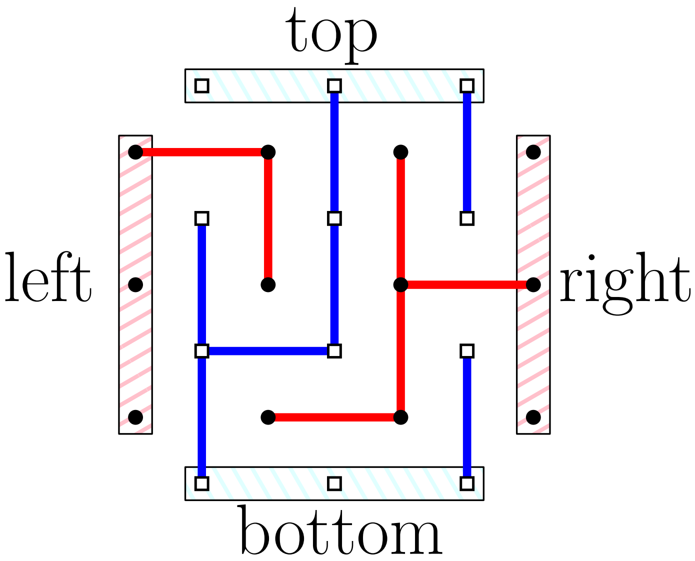
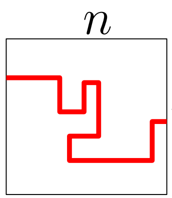
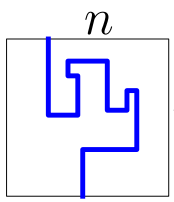


Consequence: For every n, $\mathrm{P}_p\Big[$  n $\Big] + \mathrm{P}_p\Big[$  n $\Big] = 1.$

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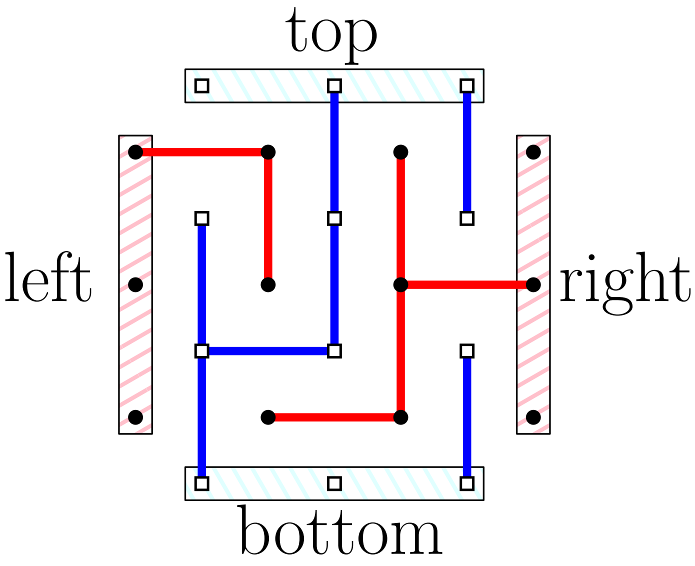
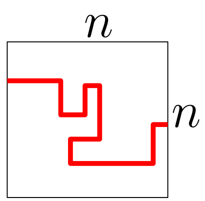
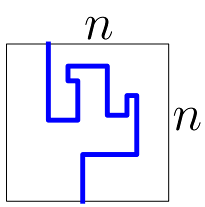
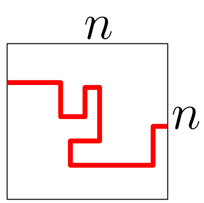


Consequence: For every n, $\mathrm{P}_p\Big[$ 

$\Big] + \mathrm{P}_p\Big[$ 

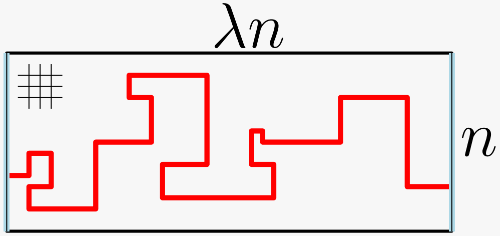
$\Big] = 1.$

➜ For $p = 1/2$, $\boxed{\mathrm{P}_p\Big[$ 

$\Big] = 1/2.}$

Conjecture: critical behavior of the crossing probabilities

Fix $\lambda \geqslant 1$. For critical Bernoulli percolation on $\mathbb{Z}^2$, we have

$\lim_{n\to\infty} \mathrm{P}_{p_c}\Big[$ 

$\Big] = \underbrace{f(\lambda)}_{\text{Cardy's formula}}.$

Properties of Cardy's formula:

Cardy's formula

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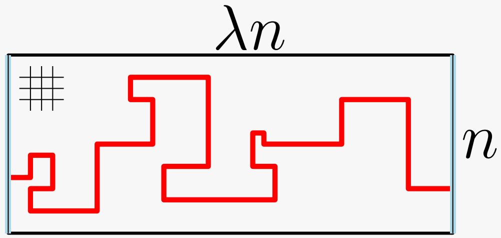

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Properties of Cardy's formula:

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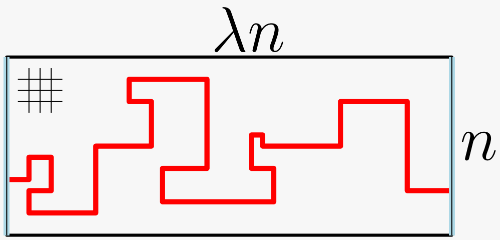

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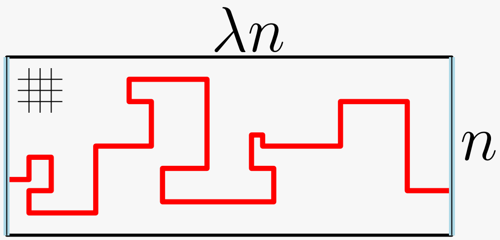

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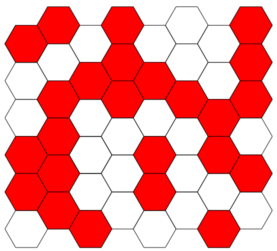
For site percolation on hexagons:



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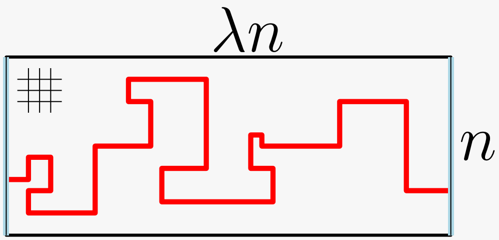

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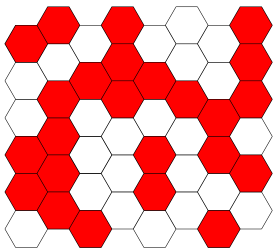

- Cardy's formula and conformal invariance. [Smirnov 01]

Cardy's formula

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λn

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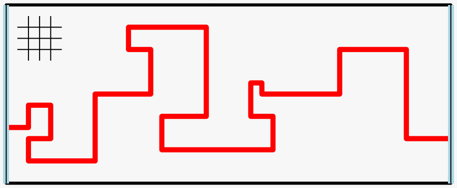
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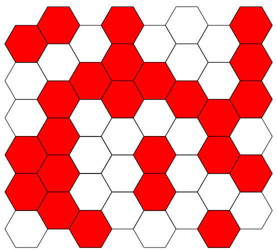
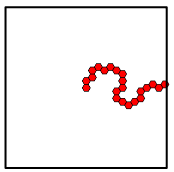


- Cardy's formula and conformal invariance. [Smirnov 01]
- Critical exponents

n

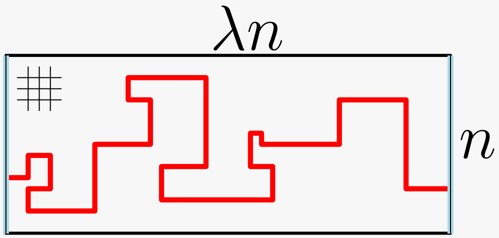
$$\mathrm{P}_{p_c}\Big[$$

$$\Big] = n^{-5/48+o(1)}.$$ [Lawler Schramm Werner 02]

Russo-Seymour-Welsh theory for Bernoulli percolation

Consider Bernoulli percolation on $\mathbb{Z}^2$ at $p_c = 1/2$.

RSW theorem [Russo 78] [Seymour Welsh 78]

Fix $\lambda \geqslant 1$. There exists $c(\lambda) > 0$ such that for every $n \geqslant 1$,

$\mathrm{P}_{p_c}\Big[$ 

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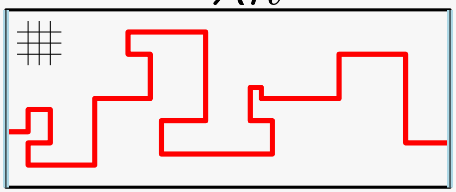
$\mathrm{P}_{p_c}\Big[$  $n\Big] \geqslant c(\lambda).$

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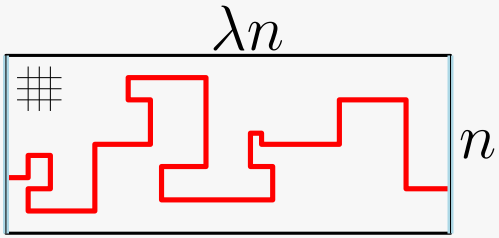
$\mathrm{P}_{p_c}\Big[$ 

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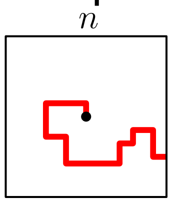
Applications:

- Bounds on critical exponents:

➜ $n^{-c_1} \leqslant \mathrm{P}_{p_c}\Big[$ 

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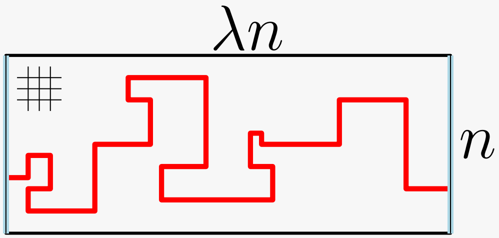
$\mathrm{P}_{p_c}\Big[$  $\Big] \geqslant c(\lambda).$

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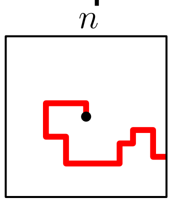
➜ $n^{-c_1} \leqslant \mathrm{P}_{p_c}\Big[$  $\Big] \leqslant n^{-c_2}$, $c_1, c_2 > 0$.

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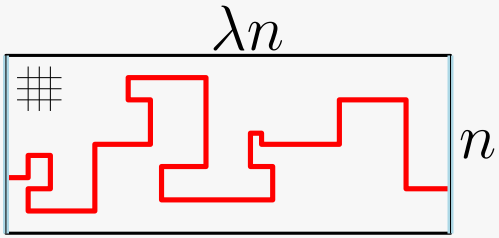
$\mathrm{P}_{p_c}\Big[$  $\Big] \geqslant c(\lambda).$

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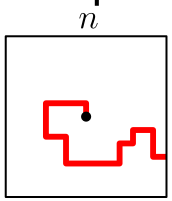
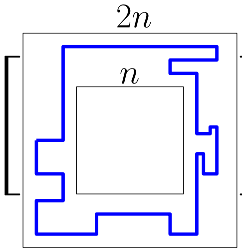
➜ $n^{-c_1} \leqslant \mathrm{P}_{p_c}\Big[$  $\Big] \leqslant n^{-c_2}$, $c_1, c_2 > 0$.

- Study of near-critical regime ($p = p_c \pm \varepsilon$).
- Tightness arguments for the scaling limit.

Annulus argument

RSW theorem: $\mathrm{P}_{1/2}\Big[$ 

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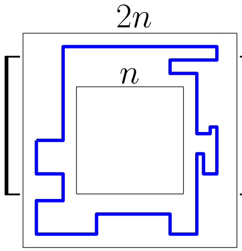
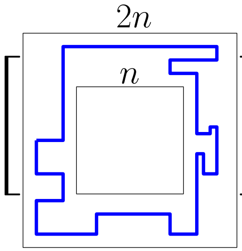
Annulus argument

RSW theorem: $\mathrm{P}_{1/2}\Big[$  $\Big] \geqslant c.$

(Rk: The proof of RSW gives $c = 0.00000000000000000000000000000000788\ldots$)

Annulus argument

RSW theorem: $\mathrm{P}_{1/2}\Big[$  $\Big] \geqslant c.$

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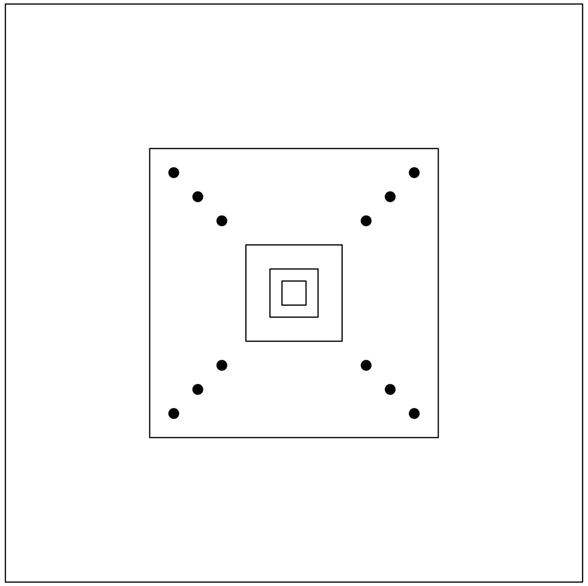
$$n = 2^k$$



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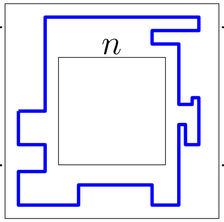
RSW theorem: $\mathrm{P}_{1/2}\Big[$  $\Big] \geqslant c.$

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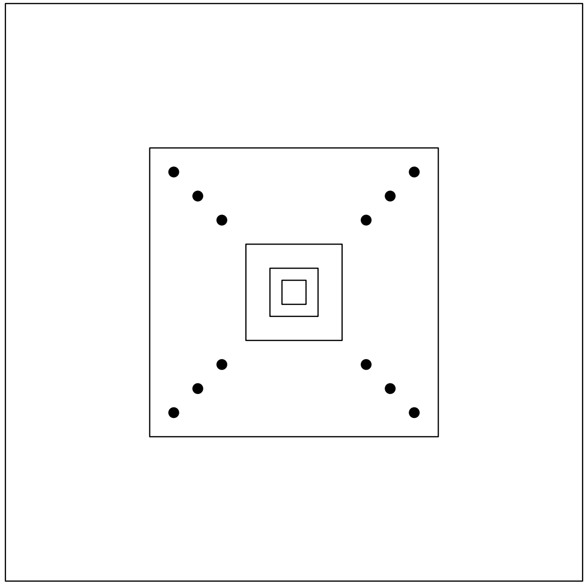
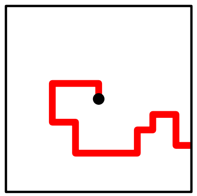
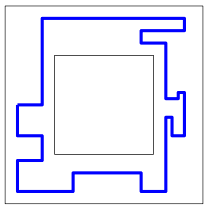
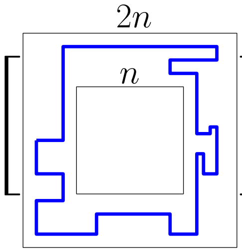


$\mathrm{P}_p\Big[$  $\Big] \leqslant \mathrm{P}_p\Big[\mathrm{NO}$  $\Big]$

Annulus argument

RSW theorem: $P_{1/2}\Big[$  $\Big] \geqslant c.$

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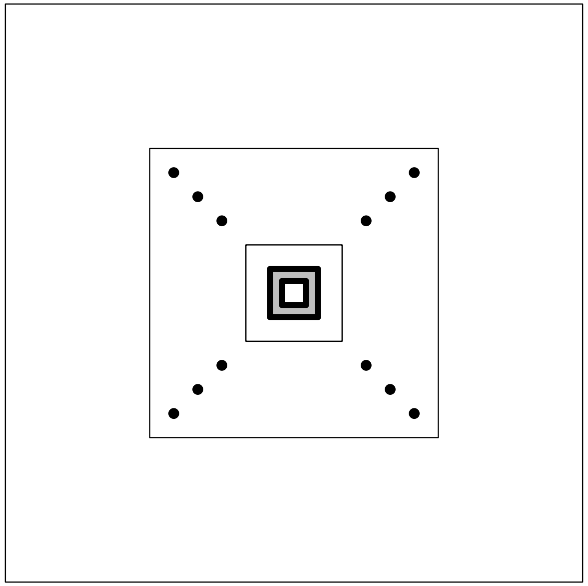
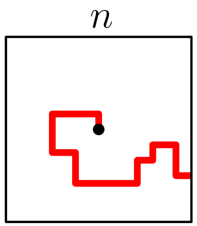
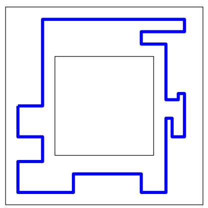
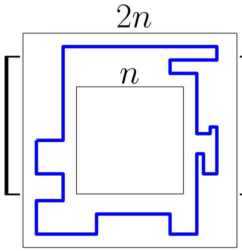


$P_p\Big[$  $\Big] \leqslant P_p\Big[\mathrm{NO}$  $\Big]$

Annulus argument

RSW theorem: $\mathrm{P}_{1/2}\Big[$  $\Big] \geqslant c.$

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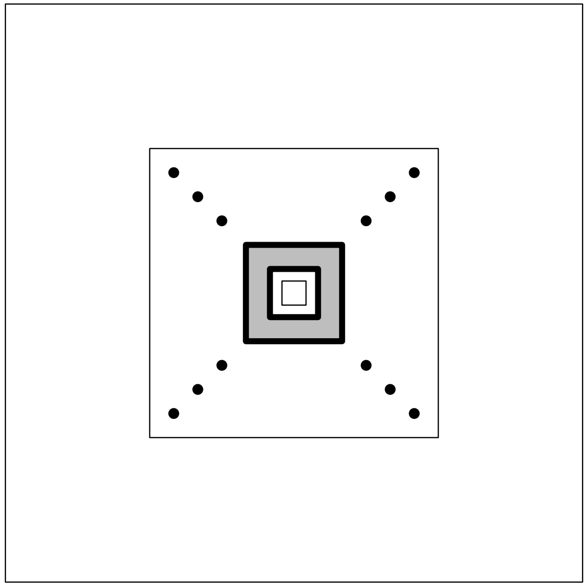
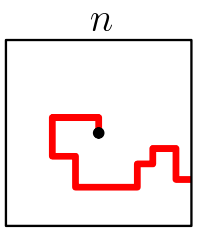
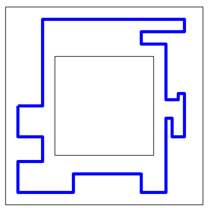


$\mathrm{P}_p\Big[$  $\Big] \leqslant \mathrm{P}_p\Big[\mathrm{NO}$  $\Big]$

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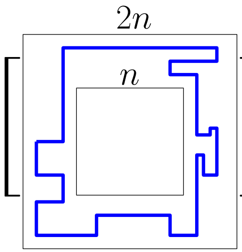
RSW theorem: $\mathrm{P}_{1/2}\Big[$  $\Big] \geqslant c.$

(Rk: The proof of RSW gives $c = 0.000000000000000000000000000000788\ldots$)

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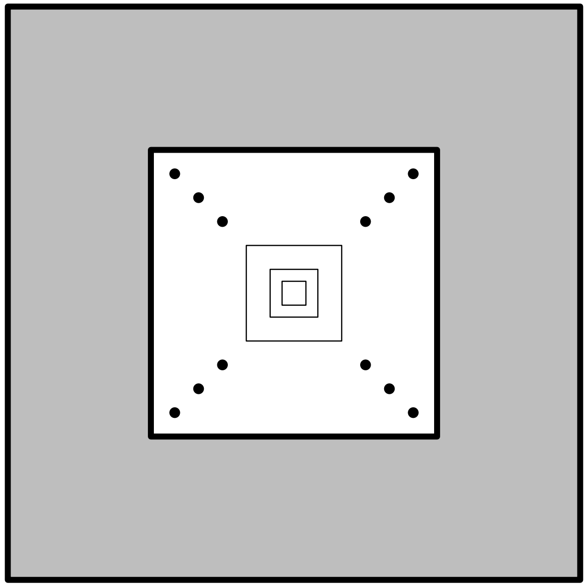
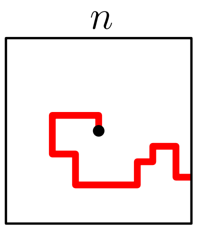
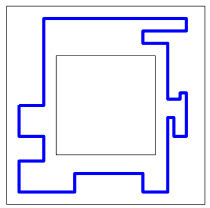


$\mathrm{P}_p\Big[$  $\Big] \leqslant \mathrm{P}_p\Big[\text{NO}$  $\Big]$

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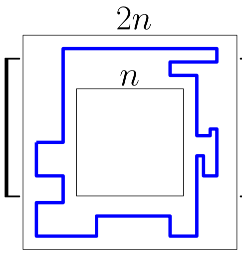
RSW theorem: $\mathrm{P}_{1/2}\Big[$  $\Big] \geqslant c.$

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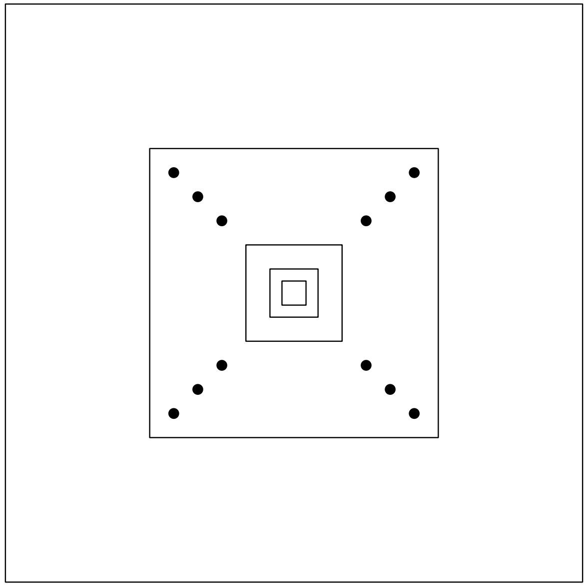
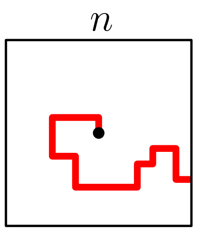
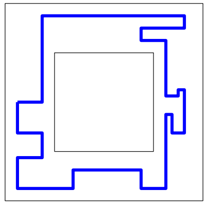


$\mathrm{P}_p\Big[$  $\Big] \leqslant \mathrm{P}_p\Big[\mathrm{NO}$  $\Big]$

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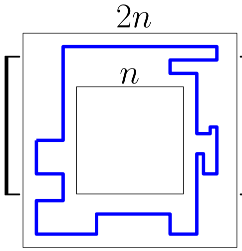
RSW theorem: $\mathrm{P}_{1/2}\Big[$ 

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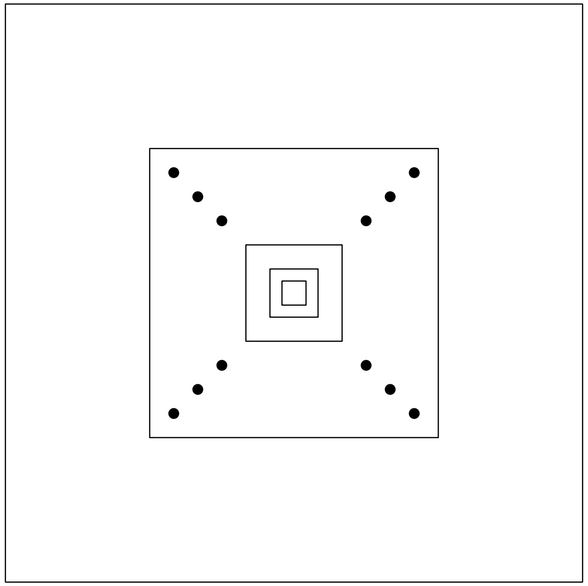
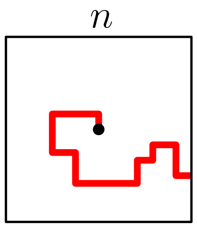
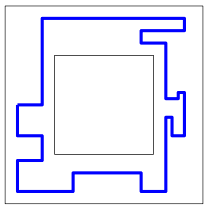
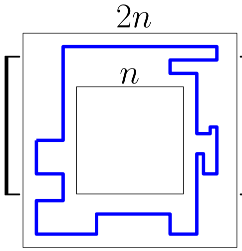


$\mathrm{P}_p\Big[$ 

$\Big] \leqslant \mathrm{P}_p\Big[\mathrm{NO}$  $\Big] \leqslant (1-c)^{\log_2 n}$

Annulus argument

RSW theorem: $\mathrm{P}_{1/2}\Big[$ 

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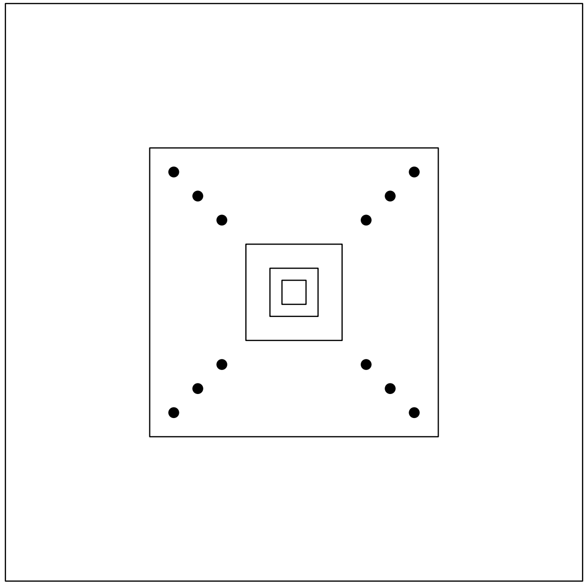
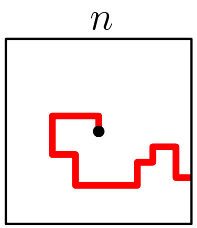
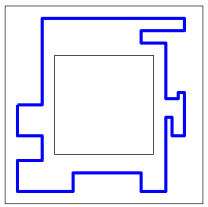
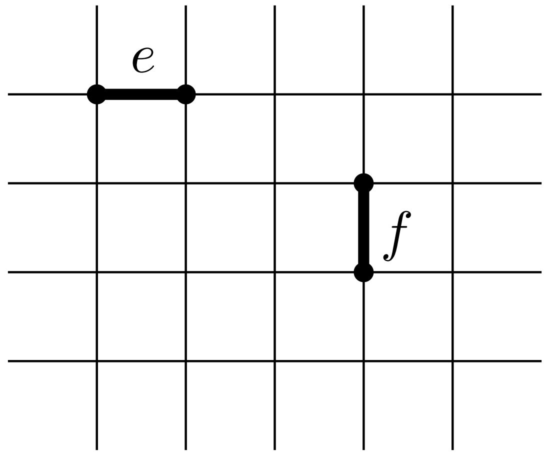


$\mathrm{P}_p\Big[$ 

$\Big] \leqslant \mathrm{P}_p\Big[\mathrm{NO}$  $\Big] \leqslant (1-c)^{\log_2 n} \leqslant \dfrac{1}{n^{c'}}.$

Bernoulli percolation: an independent percolation model



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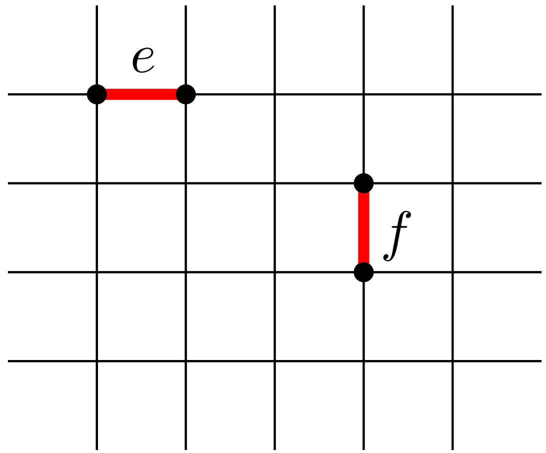

$$\mathrm{P}_p[e, f \text{ open}] = p^2$$

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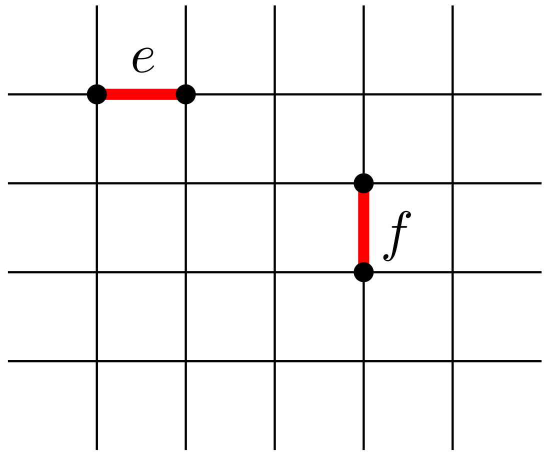

$$\boxed{\mathrm{P}_p[e, f \text{ open}] = p^2 = \mathrm{P}_p[e \text{ open}]\mathrm{P}_p[f \text{ open}].}$$

1. RSW theory for Bernoulli percolation on $\mathbb{Z}^2$

[Russo '78][Seymour Welsh '78]

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$\downarrow$

2. The RSW lemma and its generalizations

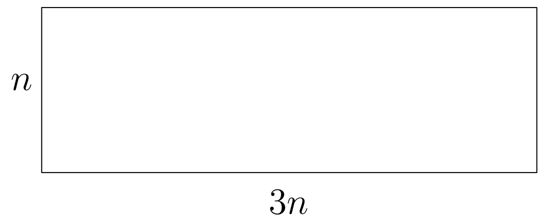
[Köhler-Schindler T. 20+]

Two important properties of P_p

Symmetries:

P_p is invariant under translations, reflections and $\pi/2$-rotation.

Positive correlations [Harris 60]**:**
Crossing events are positively correlated.



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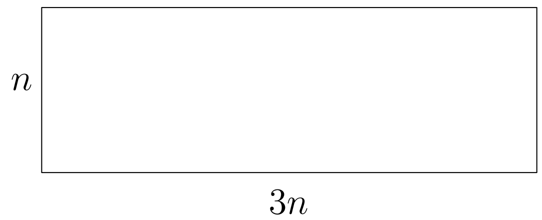

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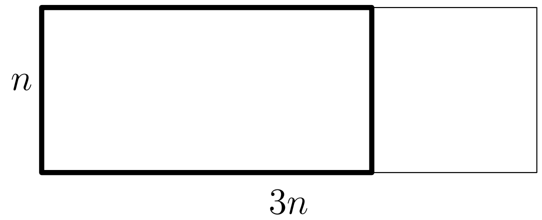

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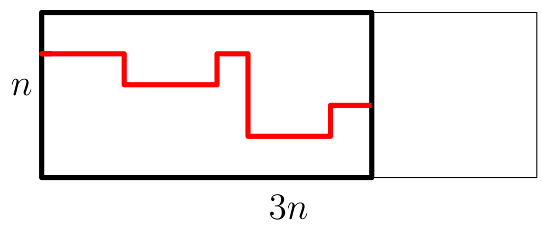

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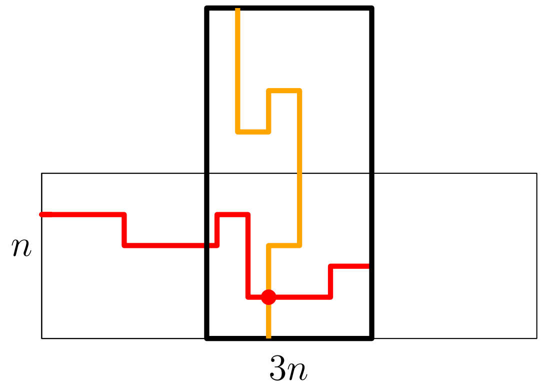

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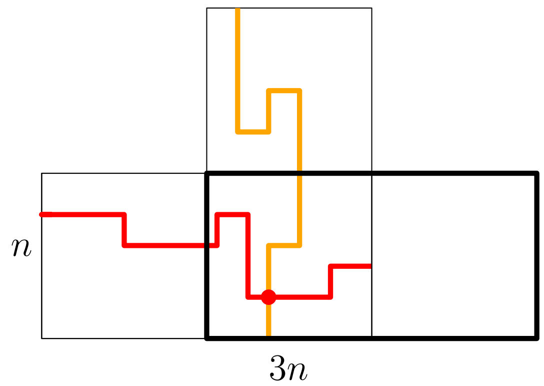

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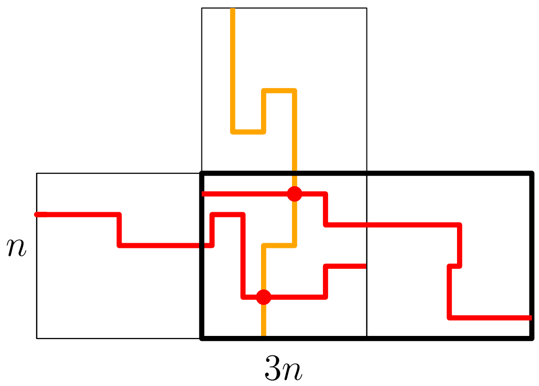

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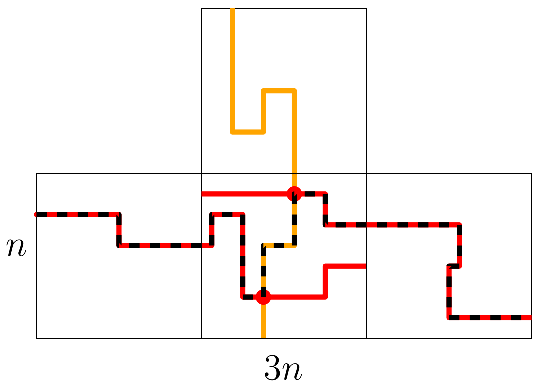

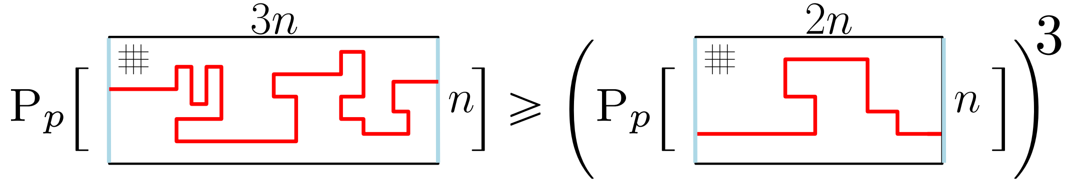

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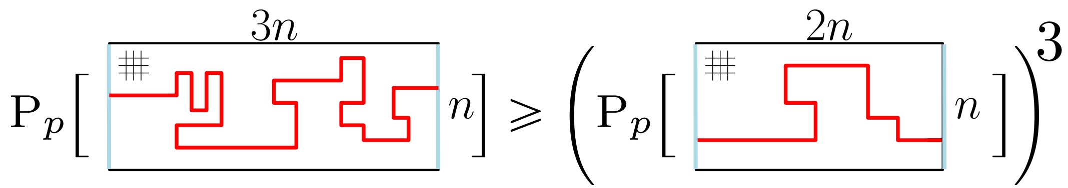
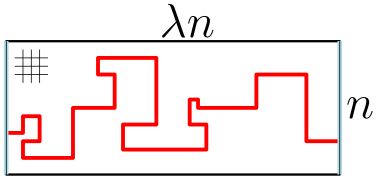
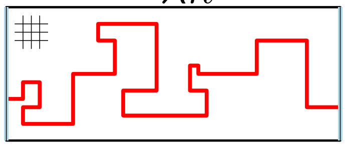


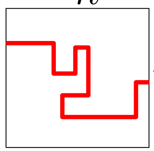
Proof of the RSW theorem

Goal: For $\lambda \geqslant 1$ and $n \geqslant 1$, $\mathrm{P}\Big[$ 

$\Big] \geqslant c(\lambda)$.

Proof of the RSW theorem

Goal: For $\lambda \geqslant 1$ and $n \geqslant 1$, $\mathrm{P}\Big[$ λn  n $\Big] \geqslant c(\lambda)$.

Step 0 (self-duality): $\mathrm{P}\Big[$ n  n $\Big] = 1/2$.

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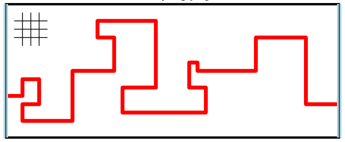

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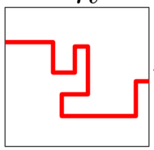
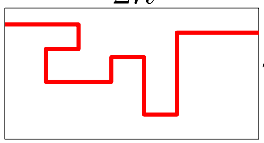
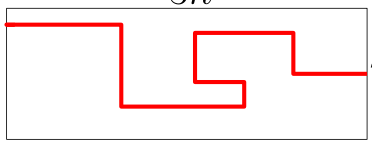
Step 0 (self-duality): $\mathrm{P}\Big[$


$\Big] = 1/2$.

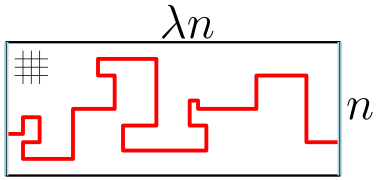
Step 2 ($\lambda = 2$ suffices):

$\mathrm{P}\Big[$


$\Big] \geqslant c \quad \overset{\text{positive correlations}}{\Longrightarrow} \quad \mathrm{P}\Big[$


$\Big] \geqslant c^3$.

Proof of the RSW theorem

Goal: For $\lambda \geqslant 1$ and $n \geqslant 1$, $\mathrm{P}\Big[$ 

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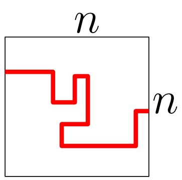
Step 0 (self-duality): $\mathrm{P}\Big[$ 

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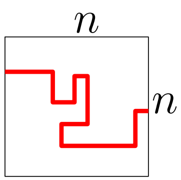
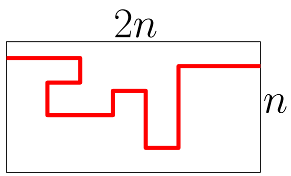
Step 1 (**RSW Lemma**): $\mathrm{P}\Big[$ 

$\Big] \geqslant c \Rightarrow \mathrm{P}\Big[$ 

$\Big] \geqslant c'$.

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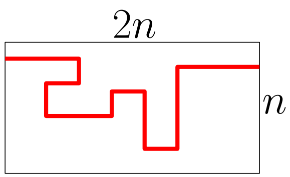
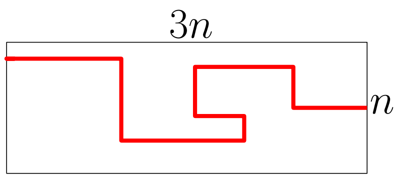
$\mathrm{P}\Big[$ 

$\Big] \geqslant c$ $\overset{\text{positive correlations}}{\Rightarrow}$ $\mathrm{P}\Big[$ 

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“Proof” of the RSW lemma

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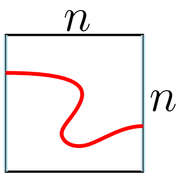
Known: $\mathrm{P}_p\Big[$ 

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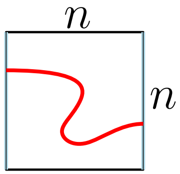
Known: $\mathrm{P}_p\Big[$ 

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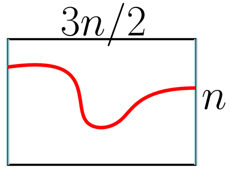
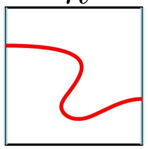
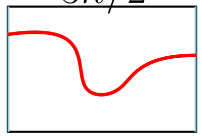
Goal: $\mathrm{P}_p\Big[$ 

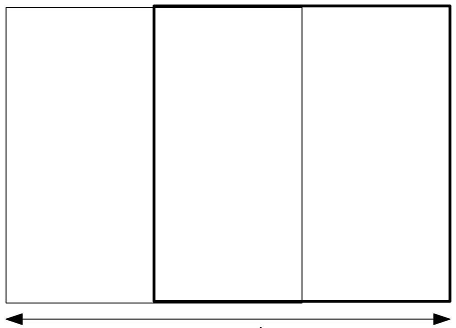
$\Big] \geqslant c' > 0.$

"Proof" of the RSW lemma

Known: $\mathrm{P}_p\Big[$  $n\Big] \geqslant c.$ (box width n)

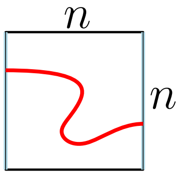
Goal: $\mathrm{P}_p\Big[$  $n\Big] \geqslant c' > 0.$ (box width $3n/2$)

Idea: "glue" square crossings together.

$\mathrm{P}_p\Big[\ n$  $\Big]$

$3n/2$

“Proof” of the RSW lemma

Known: $\mathrm{P}_p\Big[$ 

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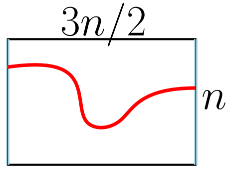
Goal: $\mathrm{P}_p\Big[$ 

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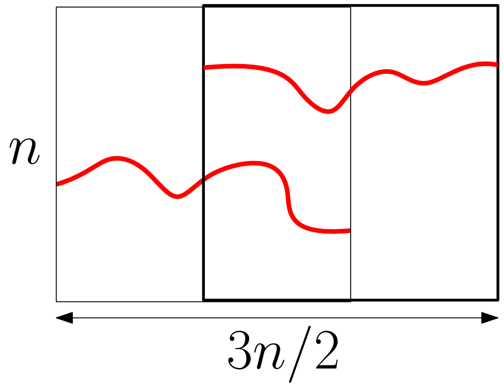
$\mathrm{P}_p\Big[$ 

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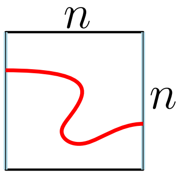
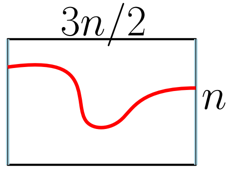
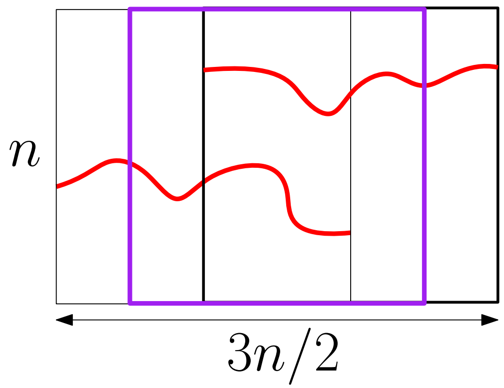
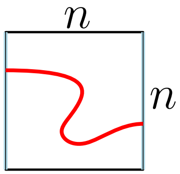
“Proof” of the RSW lemma

Known: $\mathrm{P}_p\Big[$  $\Big] \geqslant c.$

Goal: $\mathrm{P}_p\Big[$  $\Big] \geqslant c' > 0.$

Idea: “glue” square crossings together.

$\mathrm{P}_p\Big[$  $\Big]$

Known: $\mathrm{P}_p\Big[$ 

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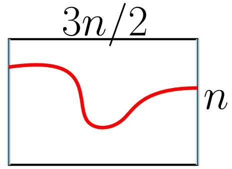
Goal: $\mathrm{P}_p\Big[$ 

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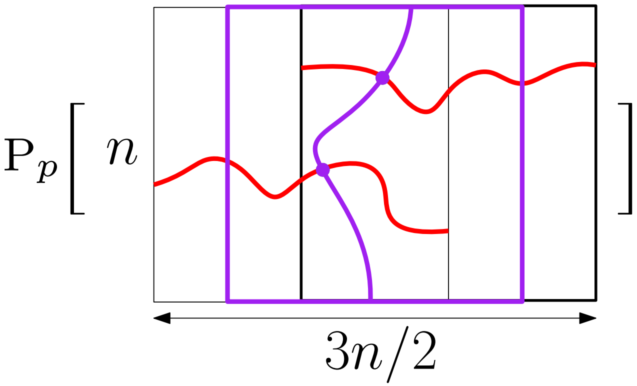

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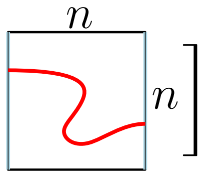
Known: $\mathrm{P}_p\Big[$ 

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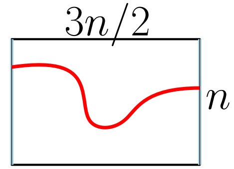
Goal: $\mathrm{P}_p\Big[$ 

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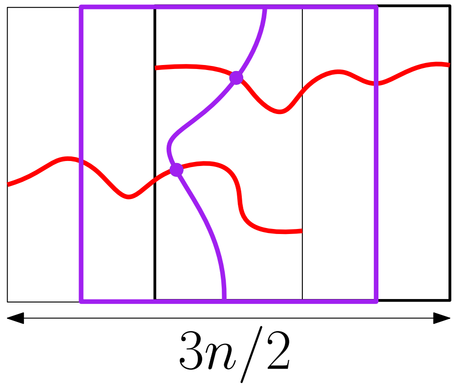
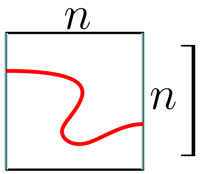
$\mathrm{P}_p\Big[\ n$ 

$\Big] \quad \geqslant \quad c^3$

positive correlations
+
symmetries

"Proof" of the RSW lemma

Known: $\mathrm{P}_p\Big[$ 

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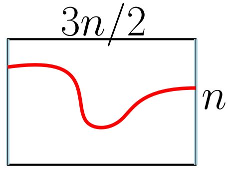
Goal: $\mathrm{P}_p\Big[$ 

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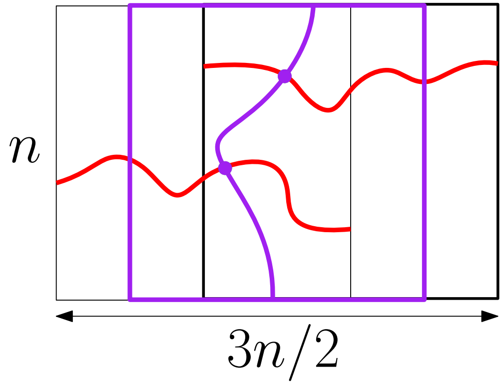
$\mathrm{P}_p\Big[$ 

$\Big] \geqslant c^3 =: c'.$

positive correlations
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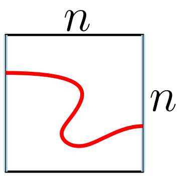
"Proof" of the RSW lemma

Known: $\mathrm{P}_p\Big[$ 

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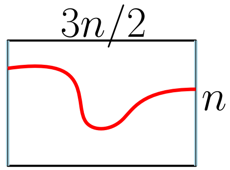
Goal: $\mathrm{P}_p\Big[$ 

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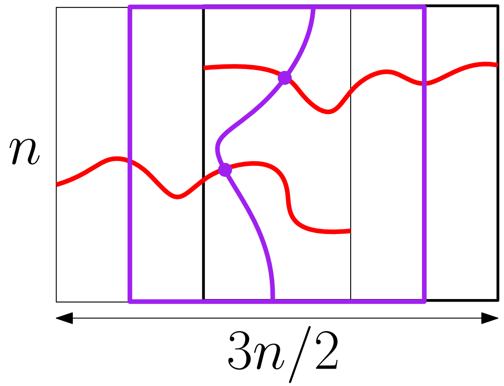
$\mathrm{P}_p\Big[$ 

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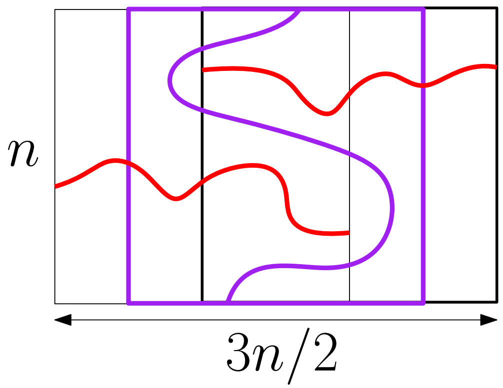
positive correlations
+
symmetries

Difficulty: "rule out" tortuous path.



The proof of RSW theorem: The key lemma

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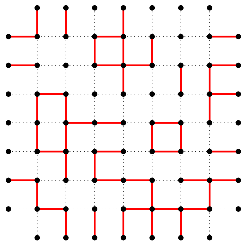
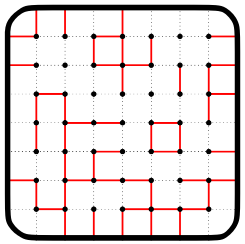
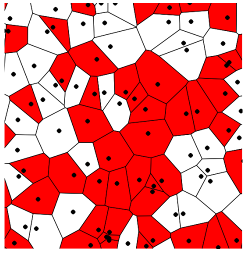
$\mathbf{P}$=general percolation process in the plane.



Bernoulli percolation
[Broadbent Hammersley, 57]



FK percolation
[Fortuin Kasteleyn 74]



Voronoi percolation
[Vahidi-Asl Wierman '90]

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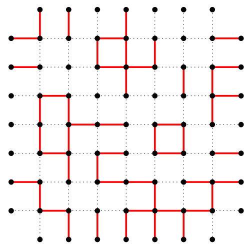

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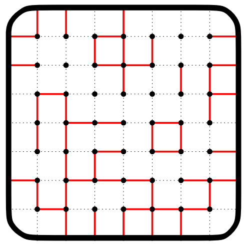

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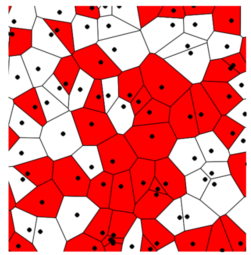

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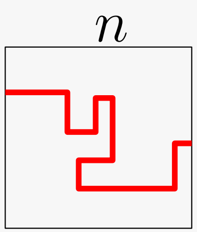
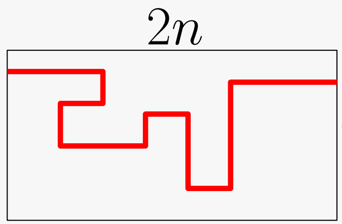
RSW Lemma

$\Big(\mathbf{P}\Big[$ 

$n\Big] \geqslant c\Big) \Rightarrow \Big(\mathbf{P}\Big[$ 

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The proof of RSW theorem: The key lemma

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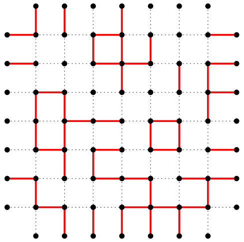

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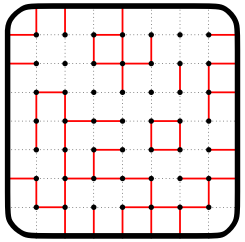

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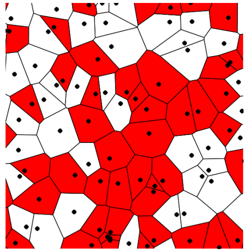

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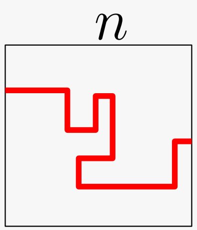
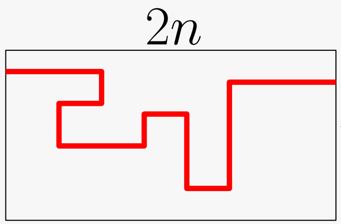
$\Big(\mathbf{P}\Big[$ 

$n\Big] \geqslant c\Big) \Rightarrow \Big(\mathbf{P}\Big[$ 

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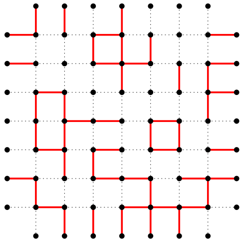

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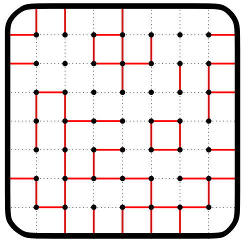

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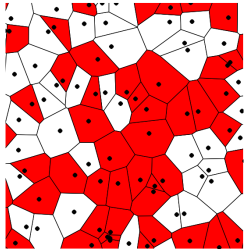

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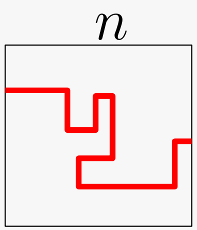
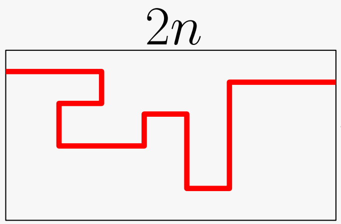
$\Big(\mathbf{P}\Big[$ 

$n\Big] \geqslant c\Big) \Rightarrow \Big(\mathbf{P}\Big[$ 

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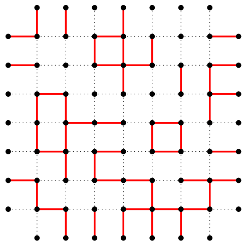

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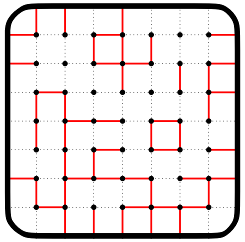

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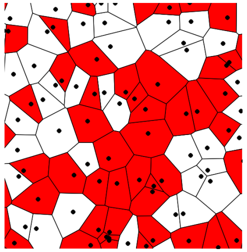

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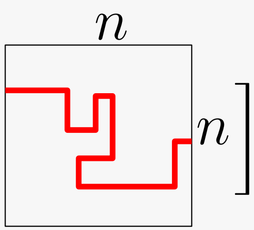
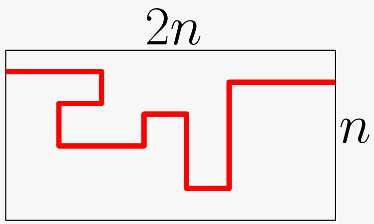
$\Big(\mathbf{P}\Big[$  $\Big] \geqslant c\Big) \Rightarrow \Big(\mathbf{P}\Big[$  $\Big] \geqslant c'\Big)$,

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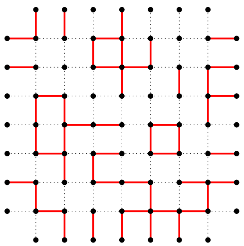

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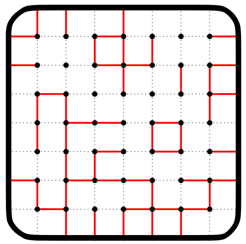

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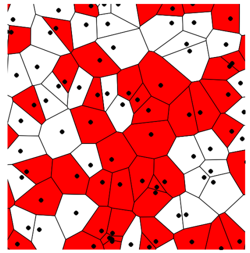

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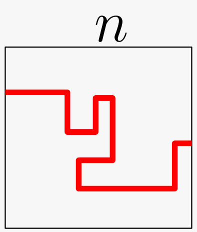
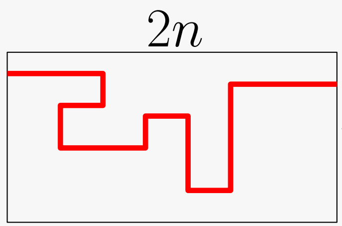
$\Big(\mathbf{P}\Big[$ 

$n\Big] \geqslant c\Big) \Rightarrow \Big(\mathbf{P}\Big[$ 

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All the proofs use **Symmetries** + **Positive correlations** + “something else”.

General RSW Lemma.

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RSW Lemma for symmetric positively correlated measures

[Köhler-Schindler T. 20+]

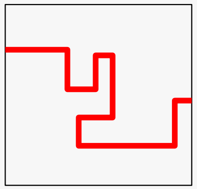
Let $\mathbf{P}$ be a planar percolation measure satisfying

- Symmetries,
- Positive correlations.

Then

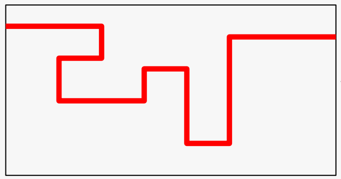
n

$$\Big(\mathbf{P}\Big[$$



$$n\Big] \geqslant c\Big) \Rightarrow \Big(\mathbf{P}\Big[$$

$2n$



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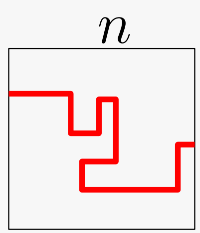
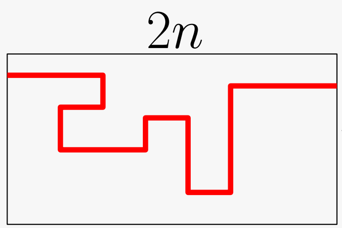
$\Big(\mathbf{P}\Big[$ 

$n\Big] \geqslant c\Big) \Rightarrow \Big(\mathbf{P}\Big[$ 

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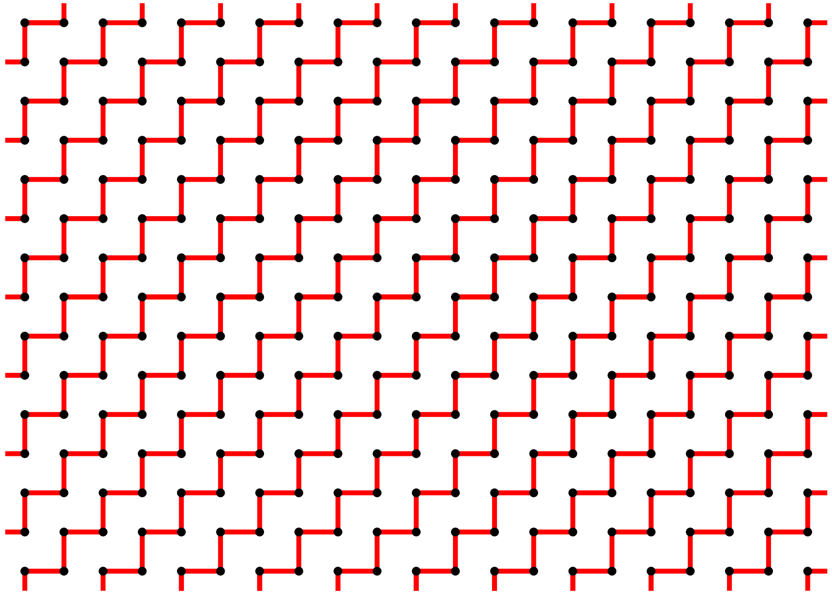
Why are symmetries important?



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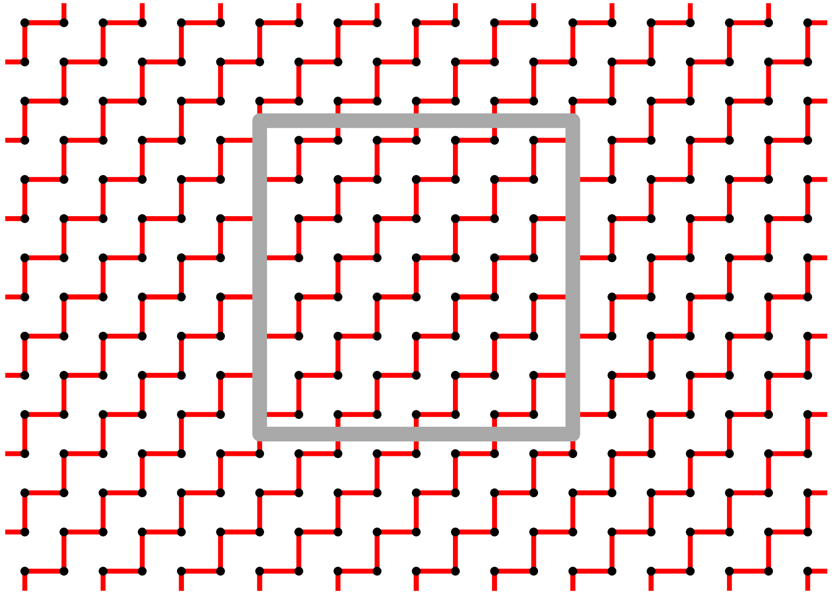

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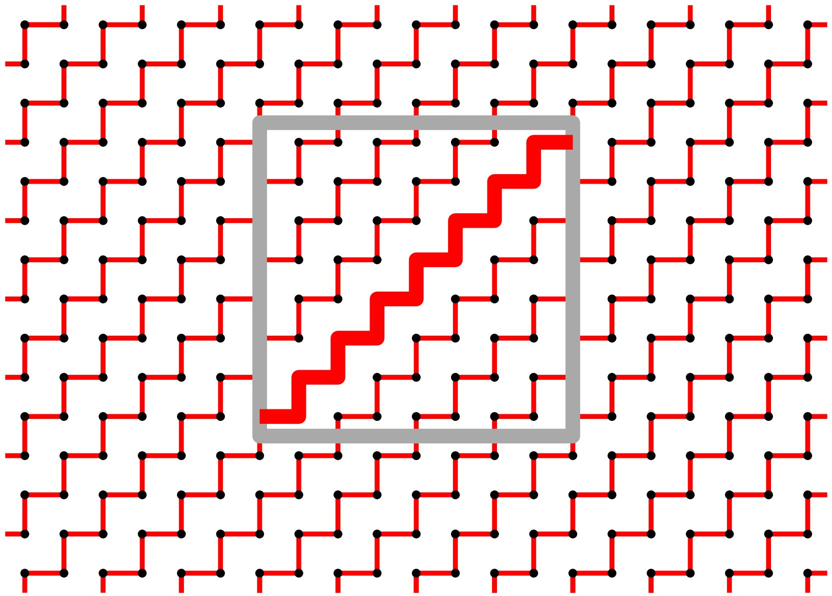

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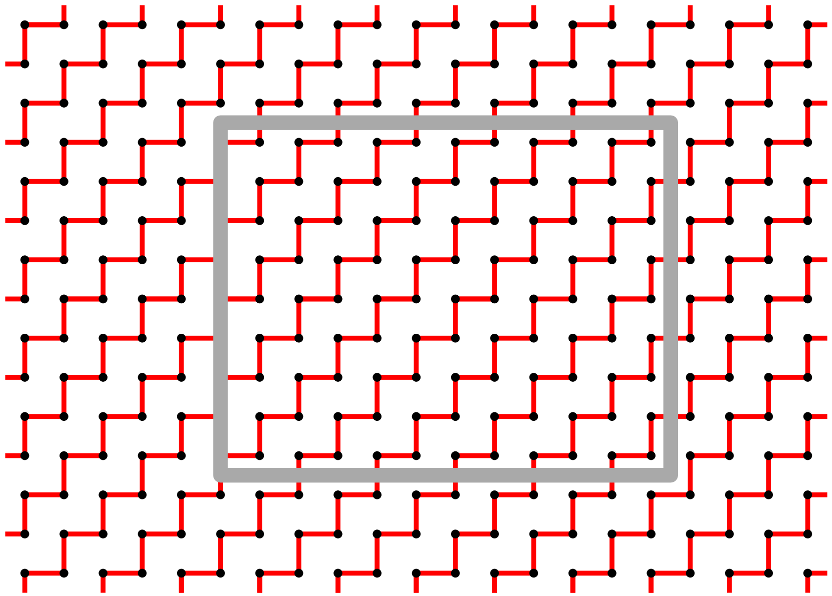

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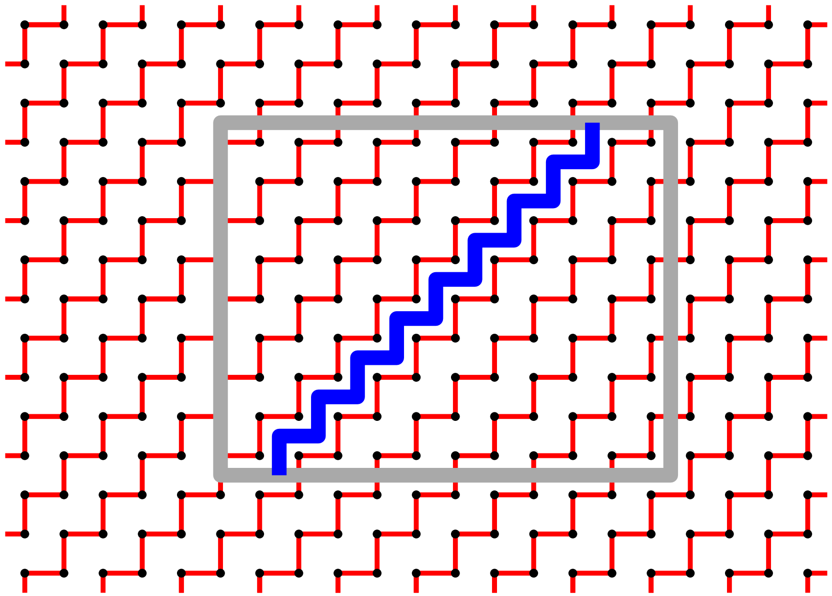

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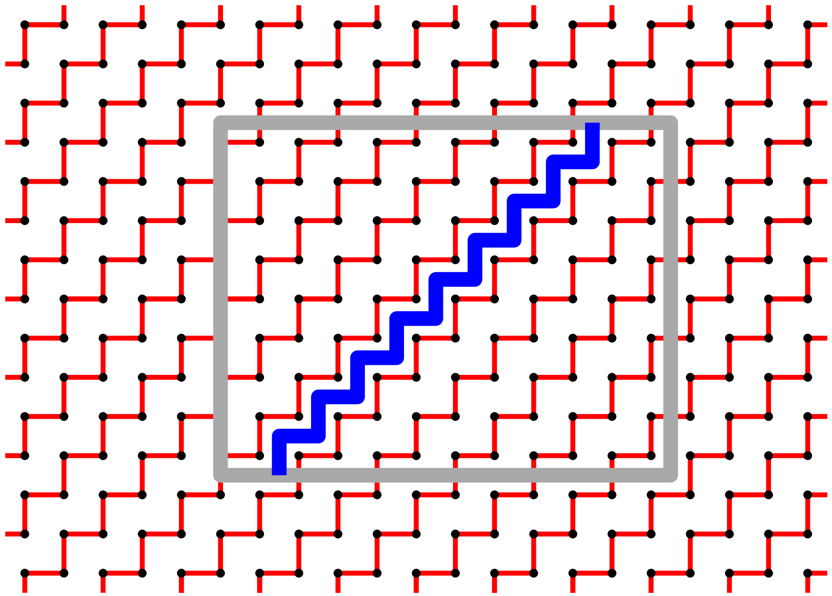

➜ Squares always crossed, long horizontal rectangles never crossed,

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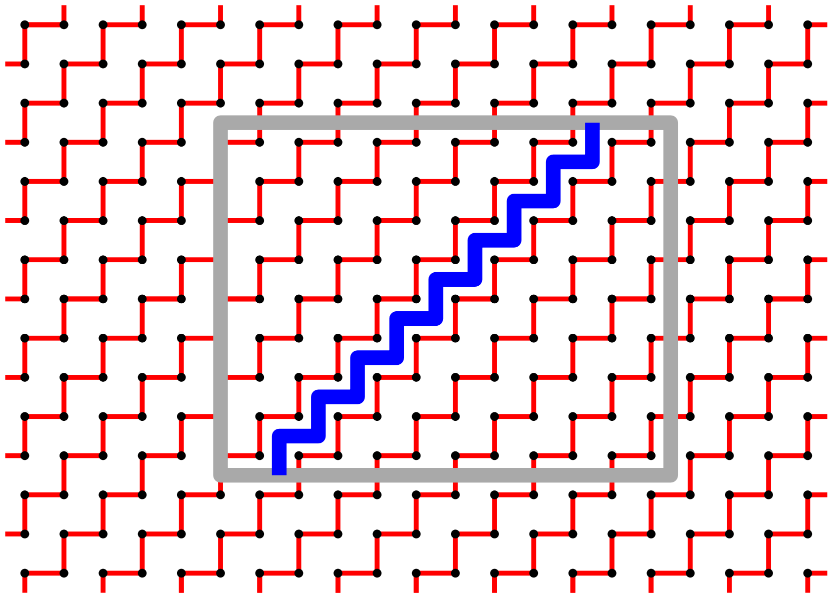
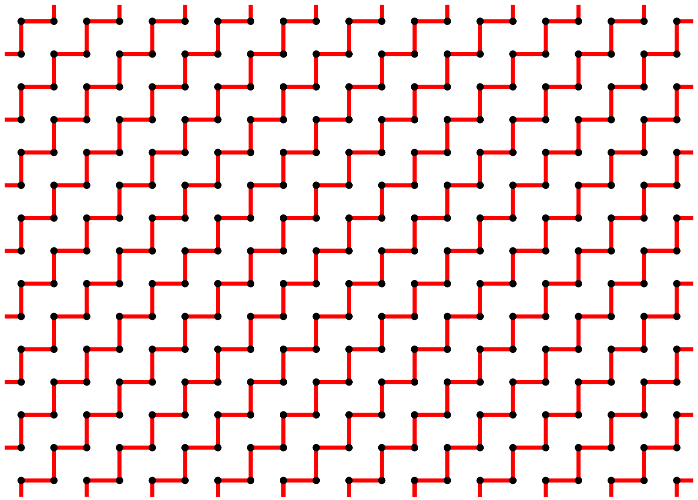


➜ Squares always crossed, long horizontal rectangles never crossed,

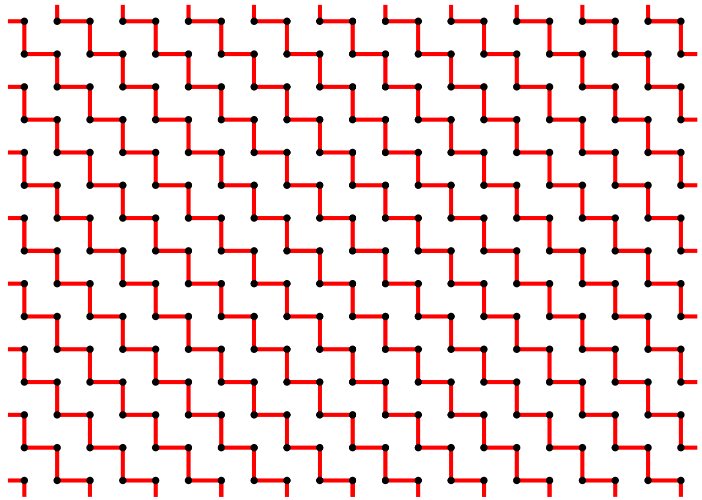
➜ Not reflection invariant.

Why are positive correlations important?

$\frac{1}{2}$



$+\ \frac{1}{2}$



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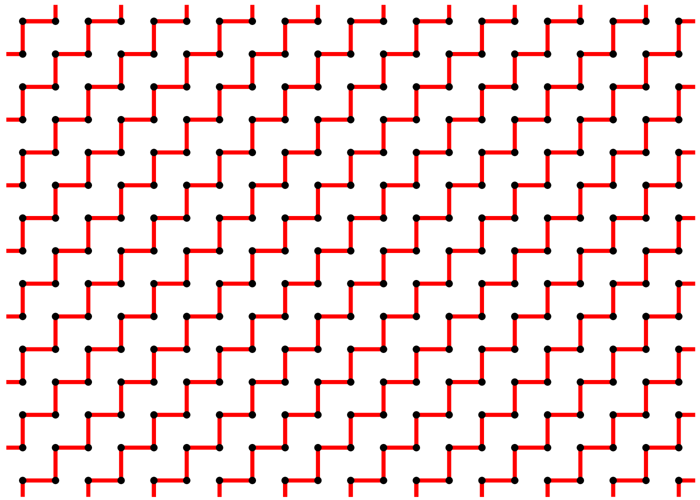
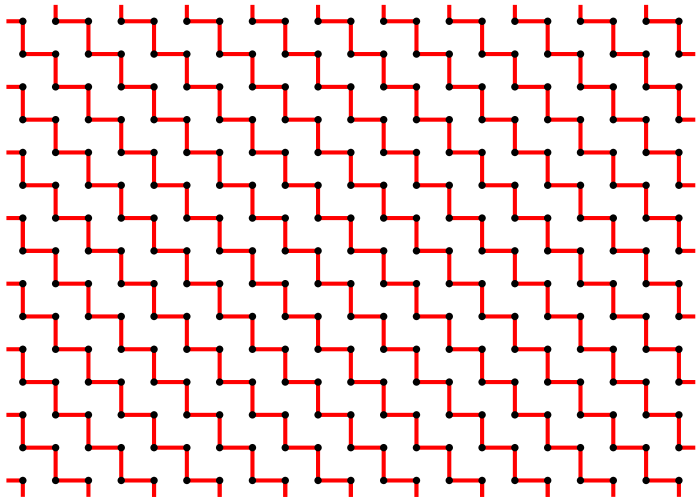
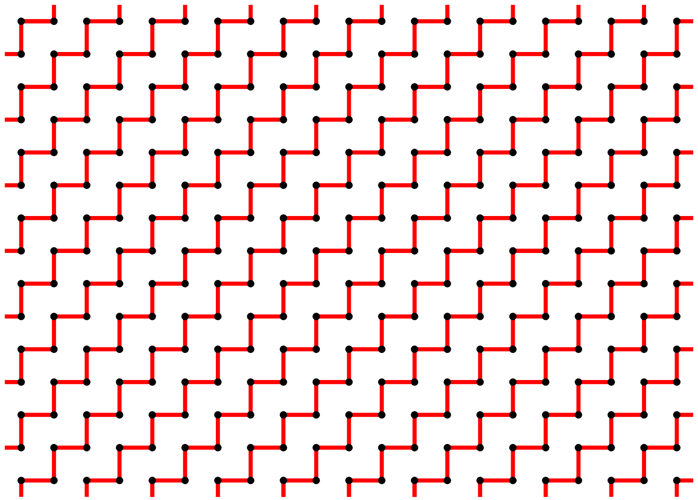
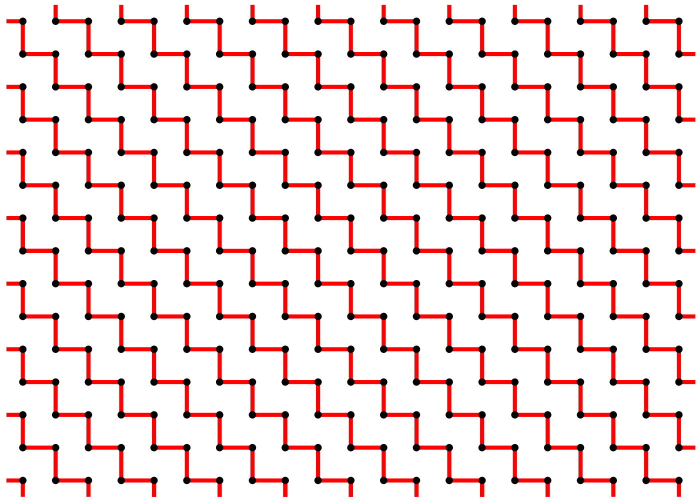
$\frac{1}{2}$  $+\ \frac{1}{2}$ 

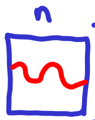
➜ Squares always crossed, long horizontal rectangles never crossed,

Why are positive correlations important?

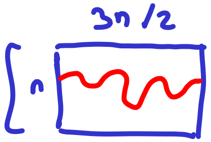
$\frac{1}{2}$  $+ \frac{1}{2}$ 

➜ Squares always crossed, long horizontal rectangles never crossed,
➜ No positive correlation.

Sketch of proof

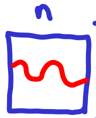
Hyp: $P\Big[$ 

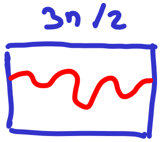
$\Big] \geq c \quad \forall n$

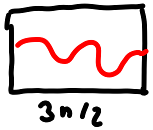
Goal: $P\Big[$ 

$\Big] \geq c'$

Sketch of proof

Hyp: $P\,[$  $] \geq c \quad \forall n$

Goal: $P\,[\,n$  $] \geq c'$

<u>Assumption</u>: $P\,[\,n$  $] \leq \varepsilon \quad \left(\begin{matrix}\varepsilon > 0 \\ \text{cst}\end{matrix}\right)$

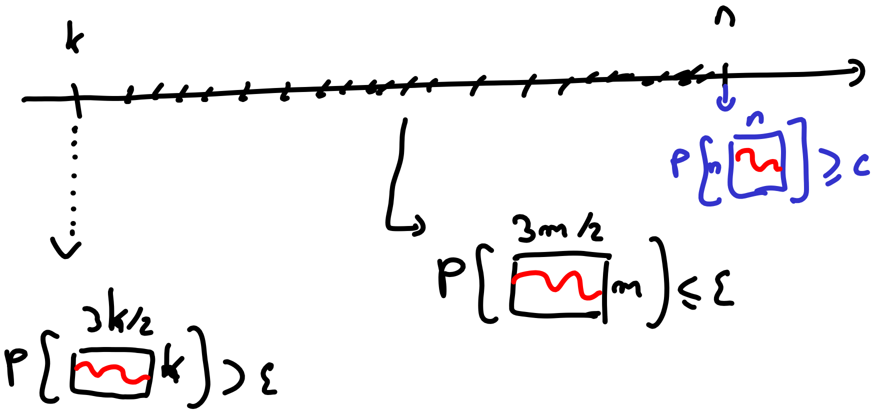
Sketch of proof

Hyp: $P\left[\overset{n}{\boxed{\sim}}\right] \geqslant c \quad \forall n$

Goal: $P\left[n\, \overset{3n/2}{\boxed{\sim\sim}} \right] \geqslant c'$

Assumption: $P\left[n\, \underset{3n/2}{\boxed{\sim\sim}} \right] \leqslant \varepsilon \quad \binom{\varepsilon > 0}{\text{cst}}$

Def: $k = \min \left\{ m \leqslant n : P\left[m\, \underset{3m/2}{\boxed{\sim\sim}} \right] \leqslant \varepsilon \right\} - 1$



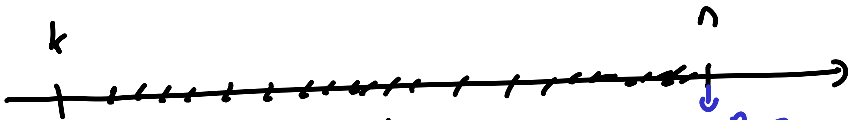
Sketch of proof

Hyp: $P\left[\,\overset{n}{\boxed{\sim}}\,\right] \geq c \quad \forall n$

Goal: $P\left[\,{}_{n}\overset{3n/2}{\boxed{\sim\sim}}\,\right] \geq c'$

<u>Assumption</u>: $P\left[\,{}_{n}\underset{3n/2}{\boxed{\sim\sim}}\,\right] \leq \varepsilon \quad \left(\begin{smallmatrix}\varepsilon > 0\\ \text{cst}\end{smallmatrix}\right)$

<u>Def</u>: $k = \min\left\{ m \leq n : P\left[\,{}_{m}\underset{3m/2}{\boxed{\sim\sim}}\,\right] \leq \varepsilon \right\} - 1$

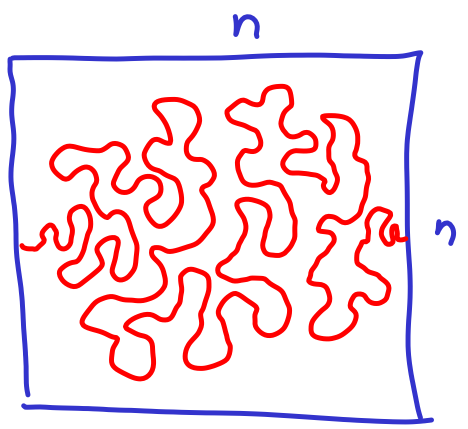


$P\left[\,{}_{n}\overset{n}{\boxed{\sim}}\,\right] \geq c$

$P\left[\,\overset{3m/2}{\boxed{\sim\sim}}\,{}_{m}\right] \leq \varepsilon$

$P\left[\,\overset{3k/2}{\boxed{\sim\sim}}\,{}_{k}\right] > \varepsilon$

→ the path is "tortuous" up to k



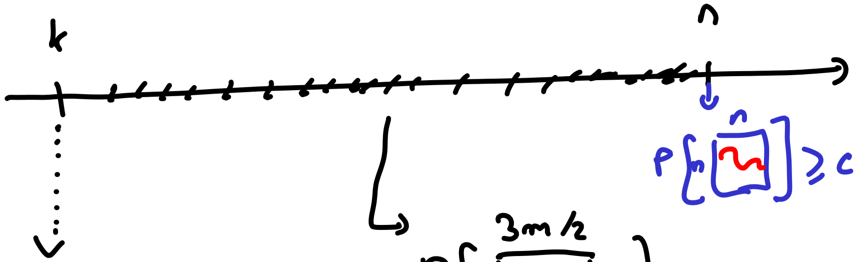
Sketch of proof

Hyp: $P\left[\ \overset{n}{\boxed{}}\ \right] \geq c \quad \forall n$

Goal: $P\left[\ n\ \overset{3n/2}{\boxed{}}\ \right] \geq c'$

Assumption: $P\left[\ n\ \underset{3n/2}{\boxed{}}\ \right] \leq \varepsilon \quad \left(\begin{matrix}\varepsilon > 0 \\ \text{cst}\end{matrix}\right)$

Def: $k = \min \left\{ m \leq n : P\left[\ m\ \underset{3m/2}{\boxed{}}\ \right] \leq \varepsilon \right\} - 1$

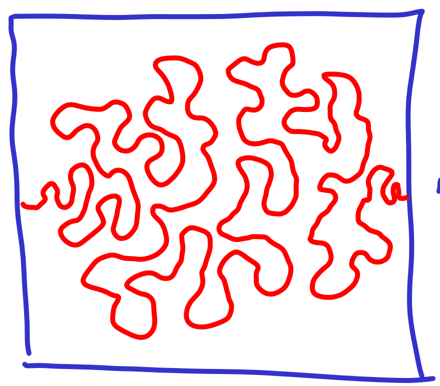


$P\left[\ n\ \overset{n}{\boxed{}}\ \right] \geq c$

$P\left[\ \overset{3m/2}{\boxed{}}\ m\ \right] \leq \varepsilon$

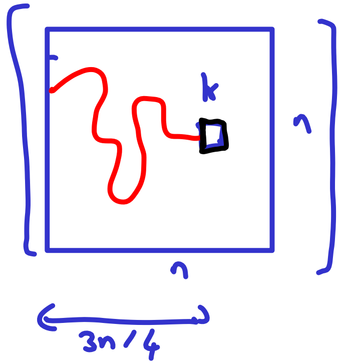
$P\left[\ \overset{3k/2}{\boxed{}}\ k\ \right] > \varepsilon$

$\rightarrow$ the path is "tortuous" up to k



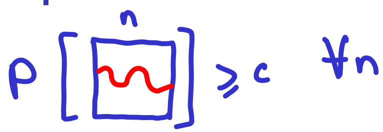
$\Leftarrow$ $P\left[\ \boxed{}\ \right] \geq \frac{c}{10}$

key lemma.

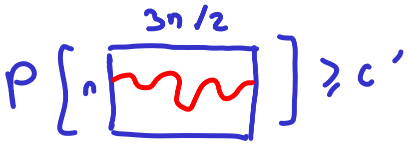


Sketch of proof

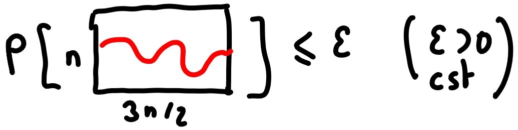
Hyp:



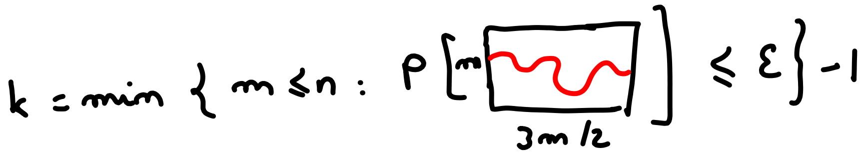
Goal:

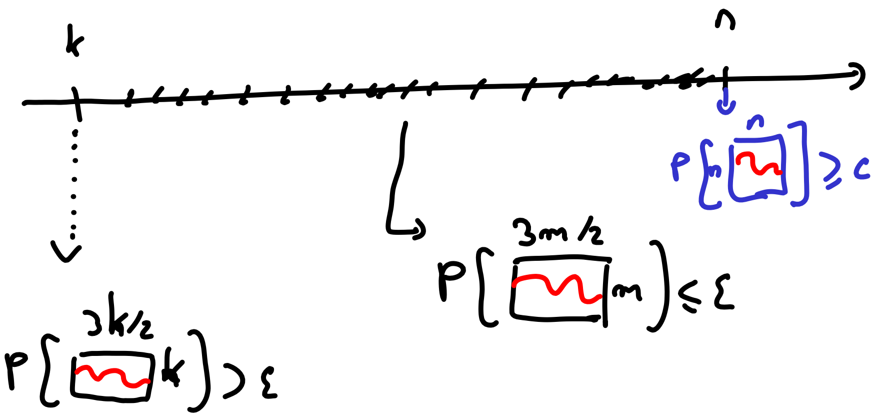


Assumption:

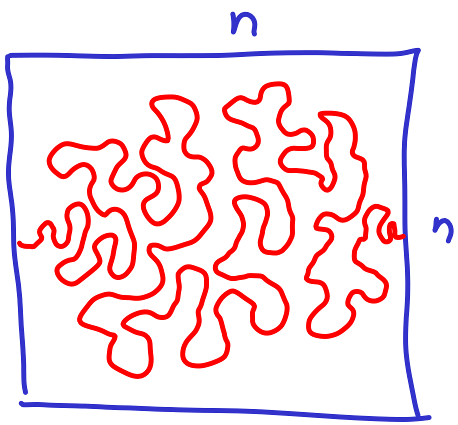


Def:

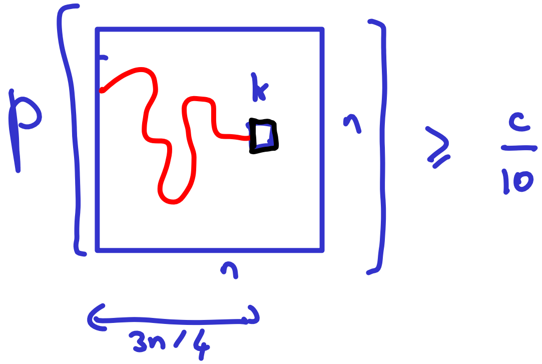




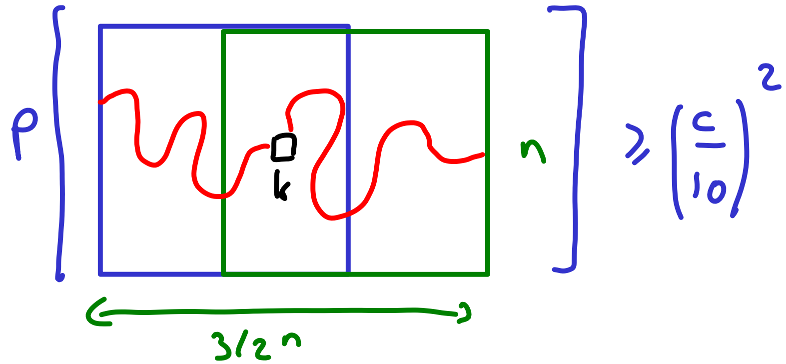
→ the path is "tortuous" up to k



⇐ key lemma.



Conclusion:



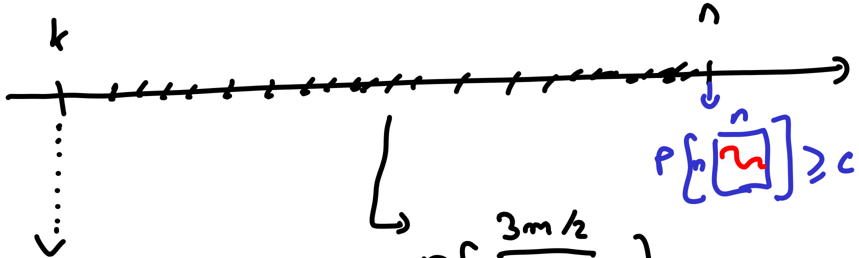
Sketch of proof

Hyp: $\mathbb{P}\left[\,n\boxed{\sim}\,\right] \geq c \quad \forall n$

Goal: $\mathbb{P}\left[\,n\boxed{\sim\sim}^{3n/2}\,\right] \geq c'$

Assumption: $\mathbb{P}\left[\,n\boxed{\sim\sim}_{3n/2}\,\right] \leq \varepsilon \quad \left(\begin{smallmatrix}\varepsilon>0\\ \text{cst}\end{smallmatrix}\right)$

Def: $k = \min\left\{ m \leq n : \mathbb{P}\left[\,m\boxed{\sim\sim}_{3m/2}\,\right] \leq \varepsilon \right\} - 1$

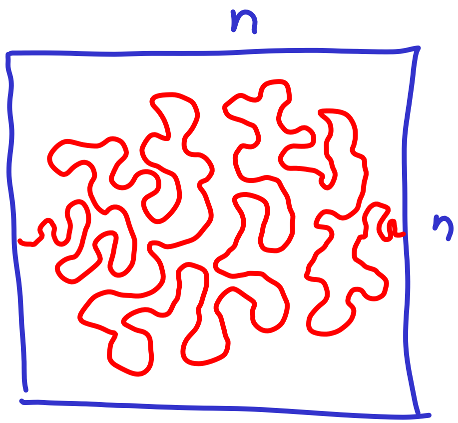


$\mathbb{P}\left[\,n\boxed{\sim}^{n}\,\right] \geq c$

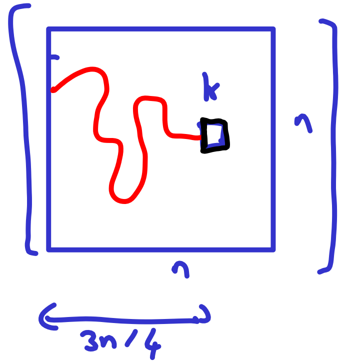
$\mathbb{P}\left[\,\boxed{\sim\sim}^{3m/2}m\,\right] \leq \varepsilon$

$\mathbb{P}\left[\,\boxed{\sim\sim}^{3k/2}k\,\right] > \varepsilon$

→ the path is "tortuous" up to k

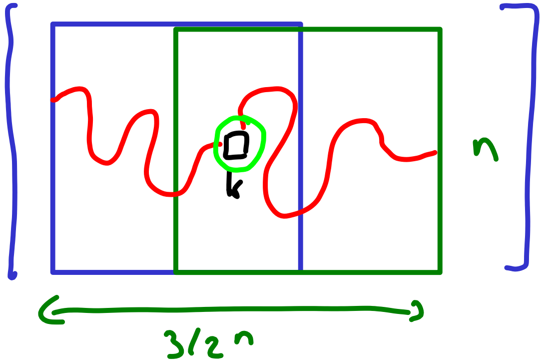


$\stackrel{\text{key lemma.}}{\Longleftarrow}$



$\mathbb{P}[\ \cdots\ ] \geq \dfrac{c}{10}$

Conclusion:

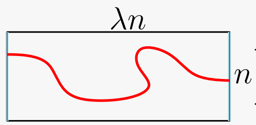


$\mathbb{P}[\ \cdots\ ] \geq \left(\dfrac{c}{10}\right)^2 \times \varepsilon^{100}$

Conclusion and outlook

Critical behavior of Bernoulli percolation

- ✔ Robust RSW theory:

$c(\lambda) \leqslant \mathrm{P}_{p_c}\big[$ 

$\big] \leqslant 1 - c(\lambda).$

Conclusion and outlook

Critical behavior of Bernoulli percolation

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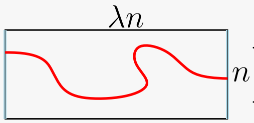
$c(\lambda) \leqslant \mathrm{P}_{p_c}\big[$ 

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Perspectives:

Conclusion and outlook

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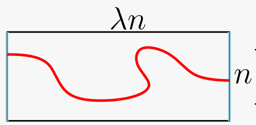
$c(\lambda) \leqslant \mathrm{P}_{p_c}\big[$ 

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Perspectives:

- RSW for non positively correlated models.

Conclusion and outlook

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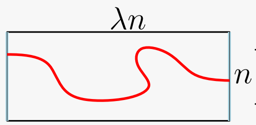
$c(\lambda) \leqslant \mathrm{P}_{p_c}\Big[$ 

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Perspectives:

- RSW for non positively correlated models.
- Prove Cardy's formula for some specific models.

Conclusion and outlook

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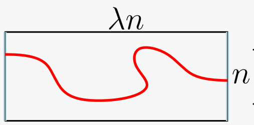

$$\Big] \leqslant 1 - c(\lambda).$$

Perspectives:

- RSW for non positively correlated models.
- Prove Cardy's formula for some specific models.
- Universality and scaling limit.

Conclusion and outlook

Critical behavior of Bernoulli percolation

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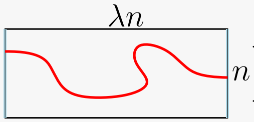
$c(\lambda) \leqslant \mathrm{P}_{p_c}\big[$ 

$\big] \leqslant 1 - c(\lambda).$

Perspectives:

- RSW for non positively correlated models.
- Prove Cardy's formula for some specific models.
- Universality and scaling limit.
- RSW theory in dimension 3.

Conclusion and outlook

Critical behavior of Bernoulli percolation

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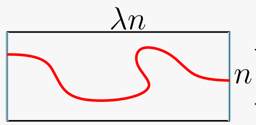

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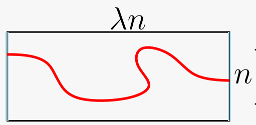
$c(\lambda) \leqslant \mathrm{P}_{p_c}\big[$ 

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