Numerical Solution of ODEs – Problem Sheet II

1. Write down the general form of a linear multi-step method for the numerical solution of the initial value problem

$$y' = f(x, y) , \qquad y(x_0) = y_0 ,$$

on the closed real interval $[x_0, x_N]$, where f is a continuous function of its arguments and y_0 is a given real number. Define the *truncation error* of the method. What does it mean to say that the method has *order of accuracy* p?

Given that α is a positive real number, consider the linear two-step method

$$y_{n+2} - \alpha y_n = \frac{h}{3} \left[f(x_{n+2}, y_{n+2}) + 4f(x_{n+1}, y_{n+1}) + f(x_n, y_n) \right] ,$$

on the mesh $\{x_n : x_n = x_0 + nh, n = 1, \ldots, N\}$ of spacing h, $h > 0$. Determine the set of all α such that the method is zero-stable. Find α such that the order of accuracy is as high as possible; is the method convergent for this value of α?

2. If $\sigma(z) = z^2$ is the second characteristic polynomial of a linear multistep method, find a quadratic polynomial $\rho(z)$ such that the order of the associated linear multistep method is 2. Use the Routh-Hurwitz criterion to determine the interval of absolute stability of the method.

3. Find the interval of absolute stability for the two-step Adams–Bashforth method

$$y_{n+2} - y_{n+1} = \frac{1}{2} h \left[3f_{n+1} - f_n \right]$$

using Schur's criterion.

4. A predictor P and a corrector C are defined by their characteristic polynomials:

$$\begin{array}{lll} P: & \rho^*(z) = z^4 - 1 , & \sigma^*(z) = \frac{4}{3}(2z^3 - z^2 + 2z) , \\ C: & \rho(z) = z^2 - 1 , & \sigma(z) = \frac{1}{3}(z^2 + 4z + 1) . \end{array}$$

   a) Write down algorithms which use P and C in the $P(EC)^m E$ and $P(EC)^m$ modes.

   b) Express the truncation errors $T_n^{P(EC)^m E}$ and $T_n^{P(EC)^m}$ of these methods in the form $O(h^r)$ where $r = r(p^*, p, m)$, with p^* and p denoting the orders of accuracy of P and C, respectively.