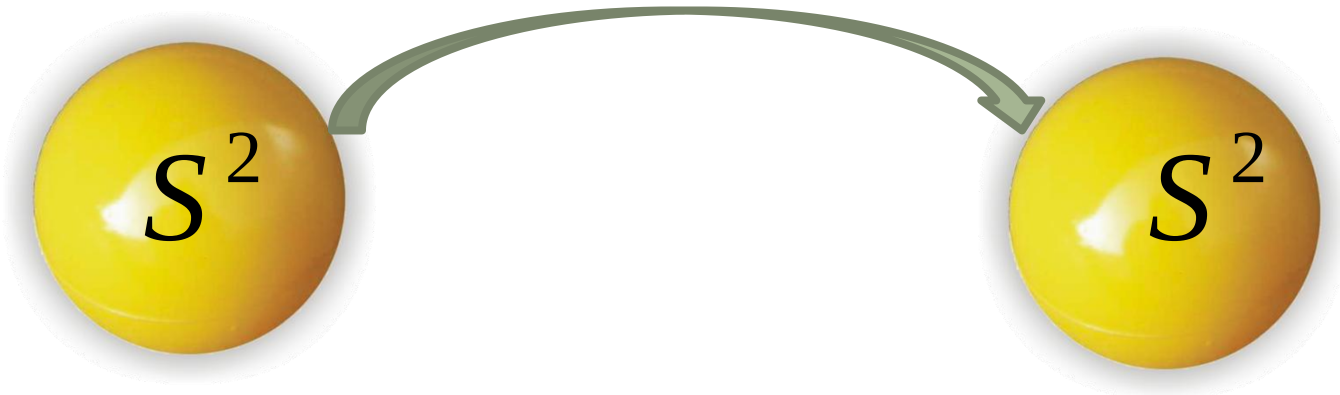
Magnetic bubbles & knots

Paul Sutcliffe
Department of Mathematical Sciences
Durham University

A topological soliton is a particle-like solution of a nonlinear partial differential equation, in which stability is due to some non-trivial topology.

Applications in
particle physics,
cosmology,
nuclear physics,
condensed matter physics,
....

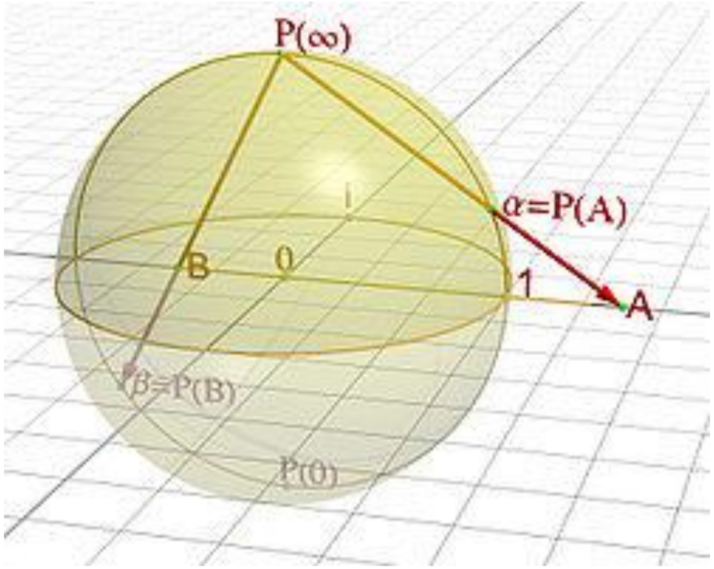
Topology of maps between spheres



$$N \in \pi_2(S^2)$$

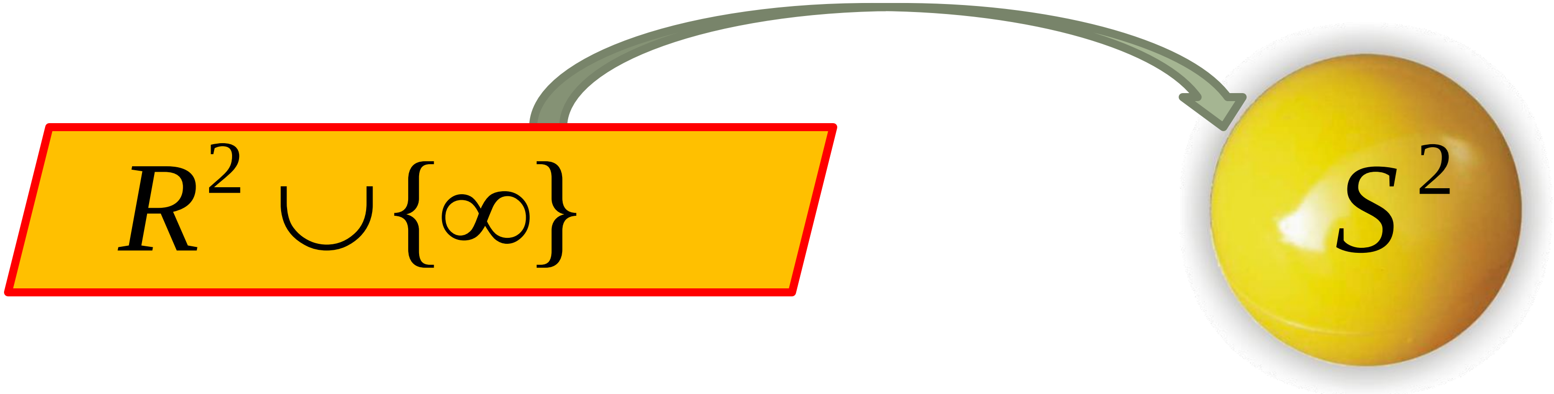
N is the degree of the map, and will be identified with the number of solitons.

Stereographic projection



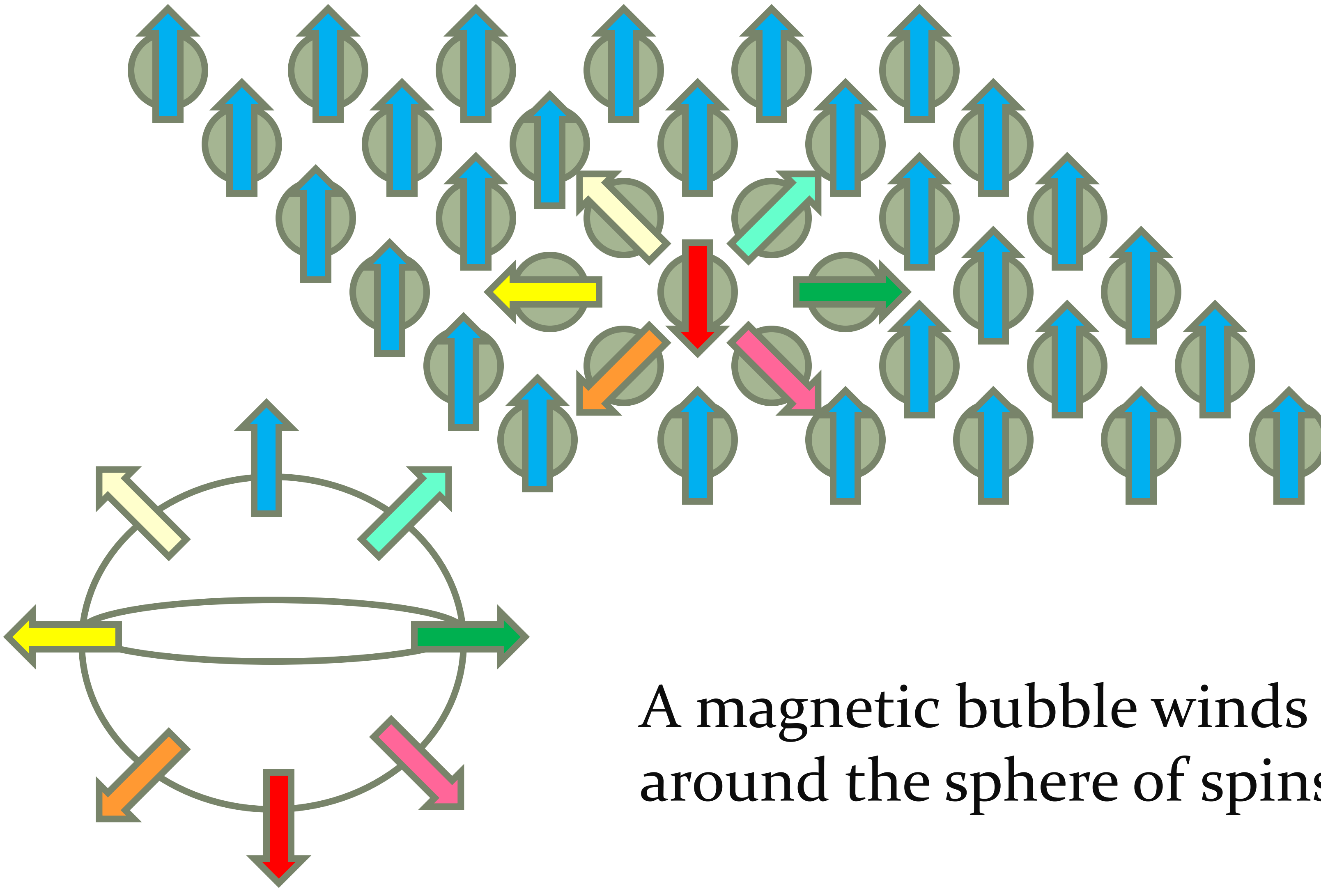
$$S^2 = R^2 \cup \{\infty\}$$

Maps from the plane to the sphere



$$N \in \pi_2(S^2)$$

Magnetic bubbles in a planar ferromagnet



A magnetic bubble winds once around the sphere of spins

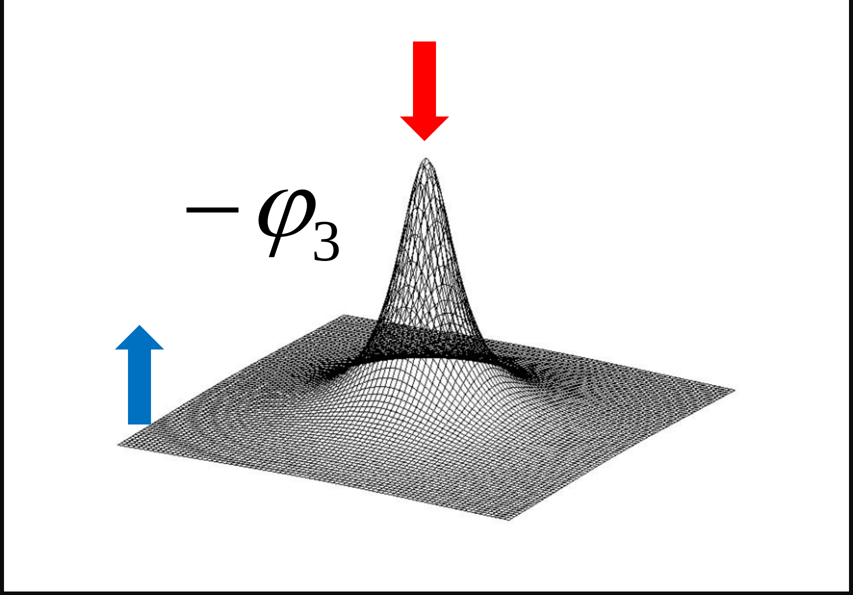
Continuum approximation

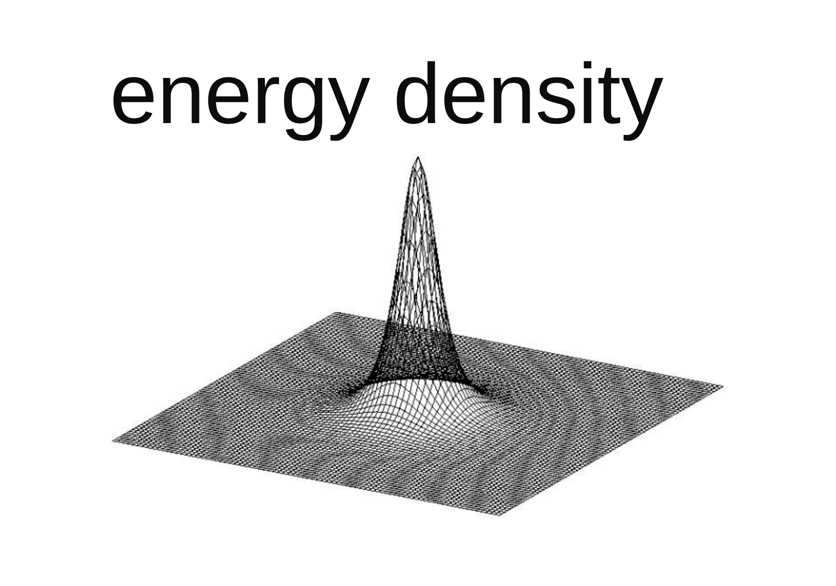
$$\vec{\varphi}(x,y) = (\varphi_1, \varphi_2, \varphi_3), \quad \vec{\varphi}\cdot\vec{\varphi} = 1$$

$$E = \int \left(\frac{\partial\vec{\varphi}}{\partial x}\cdot\frac{\partial\vec{\varphi}}{\partial x} + \frac{\partial\vec{\varphi}}{\partial y}\cdot\frac{\partial\vec{\varphi}}{\partial y} \right) dxdy$$

$$\vec{\varphi} = \frac{1}{1+x^2+y^2}\left(2x, \quad 2y, \quad x^2+y^2-1\right)$$

$$\vec{\varphi} = \frac{1}{1+x^2+y^2}\left(2x, \quad 2y, \quad x^2+y^2-1\right)$$

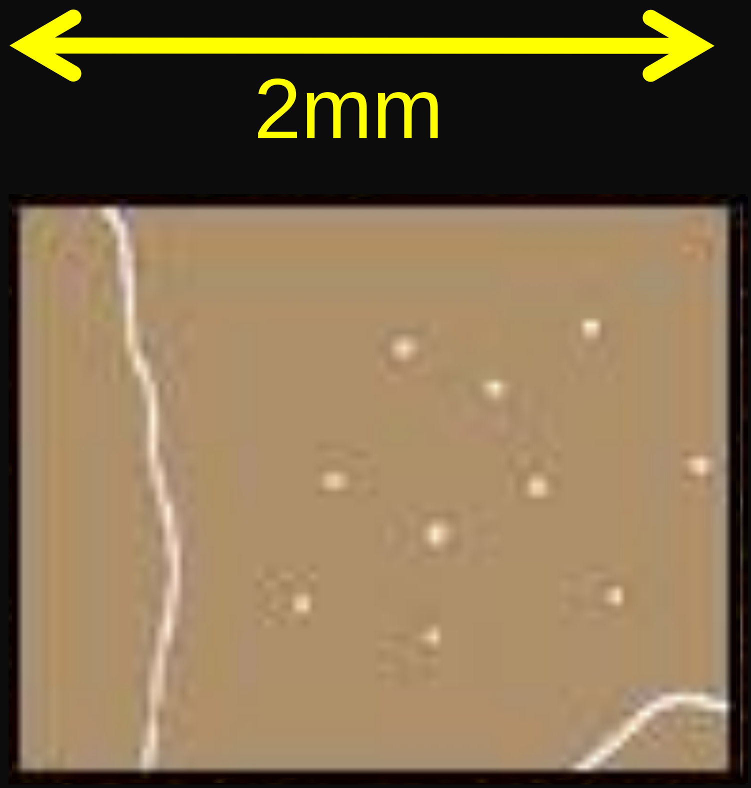




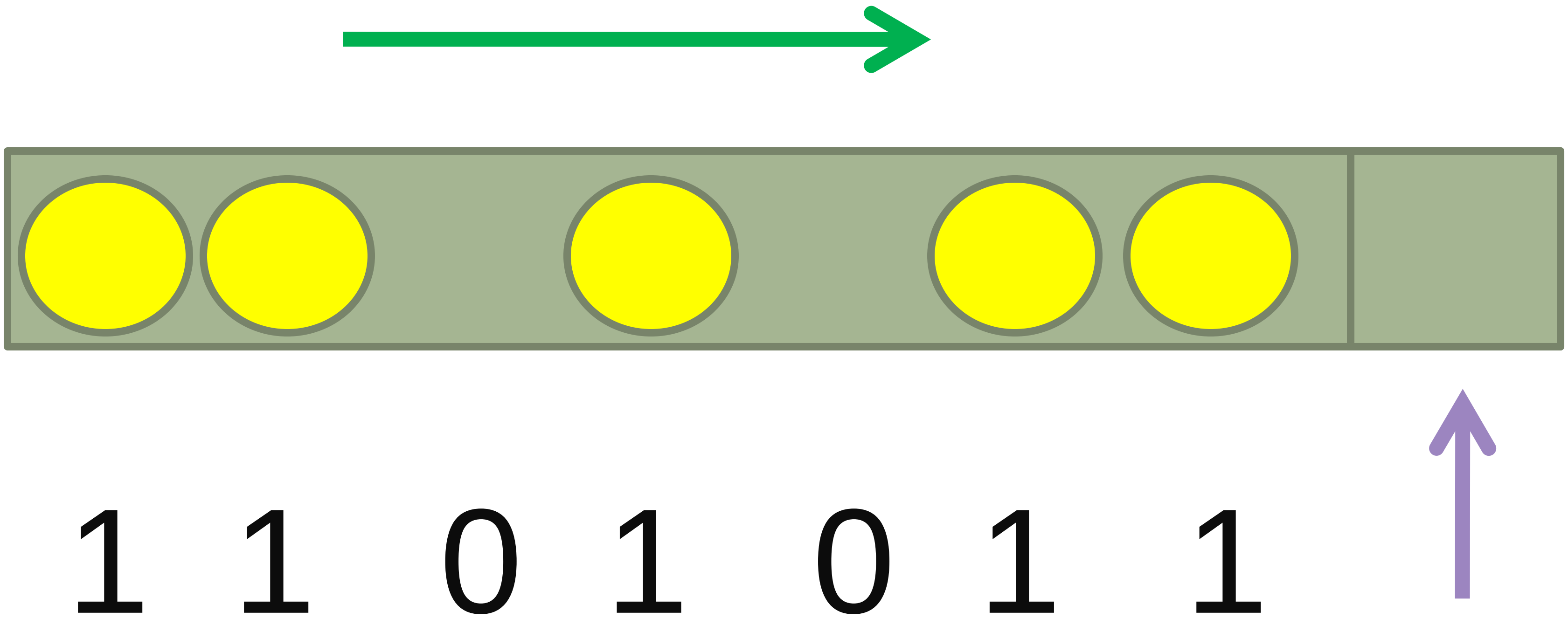
$$E = \int \left(\frac{\partial \vec{\varphi}}{\partial x} \cdot \frac{\partial \vec{\varphi}}{\partial x} + \frac{\partial \vec{\varphi}}{\partial y} \cdot \frac{\partial \vec{\varphi}}{\partial y} \right) dxdy$$

Experiments on Magnetic Bubbles

Experiments on Magnetic Bubbles



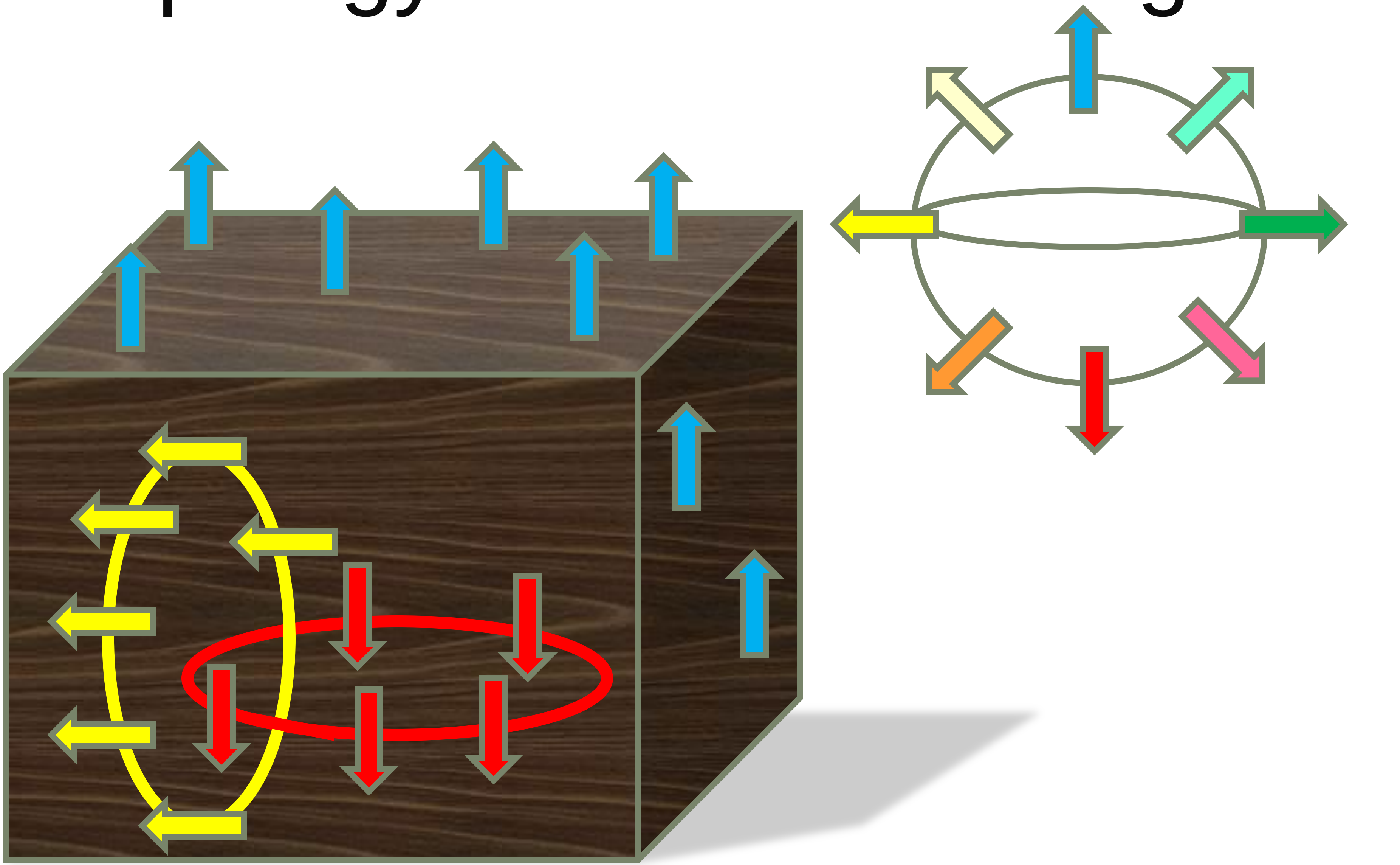
Magnetic Bubble Memory



Intel 1981

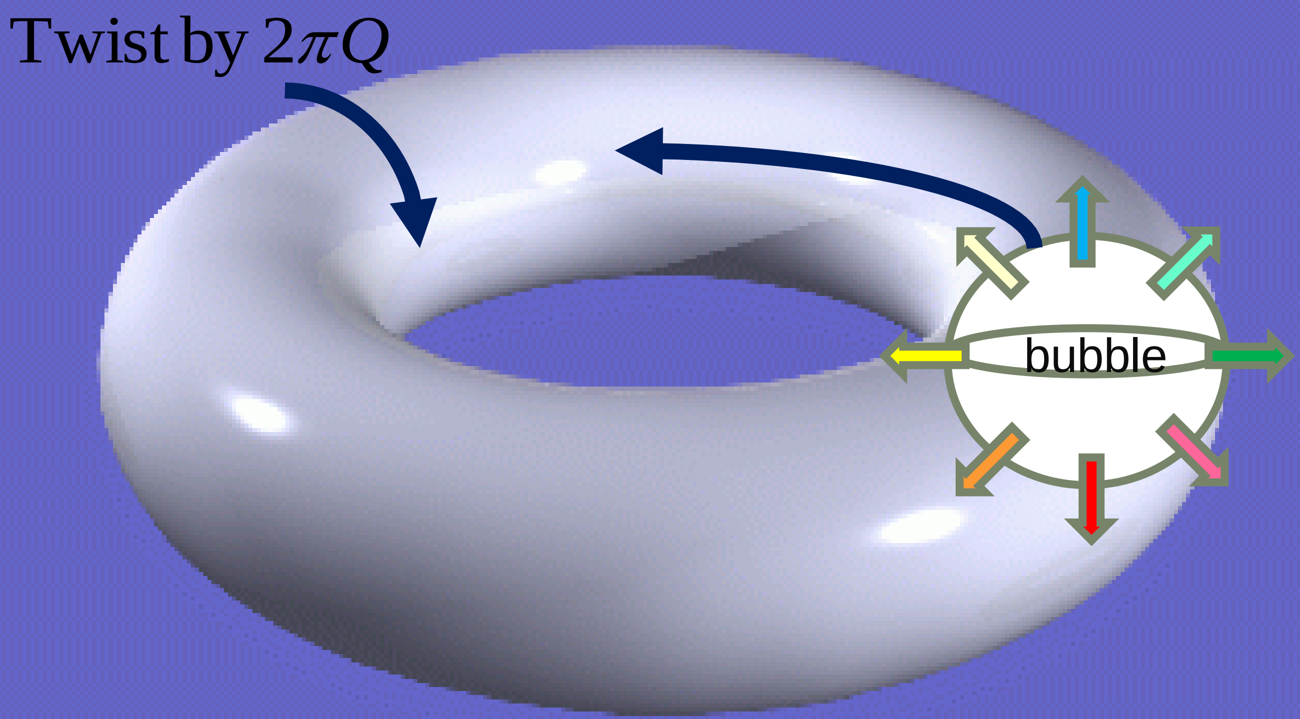

1P 0 HI 19850
GALACTIC
WARRIORS
TM
Konami®
© KONAMI 1985
CREDIT 00

Topology in a 3D ferromagnet

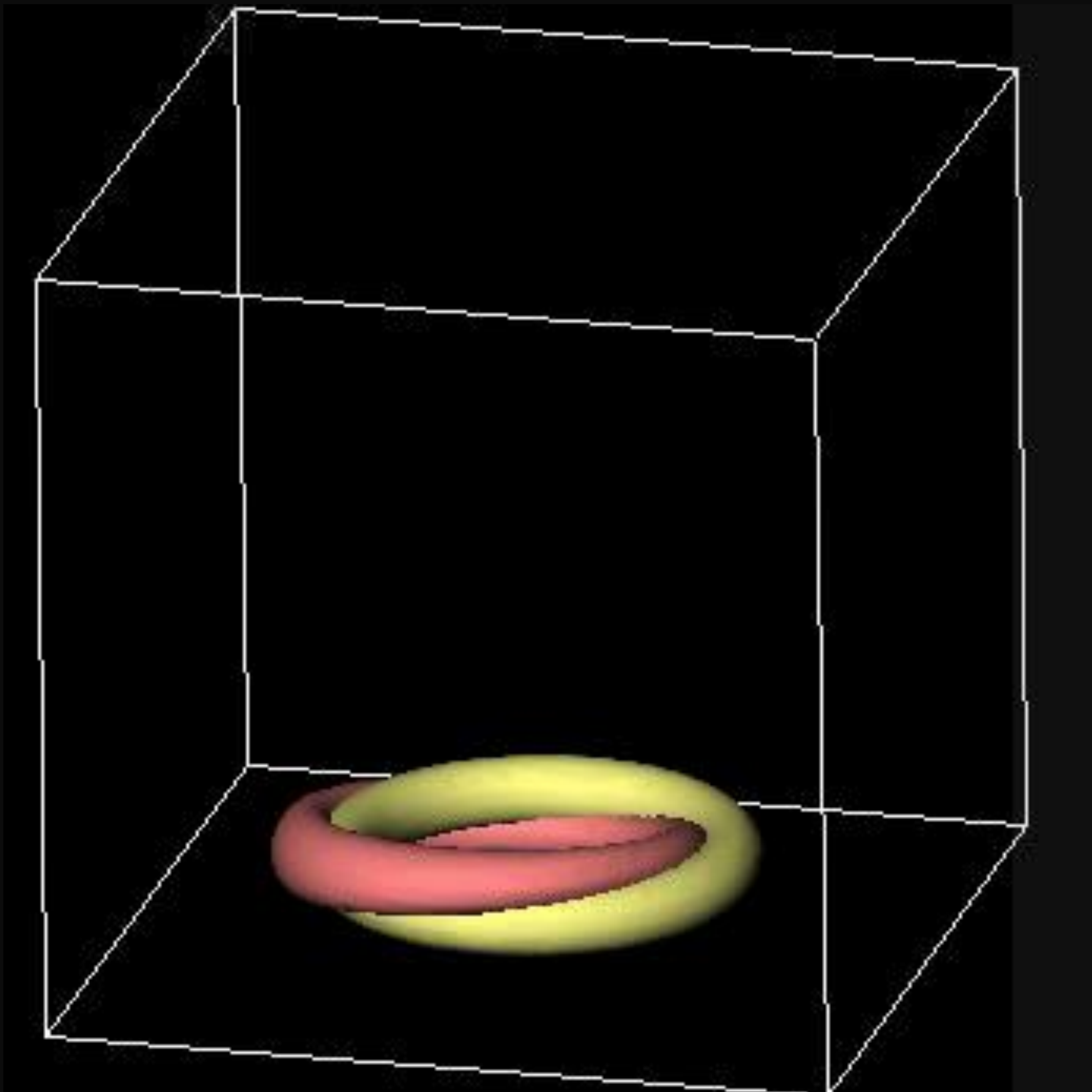


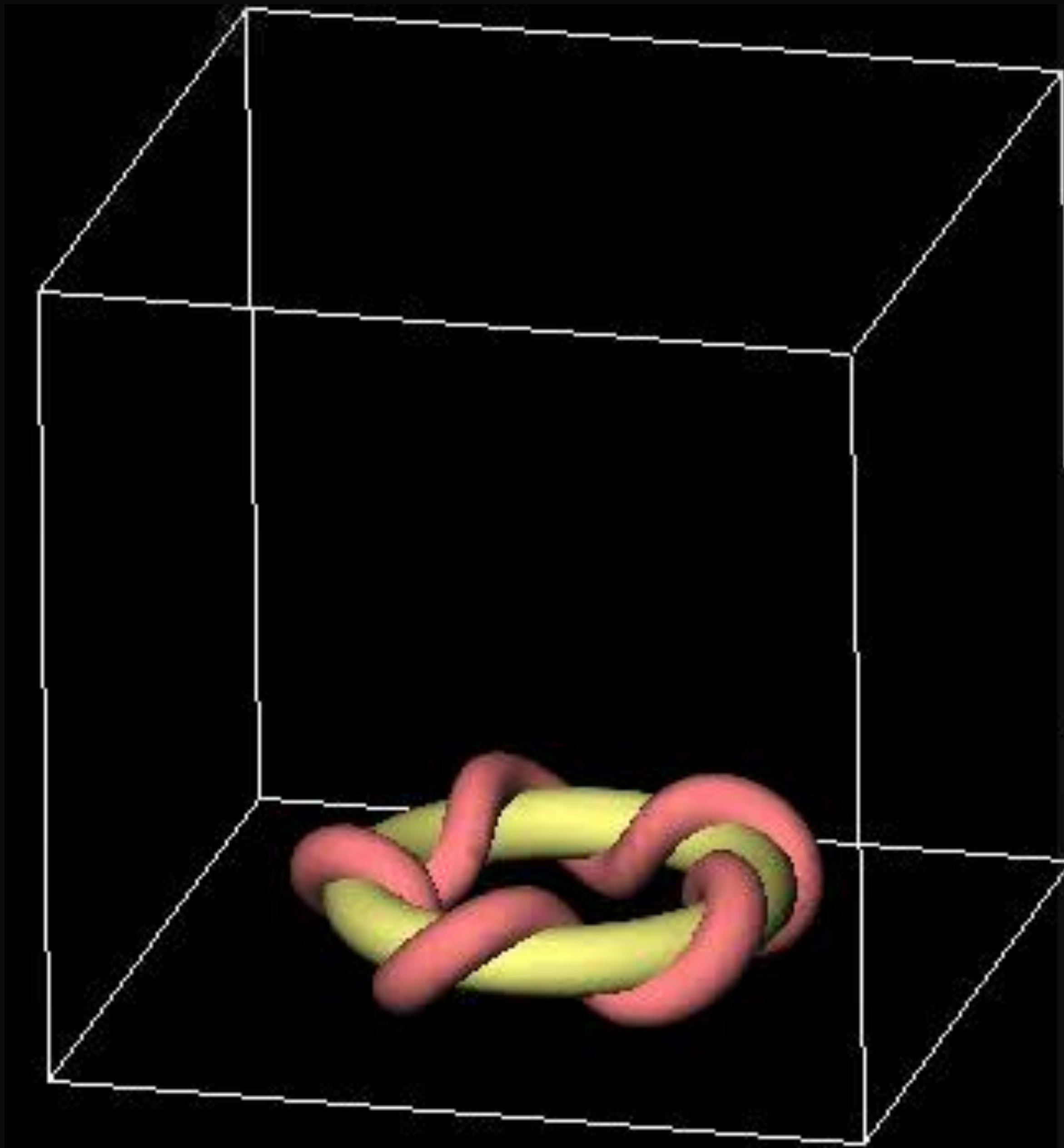
Linking number $Q \in Z$ (Hopf charge)

Magnetic smoke ring



$$\frac{\partial \vec{\varphi}}{\partial t} = \vec{\varphi} \times \nabla^2 \vec{\varphi}$$





Solitons in an *exotic* 3D ferromagnet

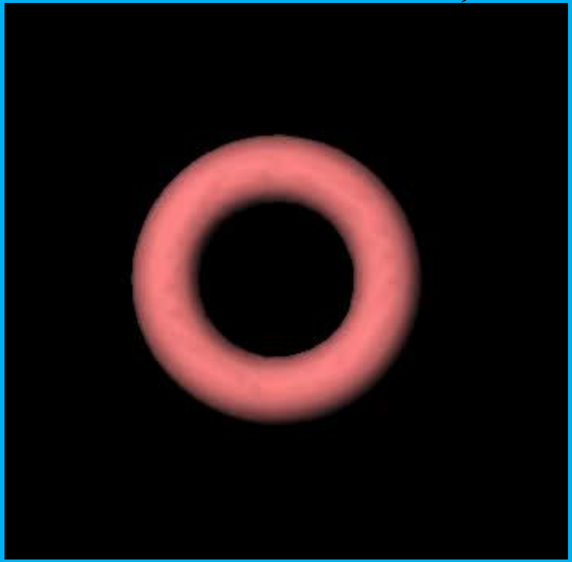
$$E = \int \left(\frac{\partial \vec{\varphi}}{\partial x} \cdot \frac{\partial \vec{\varphi}}{\partial x} + \frac{\partial \vec{\varphi}}{\partial y} \cdot \frac{\partial \vec{\varphi}}{\partial y} + \frac{\partial \vec{\varphi}}{\partial z} \cdot \frac{\partial \vec{\varphi}}{\partial z} \right) dxdydz$$

Solitons in an *exotic* 3D ferromagnet

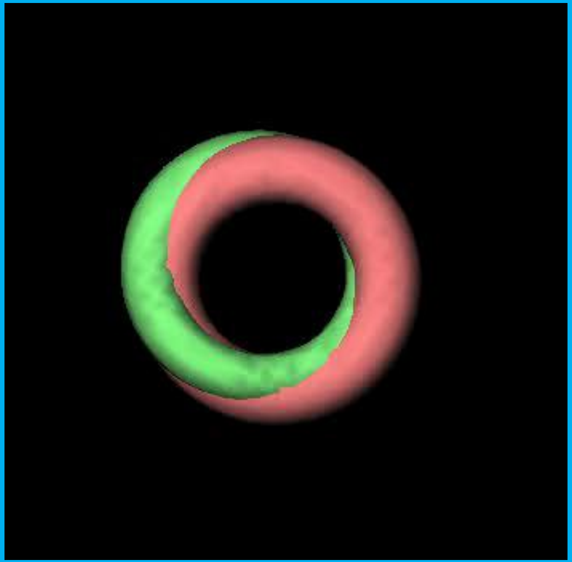
$$E=\int\left(\frac{\partial\vec{\varphi}}{\partial x}\cdot\frac{\partial\vec{\varphi}}{\partial x}+\frac{\partial\vec{\varphi}}{\partial y}\cdot\frac{\partial\vec{\varphi}}{\partial y}+\frac{\partial\vec{\varphi}}{\partial z}\cdot\frac{\partial\vec{\varphi}}{\partial z}\right)dxdydz$$

$$+\int\left(\left(\frac{\partial\vec{\varphi}}{\partial x}\times\frac{\partial\vec{\varphi}}{\partial y}\right)\cdot\left(\frac{\partial\vec{\varphi}}{\partial x}\times\frac{\partial\vec{\varphi}}{\partial y}\right)+cyclic\right)dxdydz$$

$Q=1; \quad \mathbb{A}_{1,1}; \quad E/Q^{3/4}=1.204$

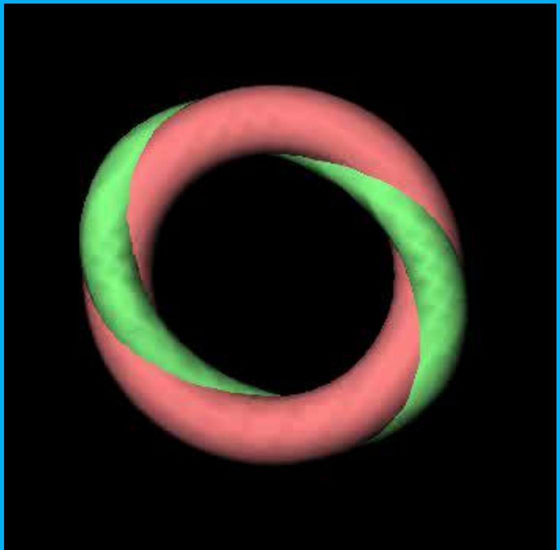


Soliton position (preimage of down).

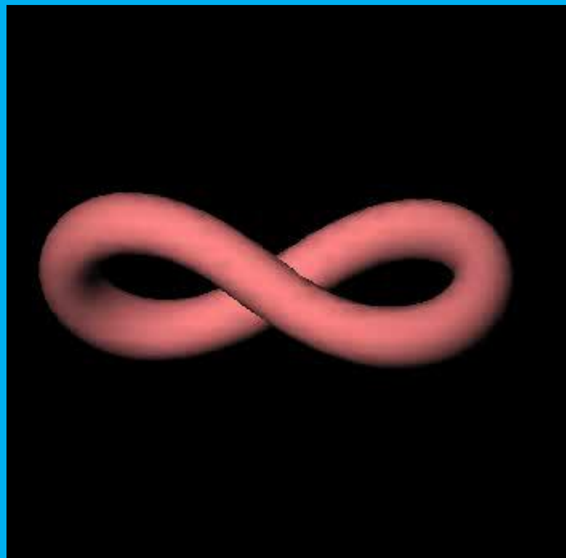


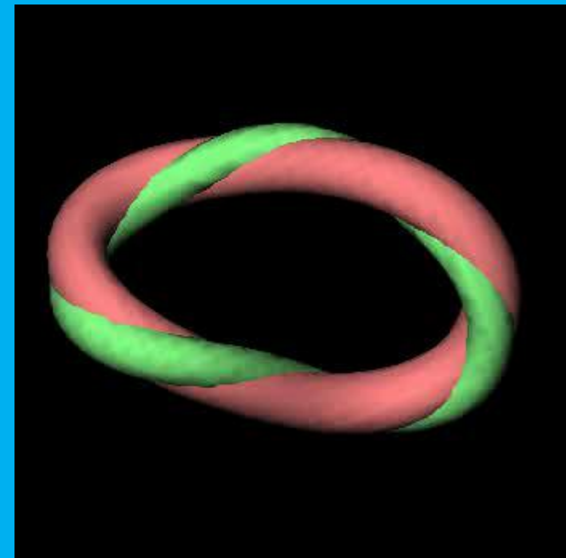
Linking (preimage of 2 points).

$Q=2; \quad \mathbb{A}_{2,1}; \quad E/Q^{3/4}=1.170$

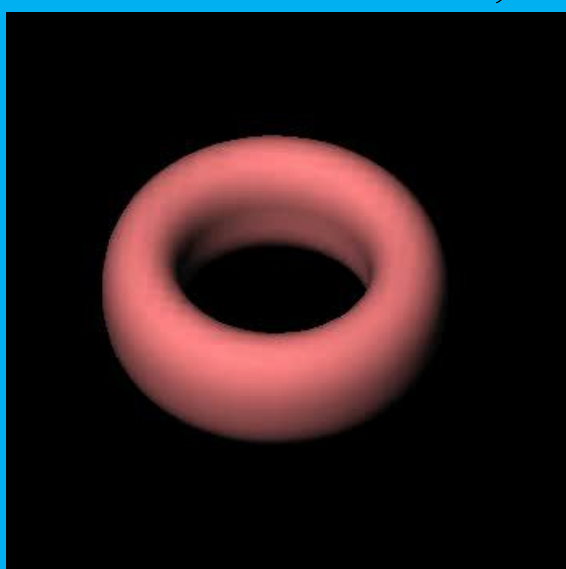


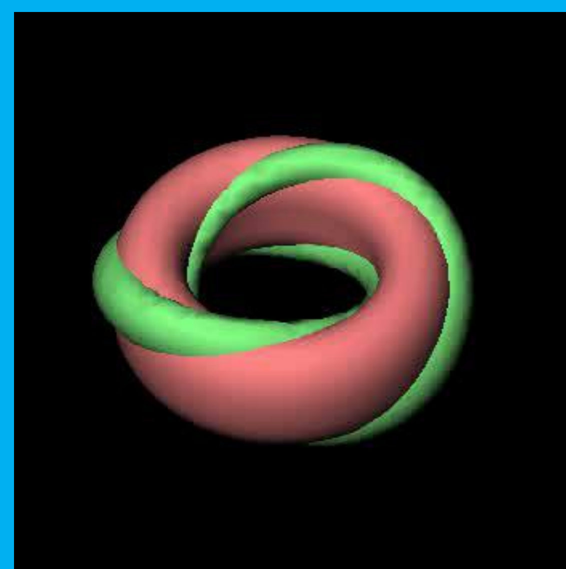
$Q = 3; \quad \widetilde{\mathbb{A}}_{3,1}; \quad E / Q^{3/4} = 1.208$



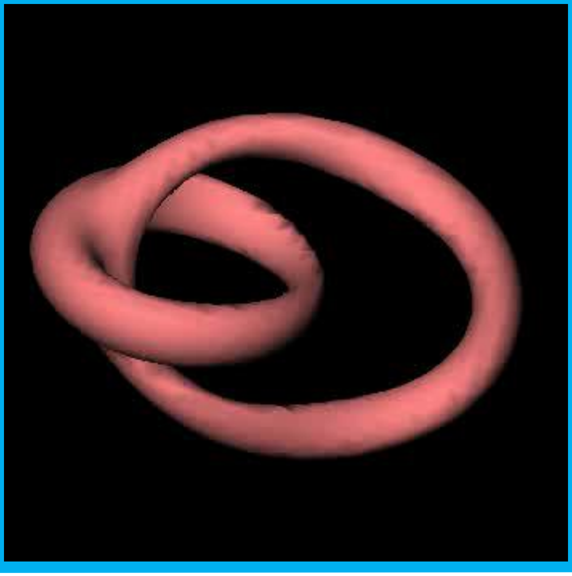


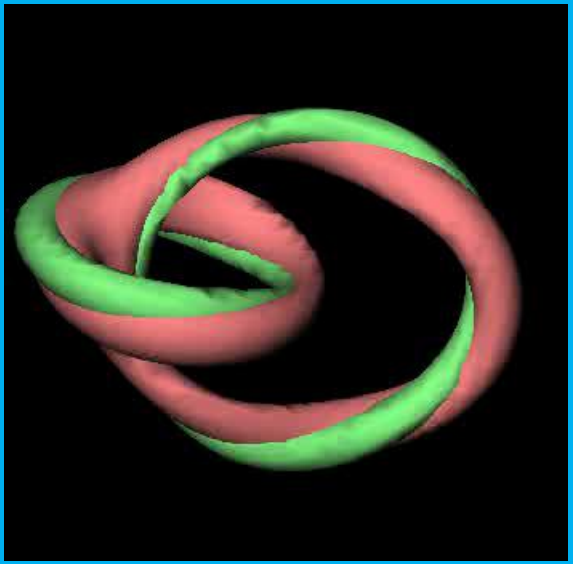
$Q = 4; \quad \widetilde{\mathbb{A}}_{2,2}; \quad E / Q^{3/4} = 1.218$



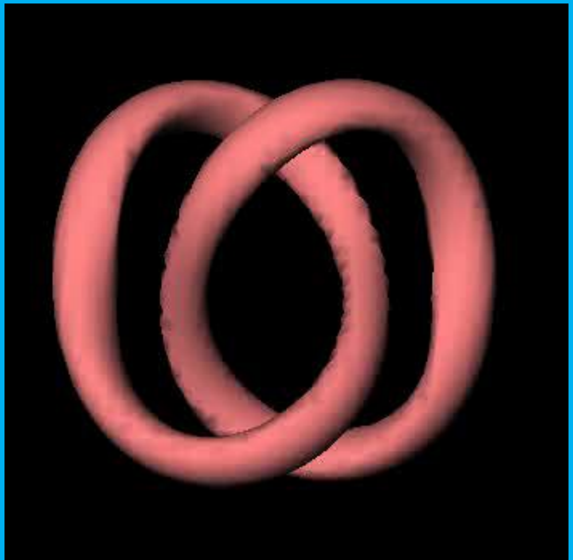


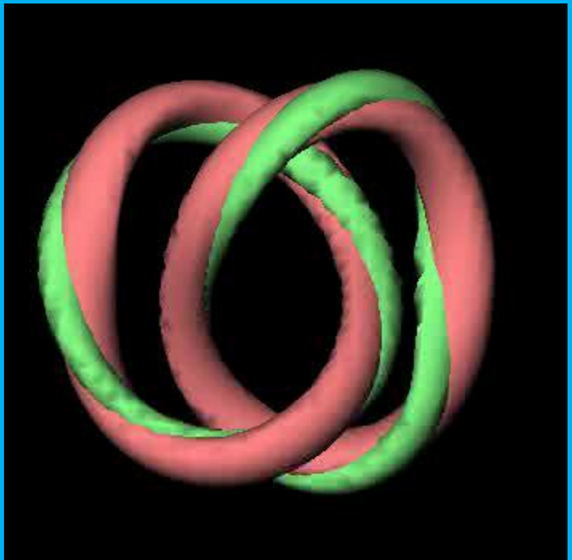
$$Q = 5; \quad \mathbb{L}^{1,1}_{1,2}; \quad E/Q^{3/4} = 1.225$$



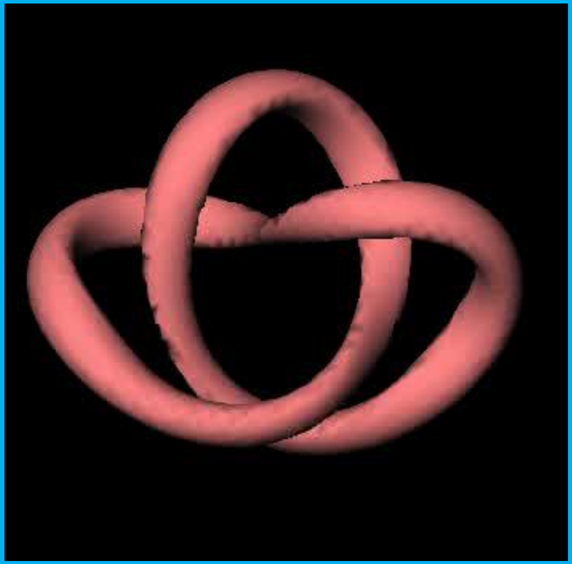


$$Q = 6; \quad \mathbb{L}^{1,1}_{2,2}; \quad E/Q^{3/4} = 1.213$$





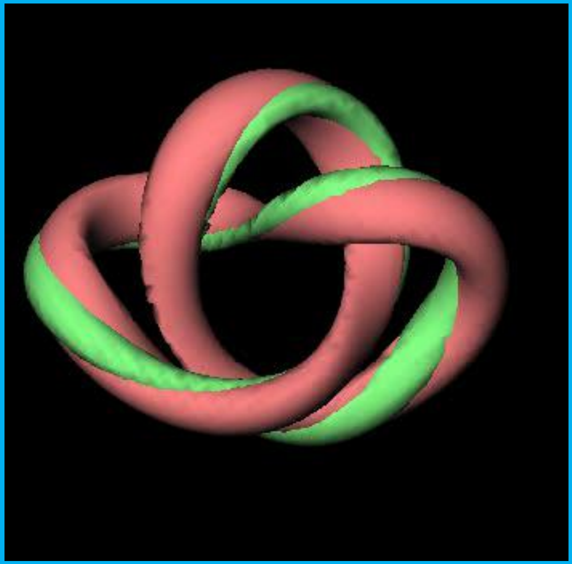
$Q = 7; \quad \mathbb{K}_{3,2}$ (trefoil knot); $\quad E / Q^{3/4} = 1.218$



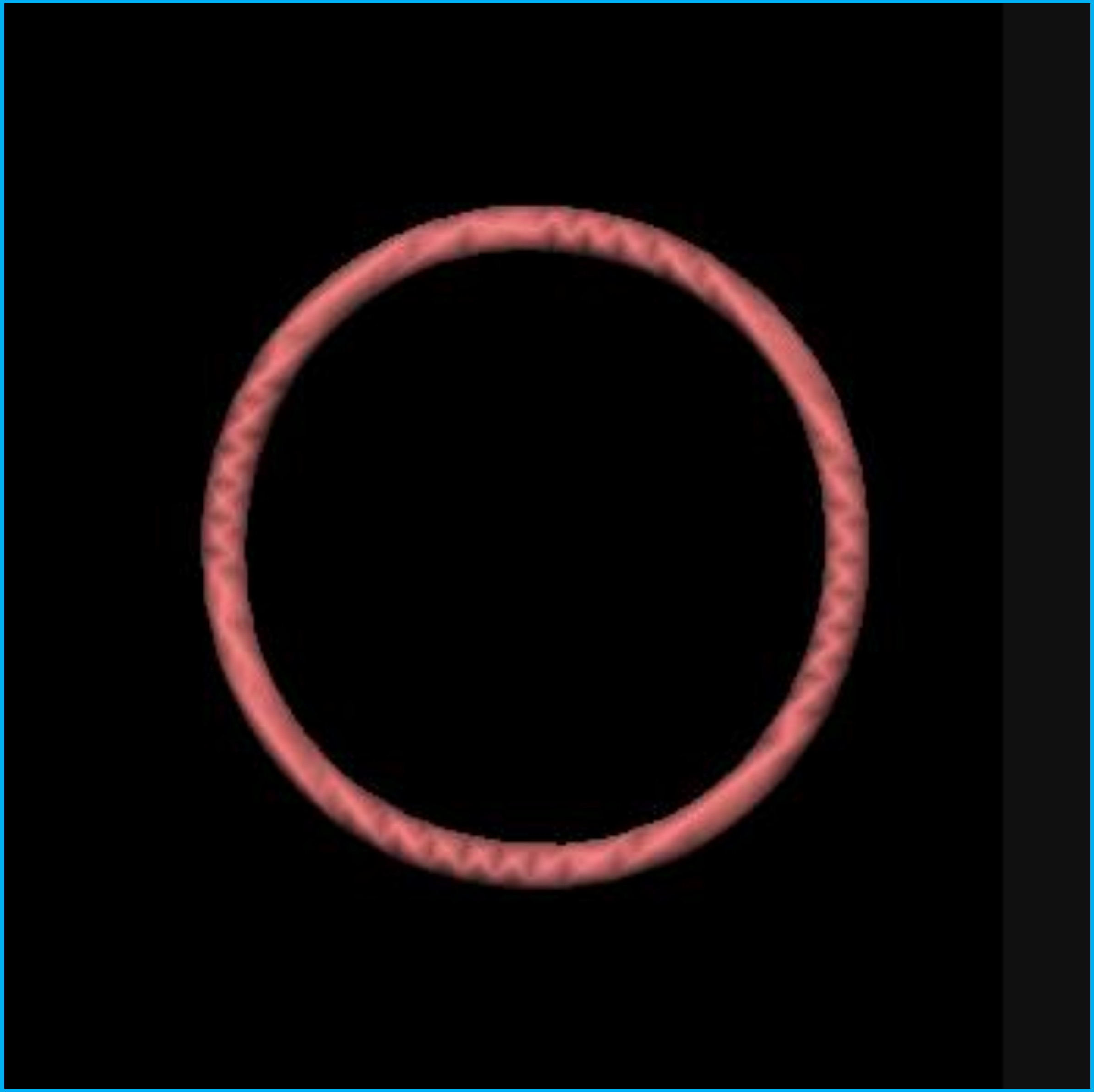
$$Q = 3 + 4 = 7$$

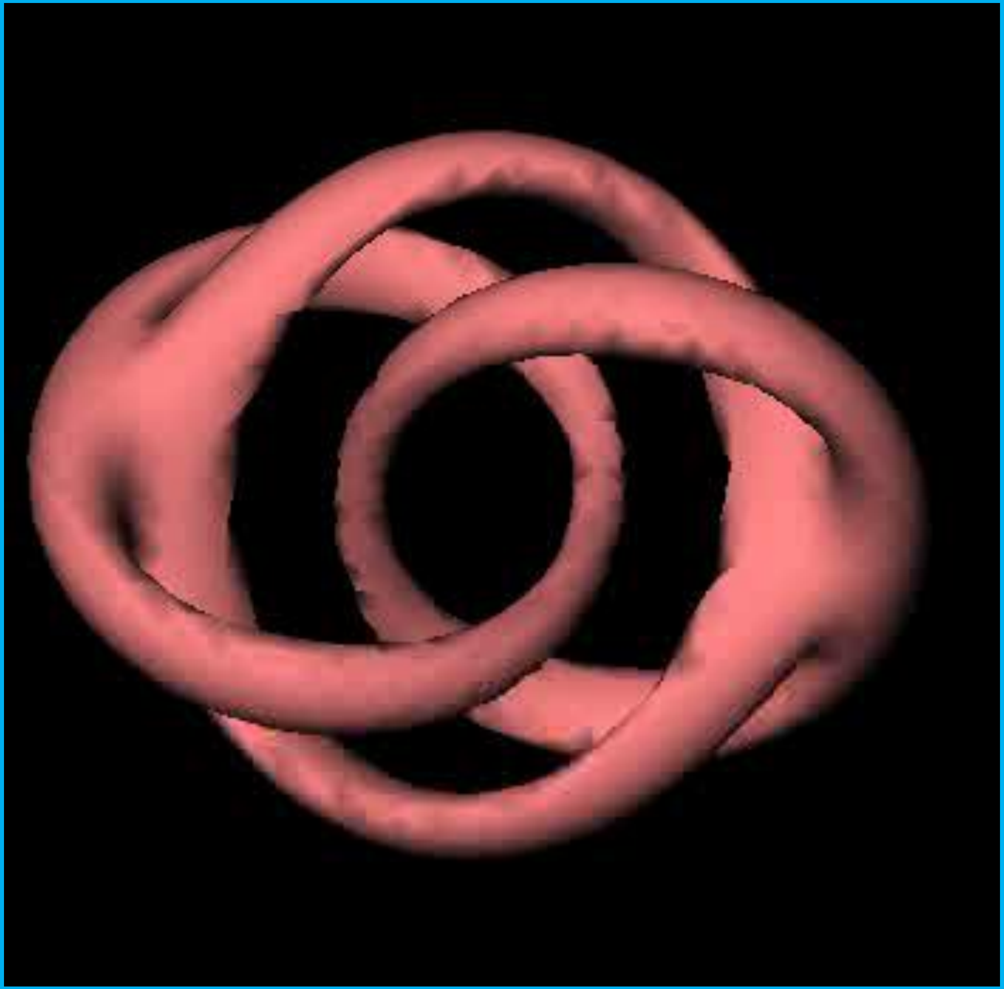
self-linking (crossing)

twist



$Q = 7; \quad \mathbb{A}_{7,1} \to \mathbb{K}_{3,2}$ (trefoil knot); energy minimization.

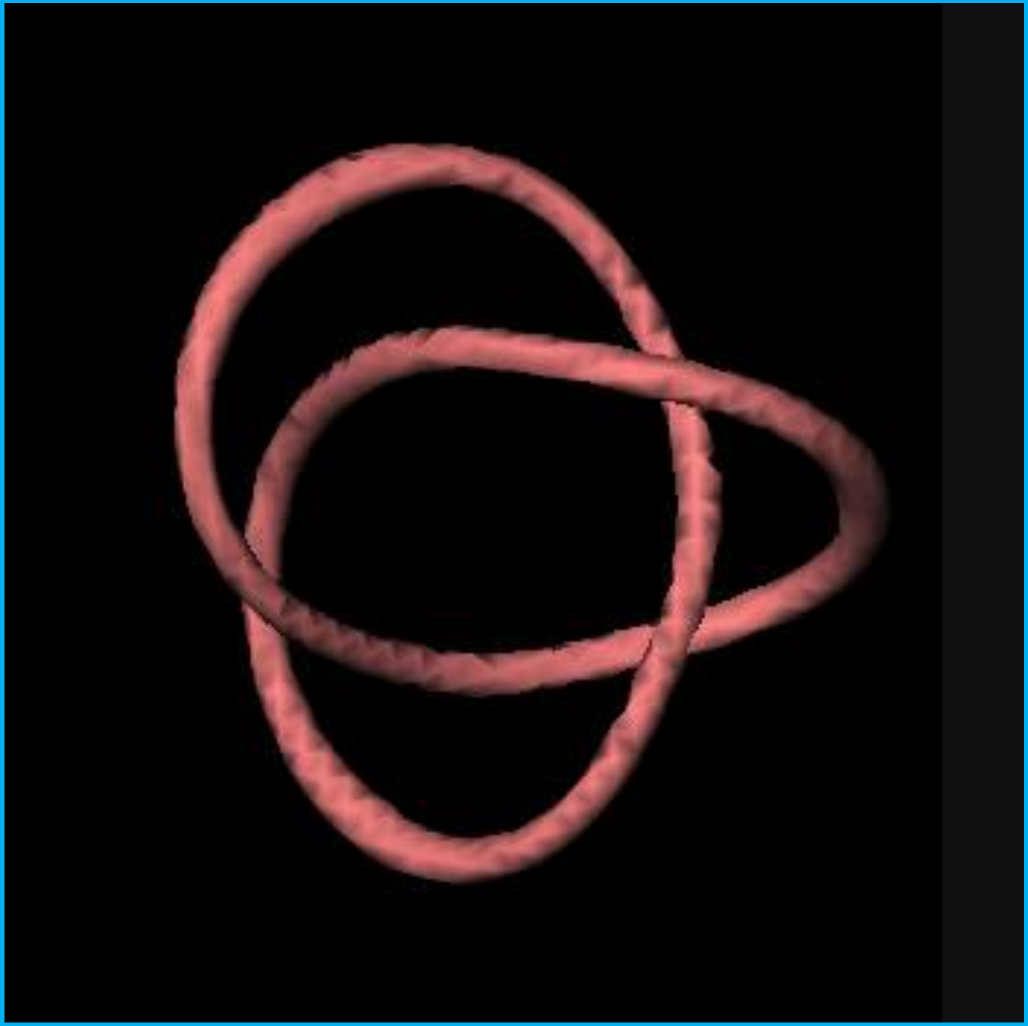


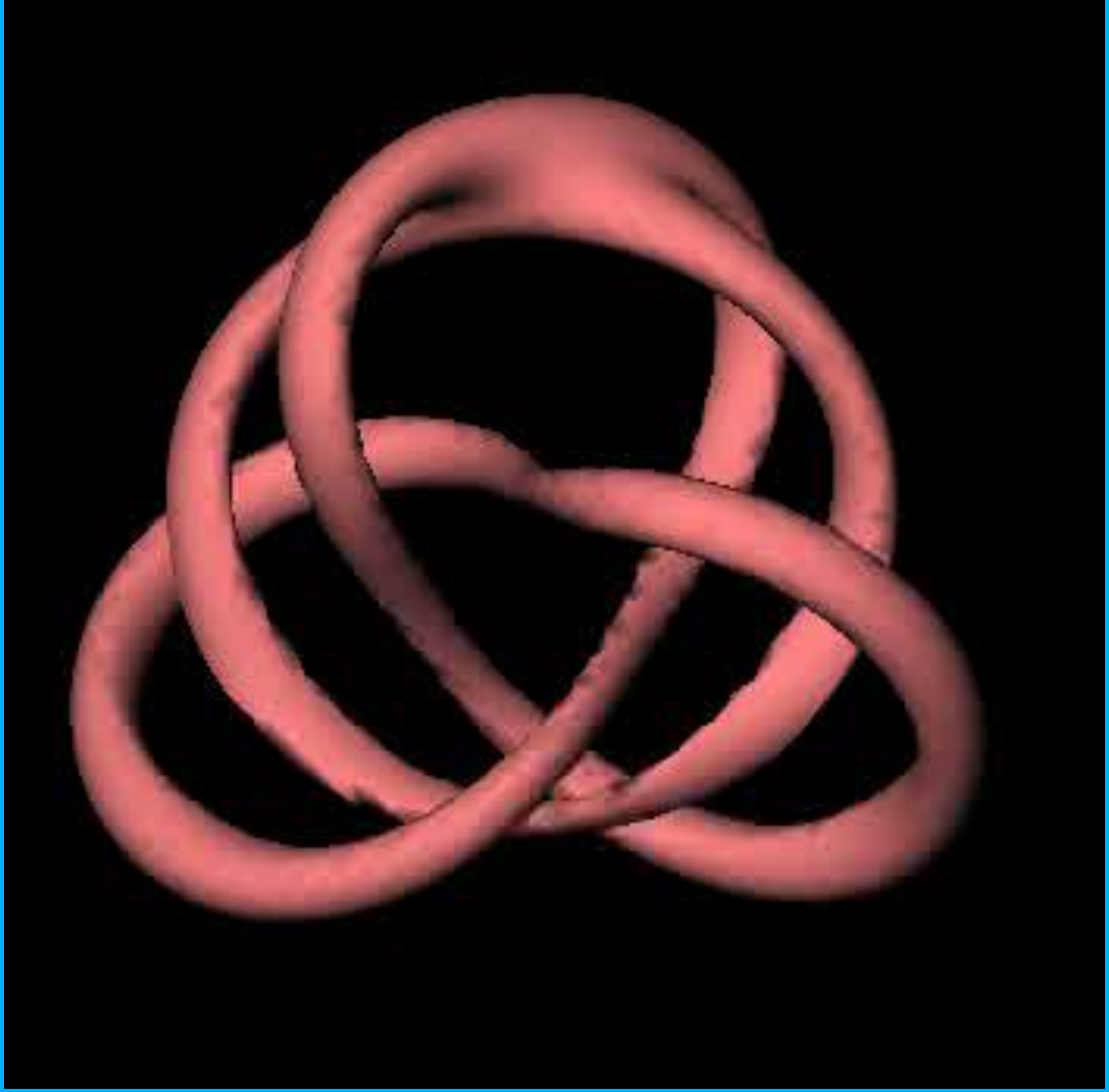


$$Q = 9, \quad \mathbb{L}^{2,2,2}_{1,1,1}$$

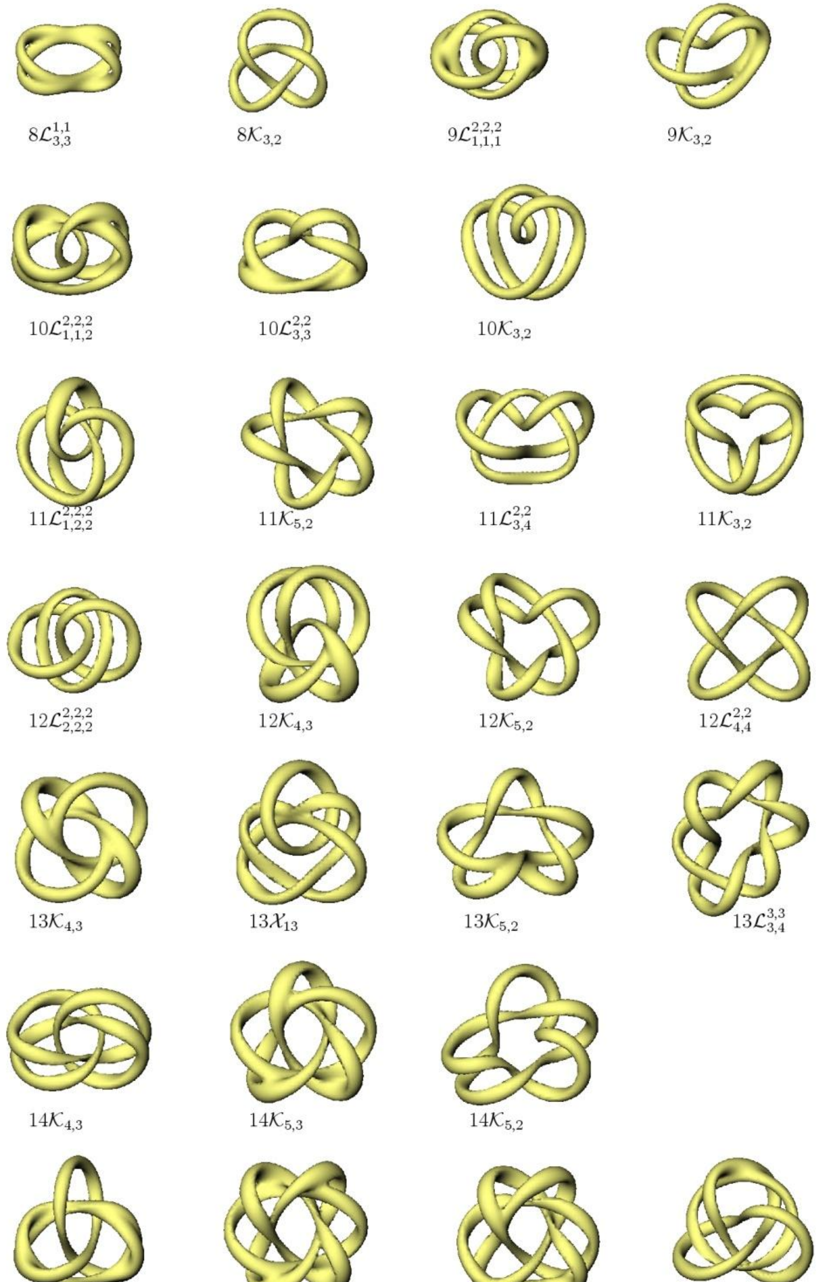
Knot transmutation

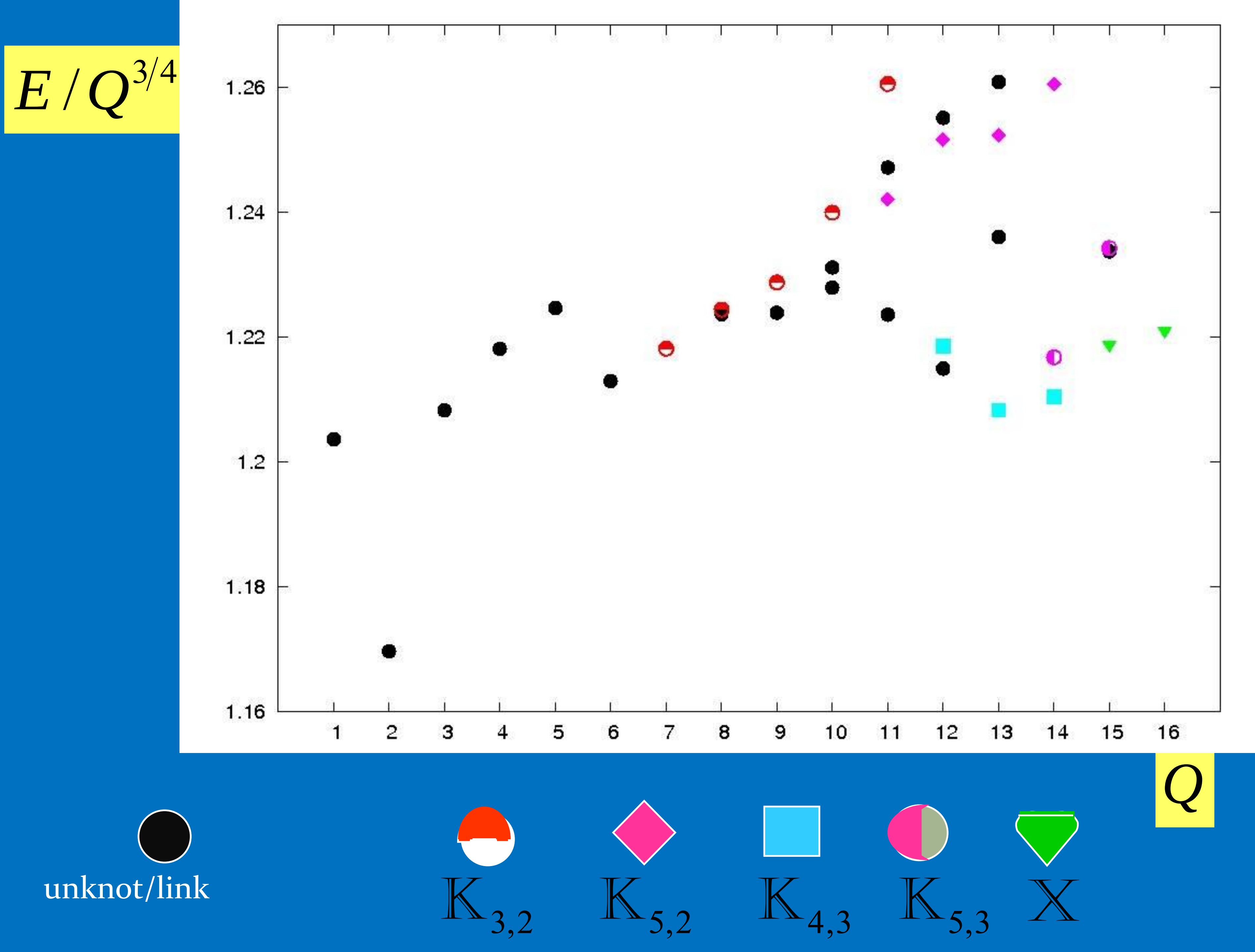
$$Q = 12, \quad \mathbb{K}_{3,2} \rightarrow \mathbb{K}_{4,3}$$





$$Q = 16, \quad \mathbb{X}_{16}$$


$8\mathcal{L}^{1,1}_{3,3}$
$8\mathcal{K}_{3,2}$
$9\mathcal{L}^{2,2,2}_{1,1,1}$
$9\mathcal{K}_{3,2}$
$10\mathcal{L}^{2,2,2}_{1,1,2}$
$10\mathcal{L}^{2,2}_{3,3}$
$10\mathcal{K}_{3,2}$
$11\mathcal{L}^{2,2,2}_{1,2,2}$
$11\mathcal{K}_{5,2}$
$11\mathcal{L}^{2,2}_{3,4}$
$11\mathcal{K}_{3,2}$
$12\mathcal{L}^{2,2,2}_{2,2,2}$
$12\mathcal{K}_{4,3}$
$12\mathcal{K}_{5,2}$
$12\mathcal{L}^{2,2}_{4,4}$
$13\mathcal{K}_{4,3}$
$13\mathcal{X}_{13}$
$13\mathcal{K}_{5,2}$
$13\mathcal{L}^{3,3}_{3,4}$
$14\mathcal{K}_{4,3}$
$14\mathcal{K}_{5,3}$
$14\mathcal{K}_{5,2}$
$15\mathcal{X}_{15}$
$15\mathcal{L}^{4,4,4}_{1,1,1}$
$15\mathcal{K}_{5,3}$
$16\mathcal{X}_{16}$



$E / Q^{3/4}$
1.26
1.24
1.22
1.2
1.18
1.16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Q
unknot/link
$K_{3,2}$
$K_{5,2}$
$K_{4,3}$
$K_{5,3}$
X

Conclusion

Topological solitons are interesting particle-like solutions of nonlinear PDEs.

They arise in many systems in condensed matter physics, particle physics, nuclear physics, cosmology, ...

Also of mathematical interest in their own right.

Solitons in Durham

18 permanent staff in the mathematics department with research interests in particle physics. Many have interests in solitons:

Solitons
Piette, Sutcliffe, Ward, Zakrzewski.

Integrable Quantum Field Theory
Bowcock, Corrigan, Dorey.

String Theory
Peeters, Zamaklar.

Cosmology
Gregory.

Solitons in the UK

Cambridge: Gibbons, Manton, Shellard, Tong.

Edinburgh: Braden, Singer.

Heriot-Watt: Schroers

Kent: Krusch.

Leeds: Speight.

Manchester (Physics): Battye.

THE END