PhDs in Algebra in the UK

Ken Brown

University of Glasgow

Prospects in Mathematics 2014
University of Oxford
18-19 December 2014

Plan

1. My current research - Hopf algebras
2. Algebra in the UK
3. Research in mathematics at Glasgow

1. Hopf Algebras

- Assume throughout that k is a field, $k = \bar{k}$, characteristic 0

Definition

An affine algebraic group G (over k) is an affine algebraic variety such that multiplication $\mu : G \times G \longrightarrow G$ and inverse $\iota : G \longrightarrow G : g \mapsto g^{-1}$ are morphisms of varieties.

1. Hopf Algebras

- Assume throughout that k is a field, $k = \bar{k}$, characteristic 0

Definition

An affine algebraic group G (over k) is an affine algebraic variety such that multiplication $\mu : G \times G \longrightarrow G$ and inverse $\iota : G \longrightarrow G : g \mapsto g^{-1}$ are morphisms of varieties.

Examples: $(k, +)$, $(k^*, \times)$, $GL(n, k)$, $SL(n, k)$, ..., every finite group.

1. Hopf Algebras

- Assume throughout that k is a field, $k = \bar{k}$, characteristic 0

Definition

An affine algebraic group G (over k) is an affine algebraic variety such that multiplication $\mu : G \times G \longrightarrow G$ and inverse $\iota : G \longrightarrow G : g \mapsto g^{-1}$ are morphisms of varieties.

Examples: $(k, +)$, $(k^*, \times)$, $GL(n, k)$, $SL(n, k)$, ..., every finite group.

We study such a group G via its k-algebra $\mathcal{O}(G)$ of polynomial functions from G to k.

1. Hopf Algebras

Example: If $G = SL(n, k)$, then

$$\mathcal{O}(G) = \frac{k[X_{11}, X_{12}, \ldots, X_{nn}]}{\langle \det(X_{ij}) - 1 \rangle}.$$

Example: If $G = SL(n, k)$, then

$$\mathcal{O}(G) = \frac{k[X_{11}, X_{12}, \ldots, X_{nn}]}{\langle \det(X_{ij}) - 1 \rangle}.$$

Multiplication in G is encoded by comultiplication, an algebra hom

$$\Delta : \mathcal{O}(G) \longrightarrow \mathcal{O}(G) \otimes_k \mathcal{O}(G) = \mathcal{O}(G \times G) : f \mapsto \Delta(f),$$

$$\Delta(f)(x, y) := f(xy).$$

1. Hopf Algebras

Example: If $G = SL(n,k)$, then

$$\mathcal{O}(G) = \frac{k[X_{11}, X_{12}, \ldots, X_{nn}]}{\langle \det(X_{ij}) - 1 \rangle}.$$

Multiplication in G is encoded by comultiplication, an algebra hom

$$\Delta : \mathcal{O}(G) \longrightarrow \mathcal{O}(G) \otimes_k \mathcal{O}(G) = \mathcal{O}(G \times G) : f \mapsto \Delta(f),$$

$$\Delta(f)(x,y) := f(xy).$$

Inverses in G are encoded by the antipode S, algebra antihom

$$S : \mathcal{O}(G) \longrightarrow \mathcal{O}(G), \;\; S(f)(x) := f(x^{-1}).$$

1. Hopf Algebras

Example: If $G = SL(n,k)$, then

$$\mathcal{O}(G) = \frac{k[X_{11}, X_{12}, \ldots, X_{nn}]}{\langle \det(X_{ij}) - 1 \rangle}.$$

Multiplication in G is encoded by comultiplication, an algebra hom

$$\Delta : \mathcal{O}(G) \longrightarrow \mathcal{O}(G) \otimes_k \mathcal{O}(G) = \mathcal{O}(G \times G) : f \mapsto \Delta(f),$$

$$\Delta(f)(x,y) := f(xy).$$

Inverses in G are encoded by the antipode S, algebra antihom

$$S : \mathcal{O}(G) \longrightarrow \mathcal{O}(G), \;\; S(f)(x) := f(x^{-1}).$$

The identity element of G is encoded by the counit, algebra hom

$$\varepsilon : \mathcal{O}(G) \longrightarrow k : f \mapsto f(1_G).$$

1. Hopf Algebras

Definition

A *Hopf k-algebra* $(H, \Delta, S, \varepsilon)$ *consists of the above data plus some axioms (which encode associativity etc). We'll always assume that* S *is bijective.*

1. Hopf Algebras

Definition

*A **Hopf k-algebra** $(H, \Delta, S, \varepsilon)$ consists of the above data plus some axioms (which encode associativity etc). We'll always assume that S is bijective.*

Notice that we *don't* require above that H is commutative.

1. Hopf Algebras

Definition

A *Hopf k-algebra* $(H, \Delta, S, \varepsilon)$ *consists of the above data plus some axioms (which encode associativity etc). We'll always assume that* S *is bijective.*

Notice that we *don't* require above that H is commutative.

Theorem

(Cartier) (Recall char $k = 0$*.) Let* $\mathcal{O}$ *be a* *commutative* *and finitely generated Hopf k-algebra. Then* $\mathcal{O} = \mathcal{O}(G)$ *for some affine algebraic group* G *over* k*.*

1. Hopf Algebras

Definition

A Hopf k-algebra $(H, \Delta, S, \varepsilon)$ *consists of the above data plus some axioms (which encode associativity etc). We'll always assume that* S *is bijective.*

Notice that we *don't* require above that H is commutative.

Theorem

(Cartier) (Recall char $k = 0$*.) Let* $\mathcal{O}$ *be a commutative and finitely generated Hopf k-algebra. Then* $\mathcal{O} = \mathcal{O}(G)$ *for some affine algebraic group* G *over* k*. So there is an equivalence of categories between algebraic groups and commutative Hopf algebras.*

1. Hopf Algebras

Why are (not necessarily commutative) Hopf algebras interesting?

1. Hopf Algebras

Why are (not necessarily commutative) Hopf algebras interesting?

- part of the general development of "noncommutative geometry";

1. Hopf Algebras

Why are (not necessarily commutative) Hopf algebras interesting?

- part of the general development of "noncommutative geometry";
- many important classes of examples

1. Hopf Algebras

Why are (not necessarily commutative) Hopf algebras interesting?

- part of the general development of "noncommutative geometry";

- many important classes of examples ...group algebras, enveloping algebras, quantum groups,...

1. Hopf Algebras

Why are (not necessarily commutative) Hopf algebras interesting?

- part of the general development of "noncommutative geometry";
- many important classes of examples ...group algebras, enveloping algebras, quantum groups,...
- allow "multiplication of representations": V, W yield $V \otimes W$.

2. Algebra in the UK

Split into three main, but intersecting, fields:

2. Algebra in the UK

Split into three main, but intersecting, fields:

Noncommutative algebra/noncommutative geometry.

2. Algebra in the UK

Split into three main, but intersecting, fields:

Noncommutative algebra/noncommutative geometry.

Group theory

2. Algebra in the UK

Split into three main, but intersecting, fields:

Noncommutative algebra/noncommutative geometry.

Group theory: finite; and infinite (= "geometric group theory").

2. Algebra in the UK

Split into three main, but intersecting, fields:

Noncommutative algebra/noncommutative geometry.

Group theory: finite; and infinite (= "geometric group theory").

Representation theory.

2. Algebra in the UK

Noncommutative algebra/noncommutative geometry.
Edinburgh, Glasgow, Lancaster, Leeds, London, Manchester, Oxford.

Group theory: finite; and infinite (= "geometric group theory").

Representation theory.

2. Algebra in the UK

Noncommutative algebra/noncommutative geometry.
Edinburgh, Glasgow, Lancaster, Leeds, London, Manchester, Oxford.

Group theory: finite; and infinite (= "geometric group theory").
Birmingham, Glasgow, Oxford, Southampton.

Representation theory.

2. Algebra in the UK

Noncommutative algebra/noncommutative geometry.
Edinburgh, Glasgow, Lancaster, Leeds, London, Manchester, Oxford.

Group theory: finite; and infinite (= "geometric group theory").
Birmingham, Glasgow, Oxford, Southampton.

Representation theory.
Edinburgh, Glasgow, London, Oxford, York.

3. PhD research at Glasgow

3. PhD research at Glasgow

Around 60 current PhD students.

3. PhD research at Glasgow

Around 60 current PhD students. Active research groups in

- pure maths: algebra; analysis; geometry and topology;

3. PhD research at Glasgow

Around 60 current PhD students. Active research groups in

- pure maths: algebra; analysis; geometry and topology;

- statistics;

3. PhD research at Glasgow

Around 60 current PhD students. Active research groups in

- pure maths: algebra; analysis; geometry and topology;

- statistics;

- applied maths: Mathematical Ecology and Physiology (Heart, Circulation, Soft Tissue Mechanics), MHD (Solar and Planetary Magnetic Fields), Elasticity.

3. PhD research at Glasgow

Around 60 current PhD students. Active research groups in

- pure maths: algebra; analysis; geometry and topology;

- statistics;

- applied maths: Mathematical Ecology and Physiology (Heart, Circulation, Soft Tissue Mechanics), MHD (Solar and Planetary Magnetic Fields), Elasticity.

More info and online application at:
http://www.gla.ac.uk/schools/mathematicsstatistics/research/postgraduate/

Lord Kelvin Adam Smith Studentship in Glasgow

What? A fully-funded 4 year interdisciplinary PhD project under the supervision of **Dr. Liam Watson** (Mathematics, Liam.Watson@glasgow.ac.uk) and **Dr. Kathryn Elmer** (Evolutionary Biology, Kathryn.Elmer@glasgow.ac.uk). The project, housed in **Mathematics**, will interact closely with biology, exploring molecular evolutionary patterns through topological methods in data analysis. A good point of entry is this paper: http://www.nature.com/srep/2013/130207/srep01236/pdf/srep01236.pdf

Closing? Applications by mid-January; expression of interest as soon as possible.

Interested? Please contact the PIs directly. More information: http://www.gla.ac.uk/schools/mathematicsstatistics/news/article/?id=99