Postrgraduate study in mathematical physics

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Outline

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Mathematical Physics research topics

An incomplete list:

- **High energy physics (string theory)**
- **General relativity**
- **Cosmology and astrophysics**
- **Quantum information theory**
- Condensed matter - cold atoms, quantum gases, superfluidity
- Biophysics
- Geophysics

Increasing overlaps between these research areas.

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Mathematics or physics?

- Contemporary physics requires sophisticated mathematics (pure and applied).
- Often the mathematics we require does not even exist: **physical mathematics.**[1]
- Physical intuition can sometimes provide new insights into longstanding mathematical problems.

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[1] http://www.physics.rutgers.edu/ gmoore/PhysicalMathematicsAndFuture.pdf

Prominent mathematical physicists

Edward Witten, Fields Medal 1990.

String theory, knot theory, positive energy in GR.

Wendelin Werner, Fields Medal 2006.

Conformal field theory

Stanislav Smirnov, Fields Medal 2010.

Proofs of conformal invariance of physical processes.

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UK research groups

The UK is very strong in mathematical physics - many string theory groups including

- Cambridge
- City
- Durham
- Edinburgh
- Imperial College, London*
- KCL
- Oxford
- Queen Mary, London*
- Southampton
- Surrey
- Swansea*

*Based in physics department. Also US/EU groups.

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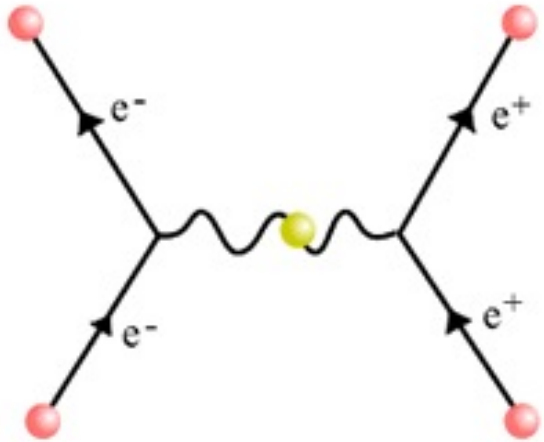
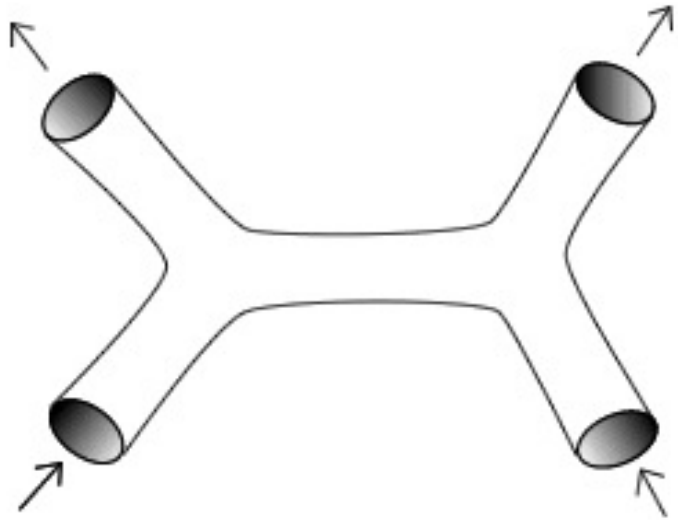
So what is string theory?





- Particles are viewed as excitations of a string; they have finite extent.
- String theory describes maps from Riemann surfaces into higher-dimensional manifolds (spacetime).

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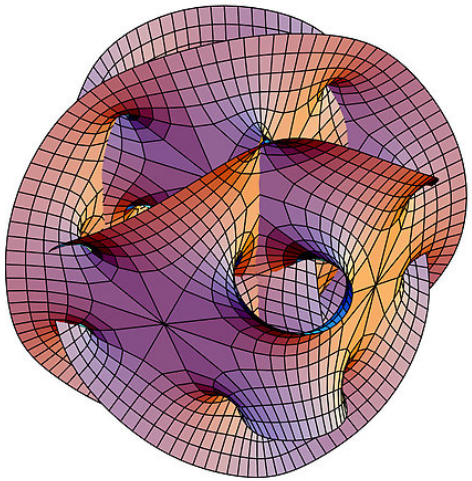
Mathematical consistency



- Mathematical self-consistency constrains the spacetimes in which strings can propagate.
- Most backgrounds are higher-dimensional (10d), with small extra dimensions.
- The extra dimensions are often Calabi-Yau manifolds (compact Kähler manifolds with trivial canonical bundle).

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Generalized geometry

- **Complex geometry** concerns structures on the tangent bundles **T** of manifolds.
- **Generalized geometry** (Hitchin et al) works with structures on a generalised tangent bundle, the sum $\mathbf{T} \oplus \mathbf{T}^*$ of the tangent and cotangent bundle (vector fields and one-forms).
- Realistic 4d physics seems to require generalized geometry.

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Duality

Duality means that there exist two distinct but equivalent descriptions of the same system.

- In physics language: equivalence between dynamical systems.
- In mathematical language: branches of mathematics are related - e.g. solving a problem in algebraic geometry may be equivalent to solving a seemingly distinct problem in group theory.

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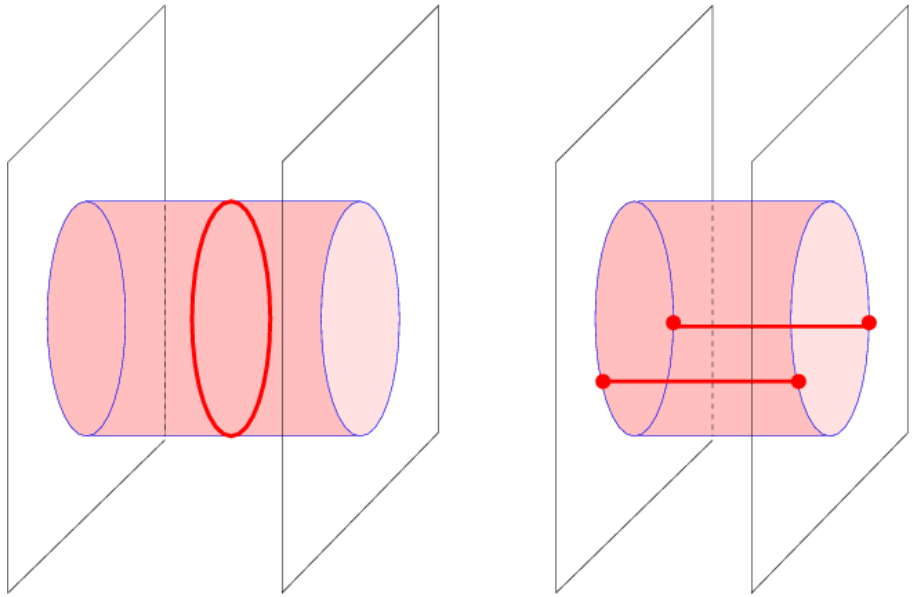
Open closed duality



- Left: closed circular string moving in time (along cylinder length).
- Right: open string (along cylinder length) doing a loop in time.

In string theory these diagrams are indistinguishable.

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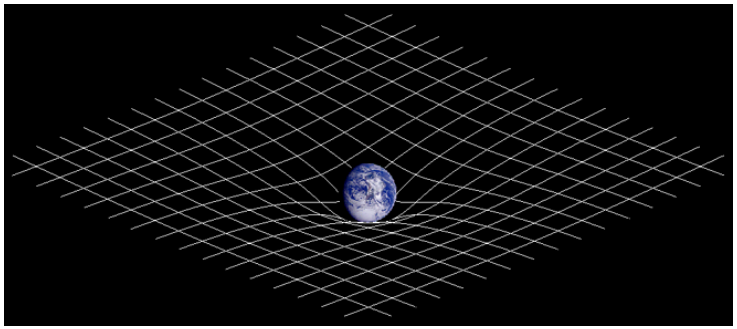
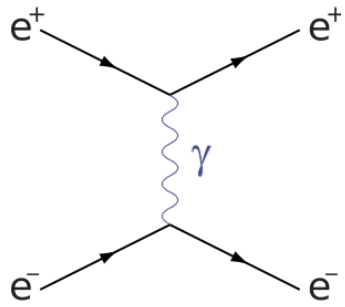
Consequences





- Closed strings describe gravity, incorporating Einstein's general relativity theory, relating curvature of spacetime to energy.
- Quantum field theory describes nuclear and electromagnetic interactions.

String theory leads to relations between general relativity and particle physics.

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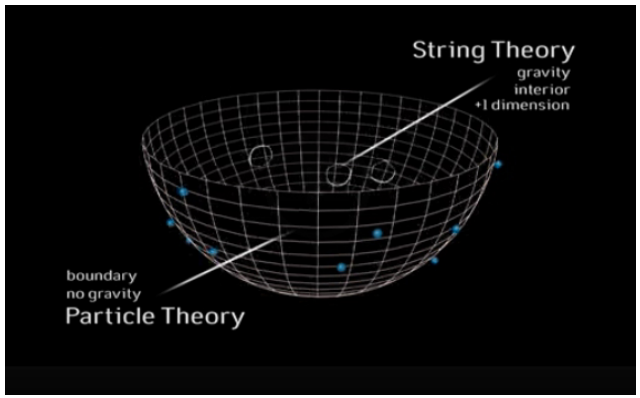
Holography

Gravity is equivalent (dual) to a particle physics theory with no gravity, in one less spatial dimension.



- Our 4d Universe can be described by 3d electromagnetism and nuclear forces, with no gravity.
- A priori unrelated dynamical systems are actually equivalent.

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Duality

Fluid/gravity duality

- Einstein's general relativity in 4d:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

with $g_{\mu\nu}$ the spacetime metric; $R_{\mu\nu}$ and R the curvature and Λ the cosmological constant.

- Non-linear second order partial differential equations.

- Hydrodynamics of the 3d particle physics system:

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = 0$$

with ρ the energy density, $\mathbf{u}$ the relativistic velocity and t time.

- Relativistic Navier-Stokes equation.

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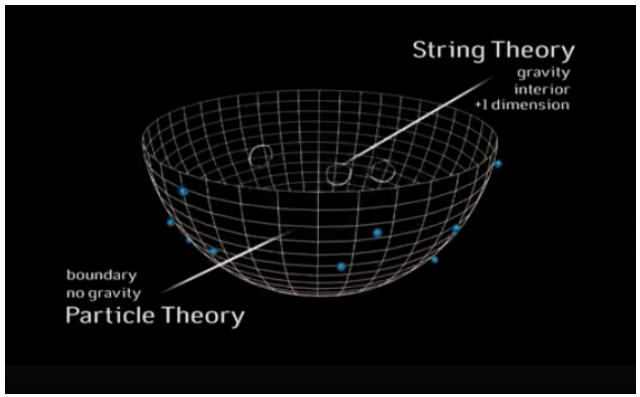
Duality



- Black hole solutions of Einstein equations capture the hydrodynamics of the dual fluid.
- Long-standing problems are mapped into each other - turbulence in fluids, stability of black holes under small perturbations.
- Progress on Clay prize problems?!

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Implications for pure mathematics

The duality also implies pure mathematics problems are mapped into each other:

- On the particle theory side: physical questions translate into questions in number theory, combinatorics et
- For example, counting of quantum states leads to

$$\sum_i m_i n_i = N$$

with m_i, n_i, N integers. (For large N, number of partitions behaves as $\exp(\sqrt{N})$.)

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Implications for pure mathematics

- The same questions translated into the gravity language turn out to be other pure mathematics problems (differential geometry, algebraic geometry).
- E.g. count the number of ways a singularity can be resolved; count the number of geodesics with certain properties.

Deep relations between a priori unconnected questions in mathematics!

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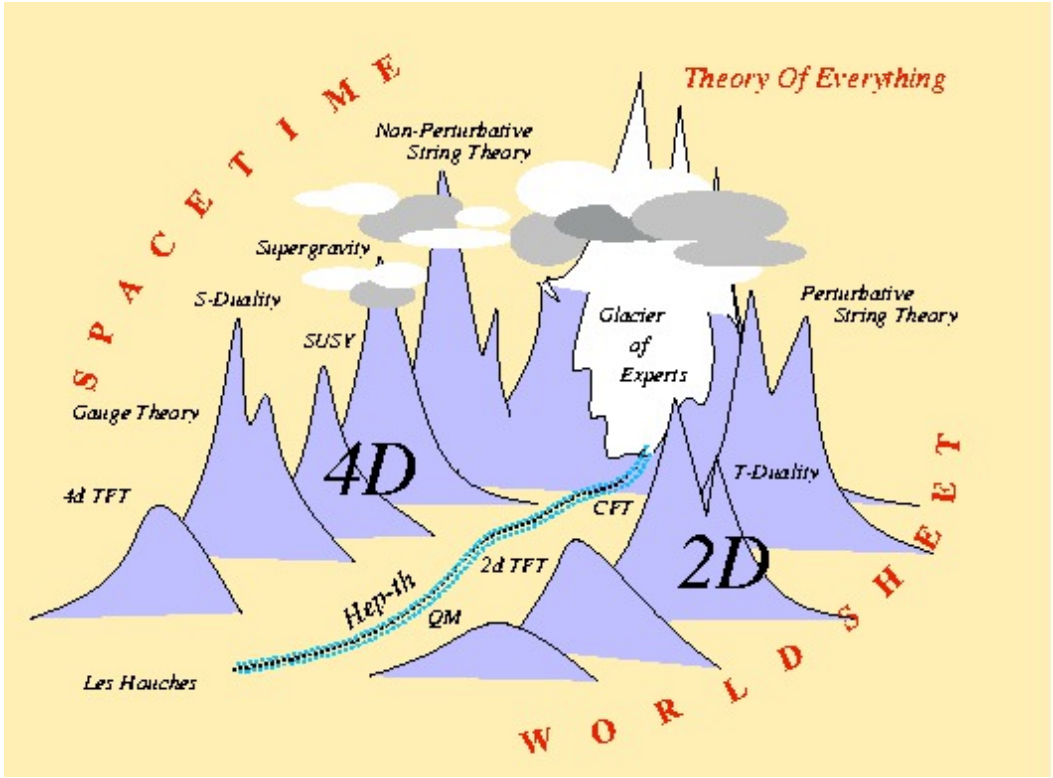
The string theory landscape



(Robbert Dijkgraaf)

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Outline

1. Introduction
2. The string theory landscape
3. **PhDs in mathematical physics**

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Applications

- PhD positions in mathematical physics are very competitive - apply widely!
- Many foreign students come in with MSc but UK students typically enter with MMath and MPhys.
- Successful applicants have strong MMath (or MPhys) results!
- Decisions are typically made between January and April, with offers conditional on MMath results.

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Funding and PhD duration

Various sources of funding are used - prospective groups will advise what applications are necessary:

- STFC studentships (quotas)
- EPSRC doctoral training grants
- Funding allocated from CDTs, where available (e.g. computational modelling in Southampton)
- Funding from European research grants

Almost always 4 or 3+1 years, even for students with MMath/MPhys.

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Prerequisites

Vary considerably within mathematical physics:

- String theory requires a strong background in both mathematics and physics - analysis, complex variable theory, differential geometry, quantum physics, relativity.
- Prerequisites for other areas are typically narrower.

Look at websites and email prospective supervisors to get a feel about their research.

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Graduate study at Southampton

- Encompasses pure mathematics, applied mathematics, statistics, operational research and mathematical physics
- MSc programmes in statistics, actuarial science and operational research.
- Around 20 PhD positions per year, six or more awarded to mathematical physics.
- New institute in theoretical physics and astronomy, STAG.
- New university graduate school (provides a wide range of training), in cooperation with DTCs.

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Application process

- Applying: http://www.southampton.ac.uk/maths/postgraduate/
- Open days: http://www.southampton.ac.uk/postgraduate/visitingtheuni/

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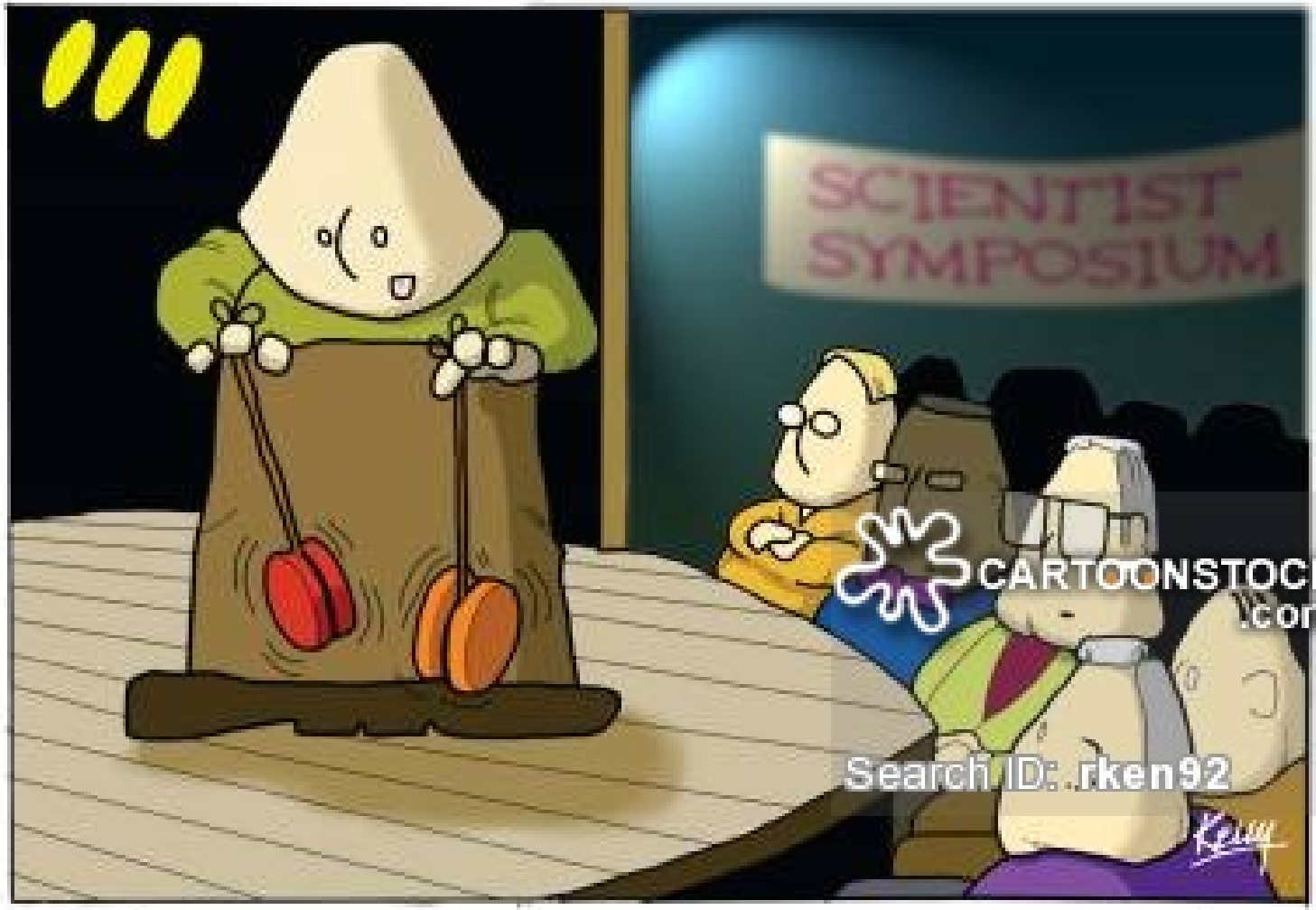

Professor Dawkins was kindly asked not to return after his presentation on "String Theory."