Stanford Symposium, July 2012
Algebraic Topology: Applications and New Developments

Titles and Abstracts

Panel Discussion
Future directions of Algebraic Topology

Panelists: Alejandro Adem (moderator), David Ayala, Bill Dwyer, Dan Freed, Kathryn Hess, Michael Hopkins, Nick Kuhn.
Scribe: Martin Palmer

Henry Adams
Evasion paths in mobile sensor networks

Suppose disk-shaped sensors wander in a planar domain. A sensor doesn't know its location but does know which sensors it overlaps. We say that an evasion path exists in this sensor network if a moving evader can avoid detection. Vin de Silva and Robert Ghrist give a necessary condition, depending only on the time-varying connectivity graph of the sensors, for an evasion path to exist. Can we sharpen this result? We'll consider an example where the existence of an evasion path depends not only on the network's connectivity data but also on its embedding.

Alejandro Adem
Equivariant K-theory and spaces of commuting elements in a compact Lie group

Let G denote a compact connected Lie group with torsion-free fundamental group acting on a compact space X such that all the isotropy subgroups are connected and of maximal rank. We derive conditions on the action which imply that the equivariant K-theory of X is a free module over the representation ring of G. This can be applied to compute the equivariant K-theory of spaces of ordered commuting elements in certain compact Lie groups. This is joint work with J.M.Gomez.

David Ayala
Higher categories are sheaves on manifolds

Many proposed higher categories come from geometric situations. This talk will demonstrate a constructive connection between a homotopy theory of local invariants of n-manifolds and that of weak n-categories in the sense of Rezk. Relations to specific topological field theories will be discussed. This is a report on joint work with Nick Rozenblyum.

Clark Barwick
Dévissage

No abstract.

Alexander Berglund
Homological stability for automorphisms of manifolds

By work of Harer and others, the homology of the diffeomorphism group of an oriented genus g surface stabilizes as $g \to \infty$. The stable homology is known by a celebrated theorem due to Madsen and Weiss.
In this talk, we will consider high dimensional cousins of genus g surfaces: g-fold connected sums of $S^d \times S^d$, for $d > 2$. In joint work with Madsen, we have obtained stability theorems for the rational homology of three kinds of automorphisms of these manifolds: homotopy self-equivalences, block diffeomorphisms and diffeomorphisms. Our proof is quite different from the proof Harer gave in the surface case; it uses rational homotopy theory and surgery theory to reduce to stability theorems for the homology of orthogonal and symplectic groups with twisted coefficients. In the process of proving the results, we constructed a new rational model for mapping spaces, which utilizes the Deligne-Getzler ∞-groupoid of a dg Lie algebra.

Julie Bergner
Diagrams encoding group actions

Many algebraic structures, such as those of monoids, groups, categories, and operads, can be understood as diagrams satisfying some kind of Segal-type condition. One could ask for such algebraic objects equipped with an action of another such algebraic objects, for example a group action on an operad. In joint work with Philip Hackney, we develop the appropriate diagram shapes and Segal conditions to understand these structures.

Andrew Blumberg
New directions for trace methods

Over the last 20 years, the development of trace methods and in particular the cyclotomic trace landing in topological cyclic homology (TC) and topological Hochschild homology (THH) revolutionized the study of algebraic K-theory.
This talk will survey a number of new questions and directions in the study of THH and TC, bringing in ideas from localization phenomena, the motivic perspective, and higher category theory.

Carl-Friedrich Bödigheimer
Homology operations for moduli spaces

The moduli spaces $\mathfrak{M}^m_{g,n}$ of Riemann surfaces of genus g with n boundary curves and m punctures admit many homology operations, not only the Dyer-Lashof operations. As far as the homology of moduli spaces is known, for example for $g = 2$

or $g = 3$, we try to describe generators by theses operations.

Tom Church
Homological stability via Koszul duality for FI-modules

FI-modules were introduced to prove representation stability, an analogue of homological stability that applies in situations when symmetries rule out classical homological stability. But it turns out that the rate of this stabilization (including the stable range) is governed by homological properties of the FI-modules, considered as algebraic objects in their own right. In this talk I will show how analogues of classical theorems in commutative algebra yield new stability results and improved stable ranges in many concrete examples. In particular, I will explain how Koszul duality for FI-modules provides a new approach to Putman's recent theorem on central homological stability for congruence subgroups of arithmetic groups. Joint work with Jordan Ellenberg, Benson Farb, and Rohit Nagpal.

Justin Curry
Cosheaves and dualities in generalized sensor networks

In this talk, I will introduce the computational framework of cellular sheaves and cosheaves, and advocate for a different perspective on Morse Theory, persistent homology, network coding and pursuit-evasion problems in sensor networks. This framework provides local-to-global results as well as generalizations of Poincar duality. To provide concrete examples, I will introduce a new model for sensing with different modalities (colours, sounds, etc.) and show how a long exact sequence of sheaf cohomology provides forcing results that allow you to infer what you don't know from what you do.

Chris Douglas
Fusion categories and field theories

I will describe a relationship between certain monoidal categories called fusion categories and 3-dimensional topological field theories, focusing on the correspondence between algebraic properties of the categories and topological properties of the associated field theories.
Fusion categories are monoidal categories that have the nice properties of the category of representations of a finite group: each object has a dual, there are finitely many simple objects, and any object decomposes into a finite sum of simples. We show that any fusion category gives rise to a 3-dimensional topological field theory. A key question about the algebraic structure of a fusion category is whether the double dual operation is trivial, as it is in the representation category of a finite group. I will explain why this question corresponds to the question of whether the 3-manifold invariants of the associated field theory depend on a spin structure.
This is joint work with Chris Schommer-Pries and Noah Snyder.

Dan Dugger
Motivic characteristic classes for quadratic bundles

The problem from the title is that of understanding characteristic classes for quadratic bundles with values in motivic cohomology. I will describe some aspects of this, and of a closely-related problem of computing the $RO(G)$-graded cohomology of real, equivariant Grassmannians for the group $G = \mathbb{Z}/2$.

Bjorn Ian Dundas
Higher topological Hochschild homology

If A is a commutative ring spectrum, consider the derived (in the appropriate sense) functor $X \mapsto X \otimes A$ from spaces to commutative ring spectra. How is the geometry of X reflected in $X \otimes A$, what are the consequences of the symmetries of X, and what is the connection to the red-shift conjecture? The talk will discuss recent calculations and structural results and relies on the insight of Carlsson, Cohen and Madsen.

Bill Dwyer
Operads and higher knots

This is joint work with Kathryn Hess. Let $K(m, n)$ be the space of smooth embeddings $\mathbb{R}^m \to \mathbb{R}^n$ which agree with a fixed linear embedding off a compact subset of $\mathbb{R}^m$; this is the space of long m-knots in $\mathbb{R}^n$. Let $E(k)$ be the little k-disk operad. Modulo a framing adjustment, the embedding calculus of Goodwillie, Klein, and Weiss leads to an identification of $K(m, n)$, $n > m + 2$, with a space of linear $E(m)$-bimodule maps $E(m) \to E(n)$. Starting from this, we identify $K(m, n)$ as the $(m + 1)$-fold loop space on the space of derived operad maps $E(m) \to E(n)$. There are some general principles involved, as well as strange constructions and interesting questions.

Dan Freed
3d TQFTs through the lens of the cobordism hypothesis

I will report on joint work in progress with Constantin Teleman about 3-dimensional topological field theories determined by modular tensor categories. I will also make a few remarks about 2-dimensional conformal field theories.

Søren Galatius
Homology of moduli spaces of high dimensional manifolds

For each n, there is a space M_g^n classifying smooth fiber bundles whose fibers are connected sums of g copies of $S^n \times S^n$. For $n = 1$, this is essentially Riemann's moduli space. I will discuss recent results with Randal-Williams about the homology of this space when g is large and $n > 2$.

Nora Ganter
Elliptic Schubert calculus

I will recall the role that equivariant cohomology theories play for Schubert calculus and in particular, how the Weyl character formula is obtained as a K-theoretic index formula. I will then speak about how to generalize this discussion to equivariant elliptic cohomology, where the "elliptic Weyl character formula" turns out to be the Weyl-Kac formula.

Teena Gerhardt

Algebraic K-theory and Witt vectors

Algebraic K-theory brings together classical invariants of rings with difficult computations in homotopy theory. The connection is through trace maps relating algebraic K-theory to fixed point spectra of topological Hochschild homology (THH). The fixed point spectra of THH are closely related to Witt vectors, and this relationship can facilitate K-theory computations. In this talk I will discuss connections between algebraic K-theory and Witt vectors. In particular I will describe new algebraic K-theory computations which naturally lead to an n-dimensional generalization of the big Witt vectors. This is joint work with Vigleik Angeltveit, Mike Hill, and Ayelet Lindenstrauss.

Boris Goldfarb

On the algebraic structure of geometric group rings

Since the work of Kropholler/Linnell/Moody in the late 80's, it has been known that for noetherian group algebras where the group has no torsion the question of (not) finding zero divisors is intimately related to computations in K-theory. I will show an extension of this relation to some discrete groups whose group algebras are not noetherian but the group has a specific kind of coarse geometry that can be exploited.

Jesper Grodal

F-isomorphism in group cohomology implies isomorphism

Recall that a map of noetherian F_p-algebras is an F-isomorphism if and only if it induces a homeomorphism of associated varieties. In this talk I will report on recent joint work with Dave Benson and Ellen Henke, where we prove that if a map $H \to G$ between finite groups with the same Sylow p-subgroup induces an F-isomorphism in mod p cohomology, then H controls p-fusion in G, when p is odd. This generalizes classical results of Quillen, who proved this when H is the Sylow p-subgroup. Our results also generalize a celebrated result of Mislin, that mod p cohomology isomorphism implies control of p-fusion, and our methods in fact provide a comparatively simple algebraic proof of his theorem, when p is odd. In my talk, I'll also discuss a $p = 2$ version, where we get a similar conclusion at the expense of using higher chromatic cohomology theories instead of mod p cohomology.

Ian Hambleton

Co-compact discrete group actions and the assembly map

A countable discrete group G acts freely and properly on some manifold M = Sphere x Euclidean space if and only if G has periodic Farrell cohomology: see Connolly-Prassidis (1989) and Adem-Smith (2001). For free co-compact actions there are additional restrictions and some interesting examples, but no general existence result is known. The talk will present some new examples and relate this problem to the Farrell-Jones assembly maps in K-theory and L-theory.

Allen Hatcher

Stable homology of spaces of graphs

Galatius' theorem on the stable homology of the automorphism group of a free group is extended in two directions: (1) a relative version where instead of finite graphs one considers spaces obtained by attaching finitely many 0-cells and 1-cells to a fixed base space. (2) a handlebody version for n-dimensional thickenings of graphs. This yields analogues of the Madsen-Weiss theorem for 3-manifolds with free fundamental group. There is also a combination of (1) and (2) which extends this to a broader class of 3-manifolds.

Richard Hepworth

String topology of classifying spaces

Let G be a Lie group. Chataur and Menichi showed that the homology of the free loop space $L(BG)$ admits a rich algebraic structure: it is part of a homological field theory, meaning that it admits operations parameterised by the homology of mapping class groups. I will discuss a new construction of this field theory that radically enlarges the class of allowable cobordisms, trading surfaces with boundaries for arbitrary spaces with the homotopy type of a finite graph. The result is a new kind of field theory related to mapping class groups of surfaces and automorphism groups of free groups with boundary. This is joint work with Anssi Lahtinen.

Kathryn Hess

The divided powers functor on symmetric sequences

This is joint work with Bill Dwyer. In the course of proving our delooping theorem for spaces of higher long knots (cf. Bill's talk), we defined and studied a family $\{\gamma_n \mid n \geq 1\}$ of "divided powers" endofunctors on the category of symmetric sequences that plays an important role in our proof. For any operad $\mathcal{O}$, the γ_n's induce endofunctors on the category $\mathcal{O}$-bimodules that we use to lift the Boardman-Vogt tensor product of operads to the tensor product of bimodules that we use in the inductive step of our proof. An intriguing, apparently new monoidal structure on symmetric sequences shows up in this context as well. Moreover, each γ_n induces a functor from the category of operads with "axial structure" to the category of nonunital operads. Applying γ_n to the little m-balls operad for $m \geq 4$, we obtain a description of the space of long links of n-strands in $\mathbb{R}^m$ as the total space of a certain looped fibration.

Lars Hesselholt
Real algebraic K-theory

No abstract.

Mike Hill
Equivariant Symmetric Monoidal Categories

The naturally occurring categories of G-objects are all symmetric monoidal (in multiple ways): G-modules have a tensor product, direct sum, and direct product; G-spaces have disjoint union and Cartesian product; and G-spectra have wedges, products, and smash products. In all of these examples, there is additional structure related to the symmetric monoidal product and coming from a G-action on the indexing set for a product. I'll talk some about the general categorical underpinnings in this talk, and then I will give computational examples in spectra showing how the language provides a natural home for (and solution to) several somewhat classical problems in equivariant homotopy theory.

Michael Hopkins
Equivariant multiplicative closure

No abstract.

Dan Isaksen
From motivic to classical homotopy theory:
Reverse-engineering the classical Adams-Novikov spectral sequence

The goal of this talk is to demonstrate how motivic homotopy theory over $\mathbb{C}$ gives new information about classical stable homotopy theory. After a brief introduction about motivic homotopy theory, I will use motivic versions of the May and Adams spectral sequence to compute the 2-complete motivic stable homotopy groups over $\mathbb{C}$ in a large range of stems. Then I will show how knowledge of motivic stable homotopy groups over $\mathbb{C}$ allows one to completely reconstruct the classical Adams-Novikov spectral sequence in the same range, including all differentials and hidden extensions.

Matthew Kahle
Topology of random flag simplicial complexes

Random flag complexes are a natural generalization of random graphs to higher dimensions, and since every simplicial complex is homeomorphic to a flag complex this puts a measure on a wide range of possible topologies. In this talk, I will discuss the recent proof that according to the Erdös-Rényi measure, asymptotically almost all d-dimensional flag complexes only have nontrivial (rational) homology in middle degree $d/2$.
The highlighted technique is originally due to Garland — what he called "p-adic curvature" in a somewhat different context. This method allows one to prove

cohomology-vanishing theorems by showing that certain discrete Laplacians have sufficiently large spectral gaps. This reduces certain questions in probabilistic topology to questions about random matrices.
Some of this depends on new results for random matrices in joint work with Chris Hoffman and Elliot Paquette. Proving central limit theorems for Betti numbers in the non-vanishing regime was done in joint work with Elizabeth Meckes.

Nitu Kitchloo
The stable symplectic category and geometric quantization

I will describe a stabilization procedure on the Symplectic Category of Lagrangian correspondences between symplectic manifolds. This category was introduced by A. Weinstein as a potential domain for the Geometric Quantization functor. The procedure of stabilization allows composition of Lagrangian correspondences to be defined in general, thereby fixing a basic drawback with Weinstein's category. The stable symplectic category is enriched over the category of modules over a certain commutative ring spectrum. I will describe this structure in detail, and introduce the stable context for Geometric Quantization.

John Klein
On the quantization of fluctuating currents: an application of algebraic topology to statistical mechanics

This talk will use algebraic topological methods to study the statistical distributions of stochastic currents generated in graphs. I'll demonstrate that, in the adiabatic and low temperature limits, quantization of current generation occurs.

Ernesto Lupercio
Non-commutative toric varieties

Traditional toric varieties are very amenable for explicit computations and while very simple they can be used as building blocks of more general geometric situations. Nevertheless they are quite rigid. In this talk I will construct the Moduli Space of toric varieties by considering their non/commutative generalizations.
This is joint work with Ludmil Katzarkov, Laurent Meersseman and Alberto Verjovsky.

Jacob Lurie
p-Divisible groups, and character theory

Let G be a finite group. One of the main theorems of representation theory asserts that the construction which assigns to each representation of G its character induces an isomorphism between the representation ring of G and the ring of conjugation-invariant functions on G. This isomorphism can be interpreted as giving a concrete description of the G-equivariant K-theory of a point "with complex coefficients". Hopkins, Kuhn, and Ravenel developed an analogous "character theory" for a large class of cohomology theories, known as Morava E-theories. In this talk, I'll review the theorem of Hopkins-Kuhn-Ravenel and explain how it can be “categorified” using some ideas from the theory of p-divisible groups.

Mike Mandell
Localization sequences in THH

For a discrete valuation ring R with quotient field F and residue field k, you have a cofibration sequence of K-theory spectra

$$K(k) \longrightarrow K(R) \longrightarrow K(F)$$

The corresponding sequence in THH is not a cofibration sequence, but both the cofiber of the map $THH(k) \to THH(R)$ and the fiber of the map $THH(R) \to THH(F)$ have an interpretation in terms of the THH of Waldhausen categories. Thinking in terms of Waldhausen categories, we therefore get two cofibration sequences for THH,

$$THH(k) \to THH(R) \to THH(F|R)$$

(first constructed by Hesselholt and Madsen) and

$$THH(\mathit{Spec}(R) \textit{ on } \mathit{Spec}(k)) \to THH(R) \to THH(F)$$

(generalizing to THH a well-known exact sequence in Hochschild homology). The first arises by looking at enrichments by connective spectra and the second by looking at enrichments in non-connective spectra. (Joint work with Andrew Blumberg, preprint arXiv:1111.4003.)

Daniel Müllner
Consistent scale selection for exploratory visualization and analysis of data sets

Choosing an appropriate scale is a frequently encountered problem in data analysis, and paradigms in the field support both the choice of strategies to make smart, definite choices and the hierarchical or persistence approach of looking at all scales at once. In the core of the Mapper algorithm for visualization and analysis of point cloud data, a scale choice must be made multiple times for overlapping fragments of the data set. We present the concept of a scale graph, where scale choices in neighboring regions are brought into conjunction to make consistent decisions at local scale, while retaining global flexibility. By selecting an optimal path through the scale graph, we can make more plausible choices, overcome existing weaknesses, validate results more easily and simplify the data analysis process for the user. (Joint work with Gunnar Carlsson, Facundo Mémoli and Gurjeet Singh.)

Monica Nicolau
Unraveling the biology of disease through data transformations and topological data analysis

The past decade has witnessed developments in the field of biology that have brought about profound changes in understanding the dynamic of diseases and

of biological systems in general. New technology has given biologists an unprecedented wealth of information, but it has generated data that is hard to analyze mathematically, thereby making its biological interpretation difficult. These challenges have given rise to a myriad of novel exciting mathematical problems and have provided an impetus to modify and adapt traditional mathematics tools, as well as develop novel techniques to tackle the data analysis problems raised in biology.
I will discuss a general approach to address some of these computational challenges by way of a combination of data transformations and topological methods, to highlight specific biologically driven questions. These methods have been applied in a wide range of settings, in particular for the study of the biology of disease. I will discuss some concrete applications to these methods, including their use to discover a new type of breast cancer and the associated biology that drives the disease, identify disease progression trends, and highlighting the driving mechanisms in acute myeloid leukemia. Much of this is joint work with Gunnar Carlsson.

Paul Norbury

Gromov-Witten invariants of the two-sphere and mirror symmetry

I will describe the well-studied problem of the Gromov-Witten invariants of the two-sphere. The Gromov-Witten invariants can be calculated via a rigorous notion of mirror symmetry using Frobenius manifolds. New insight into this problem comes from recent ideas of Eynard and Orantin who have developed a tool in complex analysis for studying enumerative problems in geometry.

Kate Poirier

Compactifying string topology

String topology studies the algebraic topology of the space of loops and paths in a manifold. Previous treatments of string topology describe algebraic structures on the homology of this space and operations parameterized by the moduli space of Riemann surfaces. One perspective is that these structures should be a shadow of a richer structure at the chain level and that the space parametrizing the operations should be compactified. In this talk, we describe a compact space of graphs giving string topology operations on the singular chains of the space of loops and paths which induce known operations on homology. This is joint work with Gabriel C. Drummond-Cole and Nathaniel Rounds.

Dan Ramras

Stable representation theory and the geometry of flat connections

In the 1960's, Atiyah and Segal studied the map $R(G) \to K(BG)$ sending a representation of G to the induced bundle over BG. We consider a natural generalization of this map to spherical families of (finite-dimensional, unitary) representations of infinite discrete groups. This topological Atiyah-Segal map is closely linked to the natural map $Hom(G, U(n)) \to Map(BG, BU(n))$, and is thereby related to various questions about (families of) flat bundles over BG. On the other hand, this map can be described in terms of a homotopy limit problem for Carlsson's deformation

K-theory functor. This brings methods from stable homotopy theory to bear, leading to various results for (products of) aspherical surfaces, tori, and flat manifolds. Parts of this work are joint with Tom Baird.

Oscar Randal-Williams

Homological stability for moduli spaces of manifolds

Harer's stability theorem states that the group homology of the mapping class group of an oriented surface of genus g is independent of g in a range of degrees (which increases with g). In recent joint work with Soren Galatius, we replace the surface of genus g with the $2n$-dimensional manifold W_g obtained as the connected sum of g copies of $S^n \times S^n$, and instead of looking at its mapping class group we study the entire topological group of diffeomorphisms of W_g which are the identity inside a fixed disc. As long as $n > 2$, we show that the classifying spaces $B\mathrm{Diff}(W_g, D^{2n})$ satisfy homological stability: the homology of this space is independent of g in degrees $* < (g-4)/2$.

Paolo Salvatore

Cellular decompositions of planar configuration spaces and the Fulton Mac Pherson operad

We construct via rational functions a cellular decomposition of planar configuration spaces and relate it to the Fulton Mac Pherson operad.

Graeme Segal

Semi-infinite homotopy theory and noncommutative geometry

I shall describe how the idea of a semi-infinite homotopy type can be motivated by quantum theory, and especially by noncommutative geometry. Roughly speaking, the nature of quantum theory presents all infinite-dimensional state spaces as nested unions of finite-dimensional ones, and noncommutativity leads to periodicity of the homotopy type. When in addition there are fermions we can expect a semi-infinite type.

Dev Sinha

Cohomology of symmetric and alternating groups

We share our understanding of the cohomology of symmetric and alternating groups, in particular ring structure and action of the Steenrod algebra, by using the geometry of the Fox-Neuwirth cell structure and the algebra of Strickland and Turner's Hopf ring structure. We emphasize graphical presentations by skyline diagrams, which generalize Young diagrams and which are also helpful in invariant theory. (Joint with Chad Giusti and Paolo Salvatore)

Primoz Skraba

Persistence of Random Points

Given points randomly distributed over a d-dimensional hypercube, I will show that if we take a distance filtration, all persistence barcodes are bounded in length depending on the dimension. The bounds are looser than other similar results on Betti numbers, but the proof is vastly simplified. These results apply to any dimension and along with it, I will present various extensions including lower bounds on barcode length in such a random sets of points.

Ulrike Tillmann
On the work of three eminent topologists

We will look at some highlights in algebraic topology from the last 30 + years.

Mikael Vejdemo-Johansson
Computation of spectral sequences of double complexes, with applications to persistent homology

In work jointly with David Lipsky, Dmitriy Morozov, and Primoz Skraba, we work out explicitly all higher differentials for the spectral sequence of a double complex C. For the case of field coefficients, this computes not only the associated graded module of the total complex $Tot(C)$, but gives us $Tot(C)$ itself.
For non-field coefficients, there is an extension problem inherent in the computation; the spectral sequence gives us information about both the associated graded module as well as the kernel of the quotient map. For the case of $k[t]$ coefficients for a field k, we give an explicit solution for the extension problem in terms of the presentation of the E-infinity page of the spectral sequence. Inherent in this solution is an explicit presentation map of $Tot(C)$ itself, which after a Smith normal form computation produces a barcode for $Tot(C)$.
We demonstrate an application of these results to work towards parallelizing persistent homology computations; the Mayer-Vietoris long exact sequence generalizes to a spectral sequence of a double complex, and our results give an approach to explicit parallel algorithms for computing persistent homology.

Nathalie Wahl
Universal operations in Hochschild homology

In this talk, I will give a construction of the natural operations on the Hochschild complex of E-algebras, where E is any algebraic structure which includes a multiplication, as for example that of Frobenius algebras, or just associative or A_∞-algebras. When E encodes the structure of open topological conformal field theory, our construction recovers a model for the closed cobordism category, thus establishing that the earlier constructed operations of Costello and Kontsevich-Soibelman on the Hochschild complex of open field theories account for all natural operations.

Michael Weiss
Smooth maps to the plane and Pontryagin classes

Rational characteristic classes for fiber bundles where the fiber is a euclidean space of dimension n are not very well understood. I will talk about the following: the rational Pontryagin classes of Novikov and Thom satisfy all the relations (together with the Euler classes) that they satisfy for vector bundles of dimension n. Smoothing theory leads to a reformulation of this hypothesis which is about spaces of smooth regular maps to the plane. I shall describe an approach to this which introduces spaces of smooth maps to the plane with only mild singularities, in the tradition of concordance theory (e.g. J. Cerf).

Craig Westerland

A higher chromatic analogue of the image of J

Different cohomology theories "see" different parts of the stable homotopy groups of spheres. Singular cohomology, for instance, detects all maps from a sphere to itself using the notion of degree. K-theory detects a large swath of homotopy known as the "image of J," which can be described very geometrically using the relation between a vector bundle and its unit sphere-bundle. In this talk, I will discuss an analogue of the image of J for higher chromatic homotopy theory – the part seen by the Morava K-theories, $K(n)$. The result lacks the charming geometry of the J-homomorphism, and stretches the notion of a homotopy group, but will hopefully give us insight into the $K(n)$-local homotopy category.

Kirsten Wickelgren

Investigating the section conjecture

Grothendieck's anabelian conjectures predict that the etale fundamental group is a fully faithful functor from certain anabelian schemes to profinite groups with Galois action, or equivalently that the maps between anabelian schemes are the same as maps between their etale homotopy types. The case of maps from Spec $\mathbb{R}$ follows from the equivalence between fixed points and homotopy fixed points for $\mathbb{Z}/2$-actions on finite complexes, which was shown by Gunnar Carlsson and Haynes Miller independently. We will discuss the anabelian conjectures and their topological analogues, and talk about some work in the case of maps from the spectrum of a field.