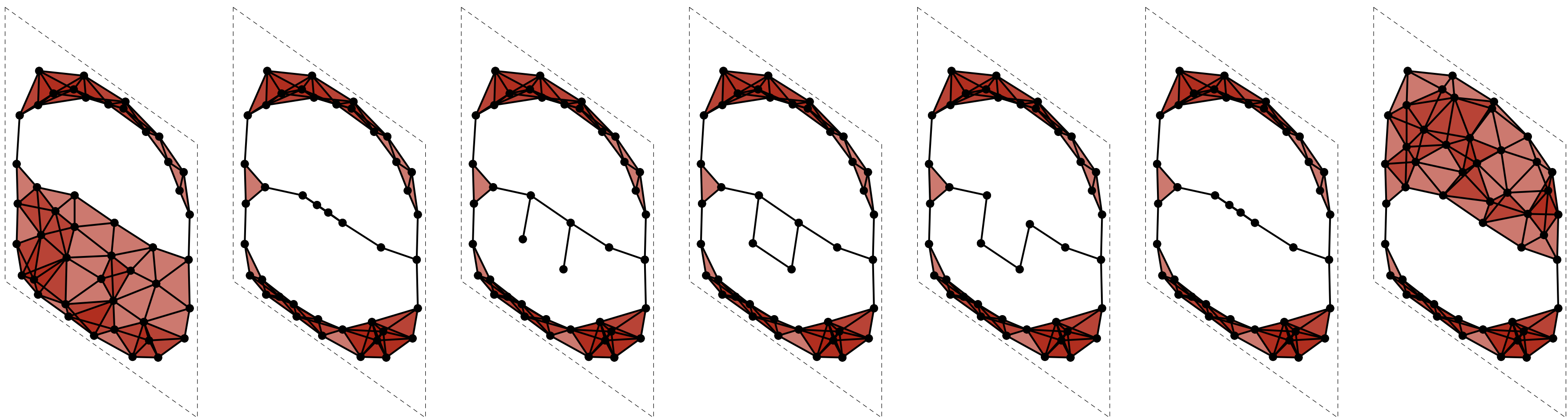
Evasion paths in mobile sensor networks



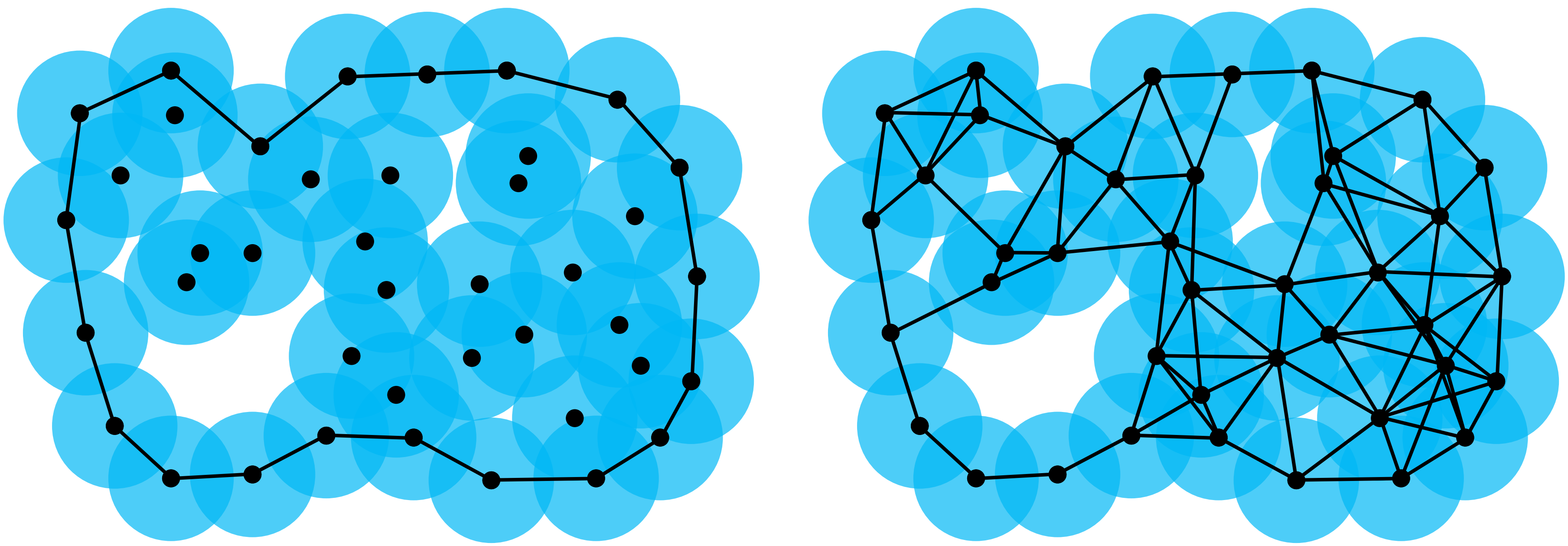
Henry Adams and Gunnar Carlsson
Stanford
July 26, 2012

Abstract

Suppose disk-shaped sensors wander in a planar domain. A sensor doesn't know its location but does know which sensors it overlaps. We say that an evasion path exists in this sensor network if a moving evader can avoid detection. Vin de Silva and Robert Ghrist give a necessary condition, depending only on the time-varying connectivity graph of the sensors, for an evasion path to exist. Can we sharpen this result? We'll consider an example where the existence of an evasion path depends not only on the network's connectivity data but also on its embedding.

Evasion problem

- Sensors move in a bounded, simply-connected domain $\mathcal{D} \subset \mathbb{R}^2$ over time interval I.
- Fixed sensors cover $\partial\mathcal{D}$.
- Sensors measure only local connectivity.



V. de Silva and R. Ghrist, *Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology*, Int. J. Rob. Res. 25 (2006), 1205-1222.

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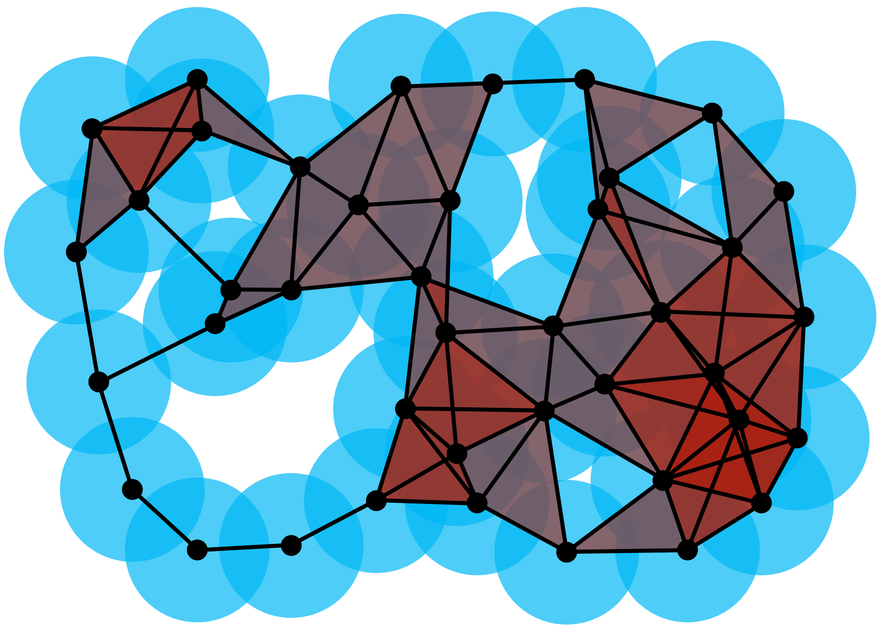
- The Čech complex is the nerve of the disks.



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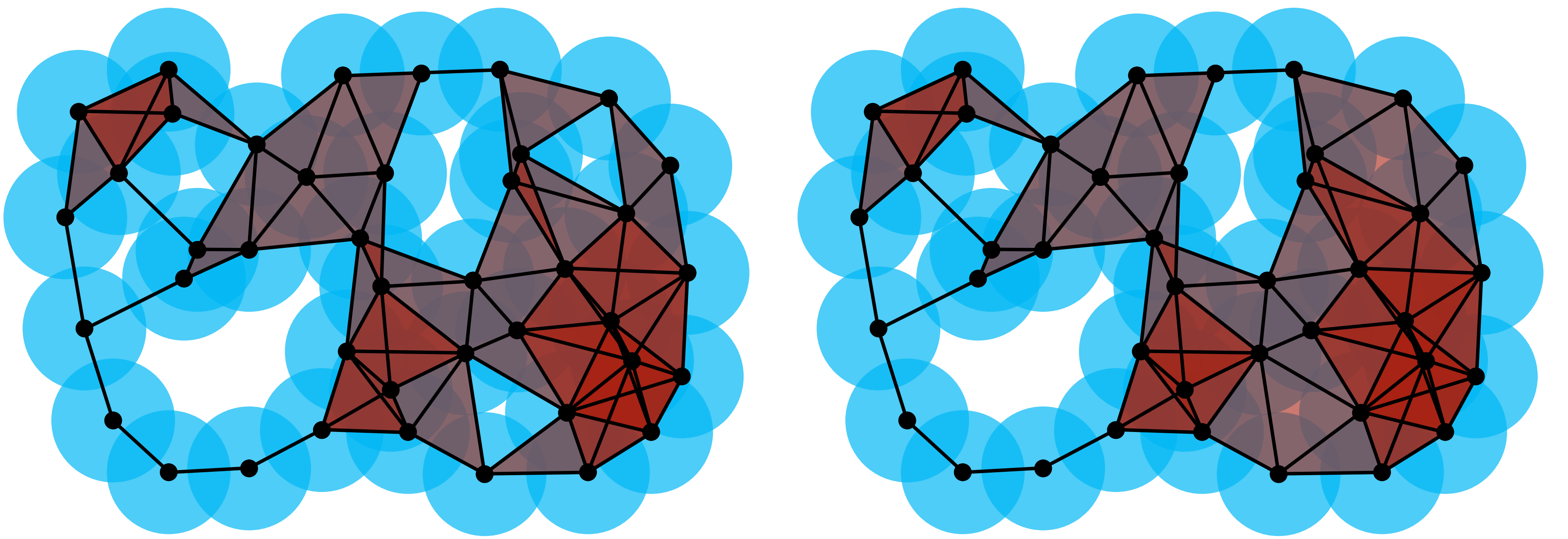

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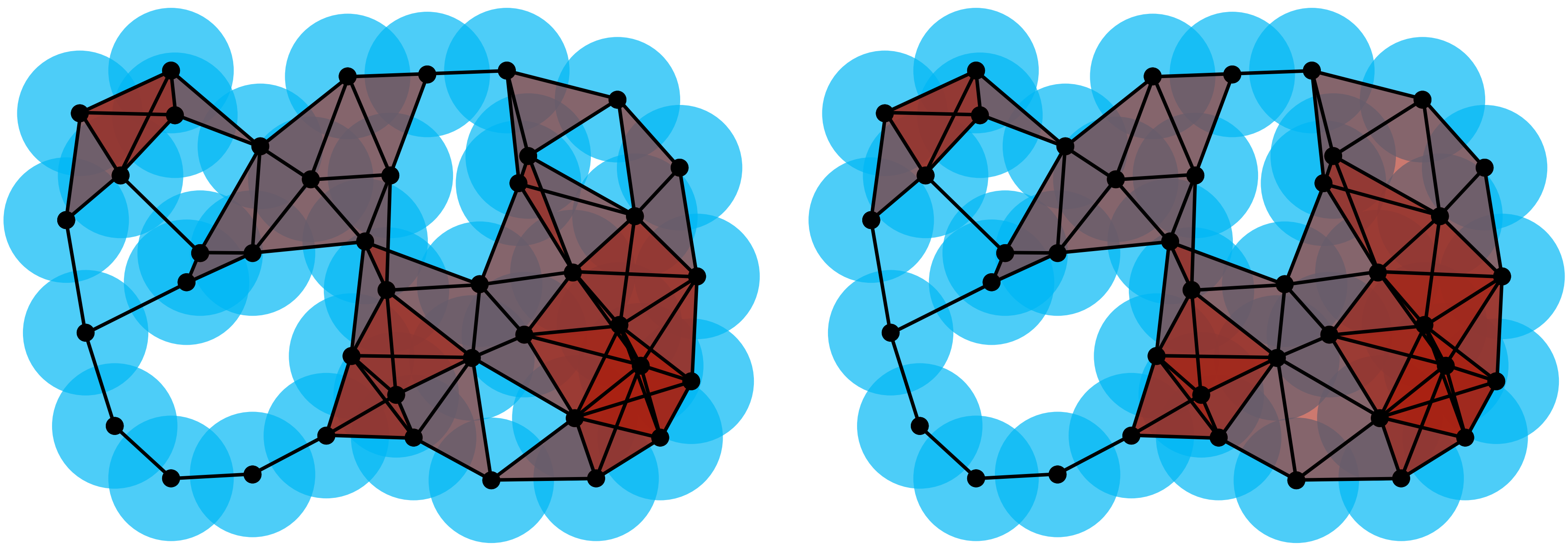

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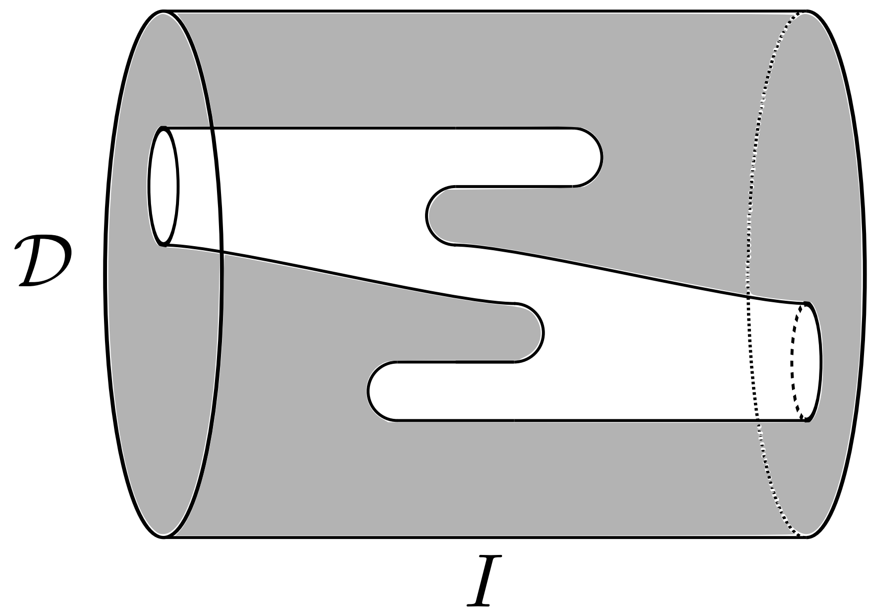
- Let $X \subset \mathcal{D} \times I$ be the covered region.
- $X \to I$ and $X^c \to I$ are fibrewise spaces.



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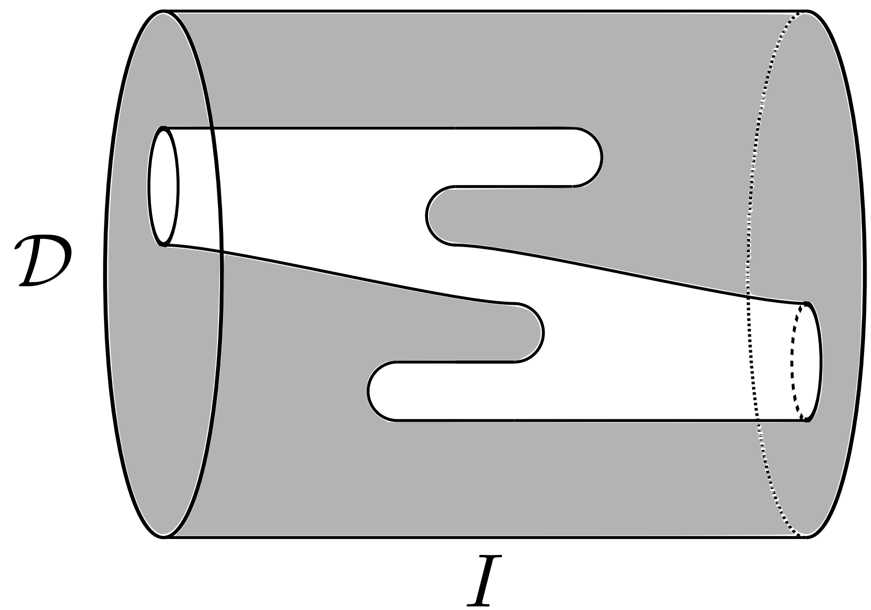

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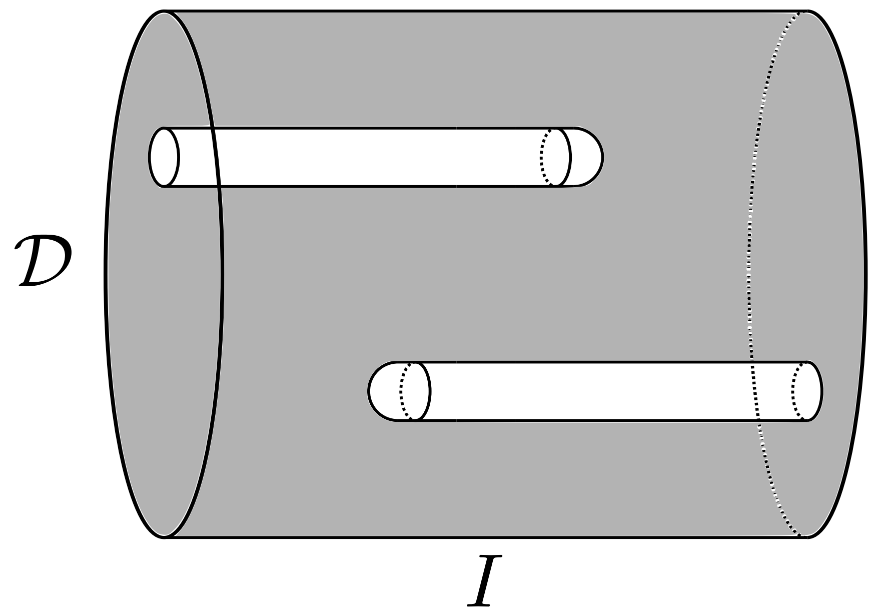

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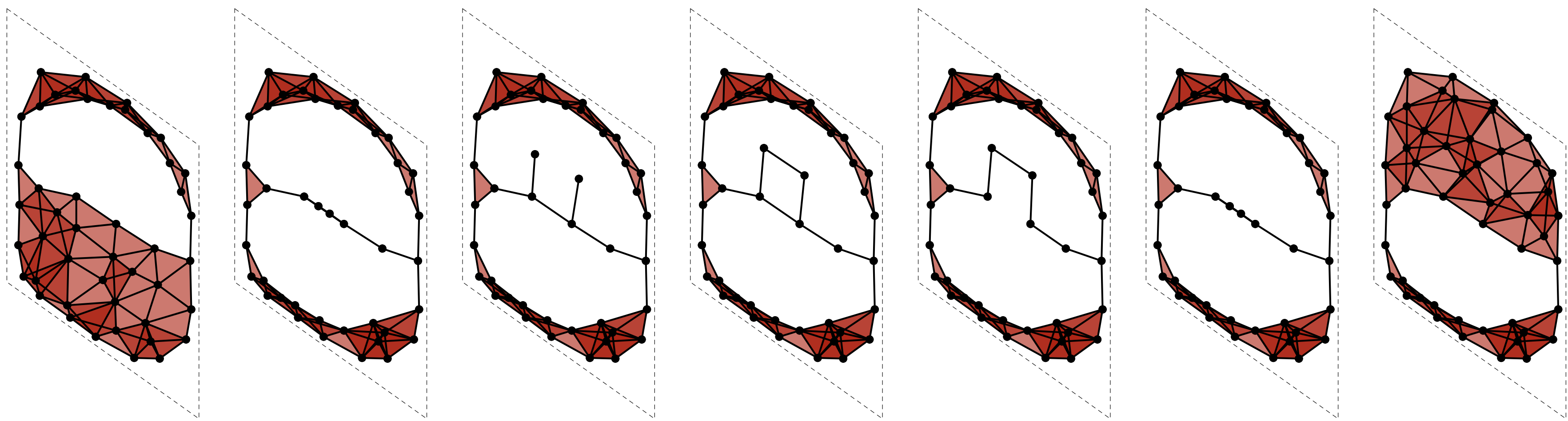

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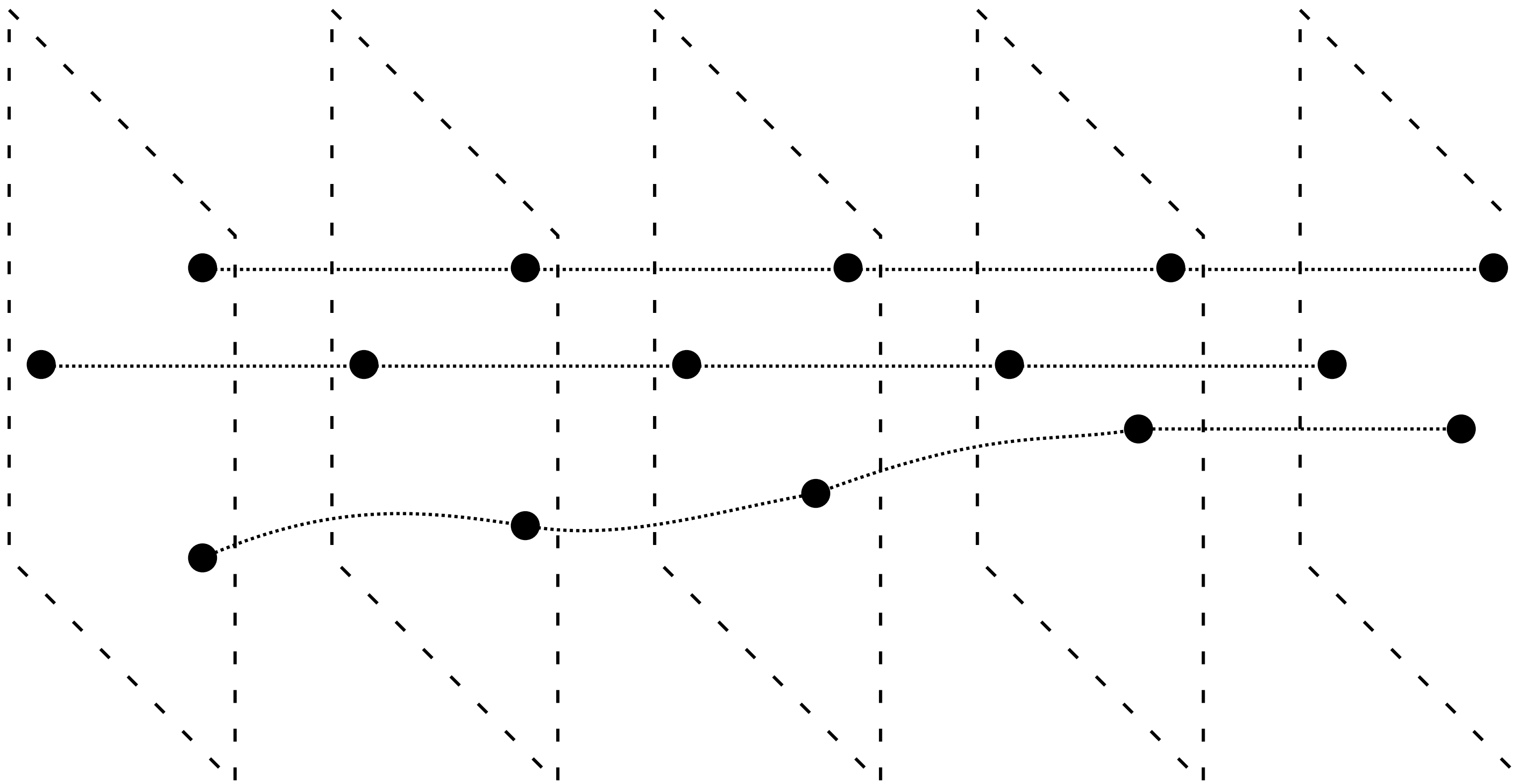

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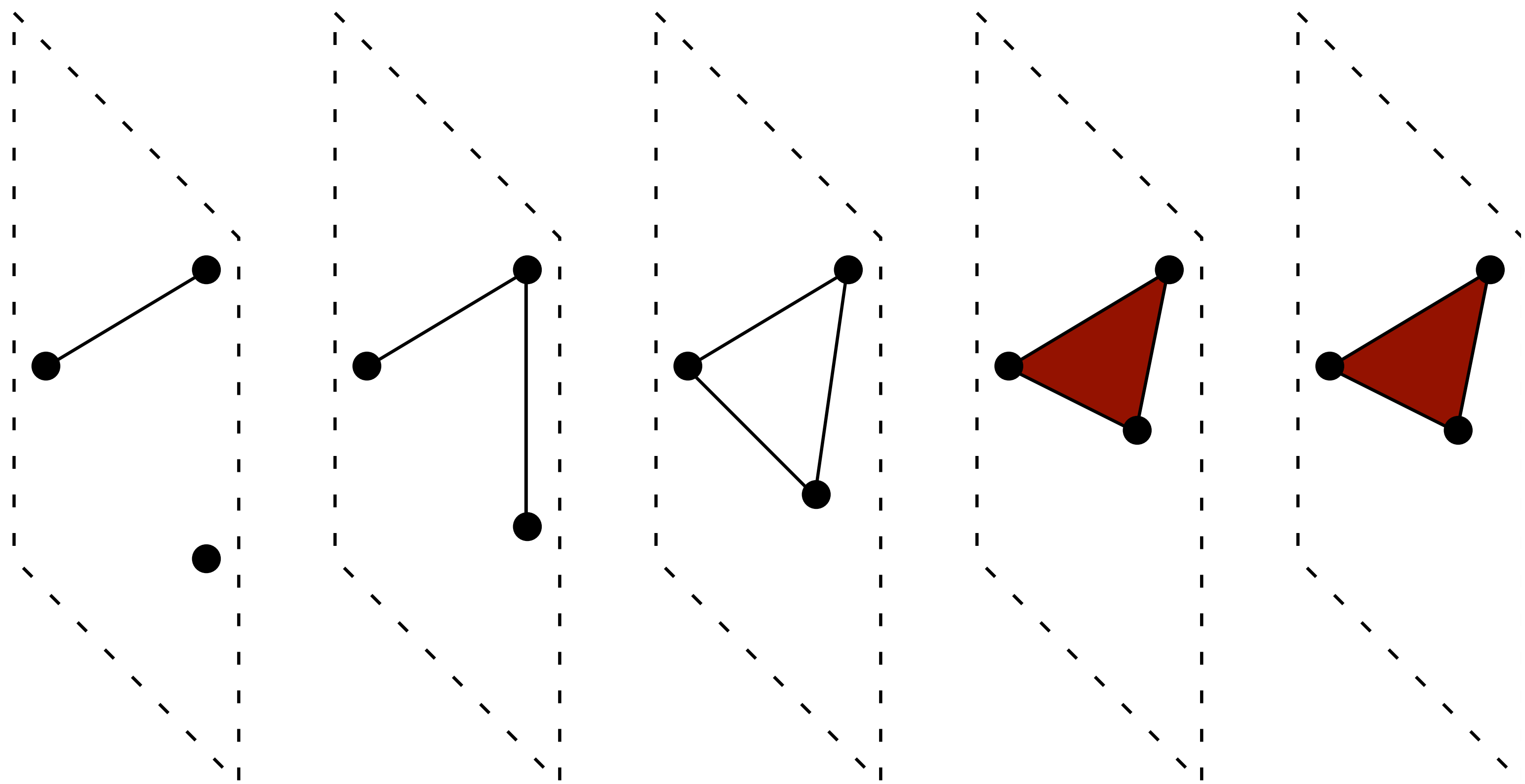

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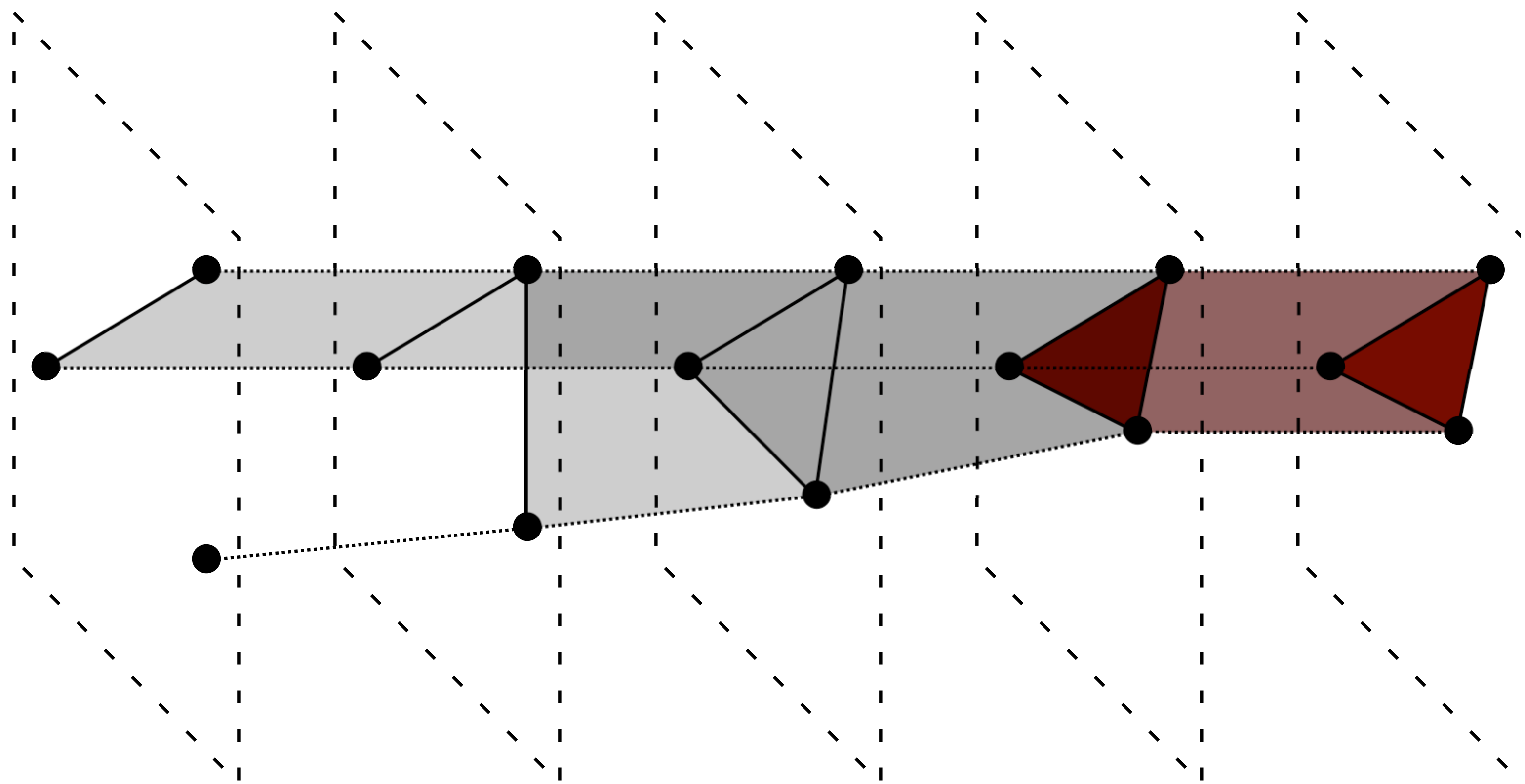

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- <u>Theorem 7 (reformulated)</u>
  If there is an $\alpha \in H_2(\mathcal{SC}, \partial\mathcal{D} \times I)$ with
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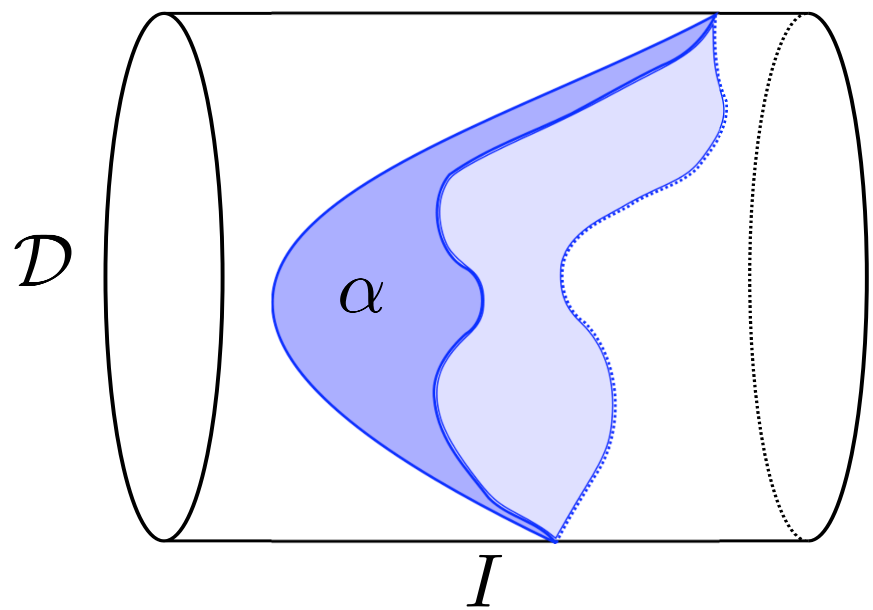

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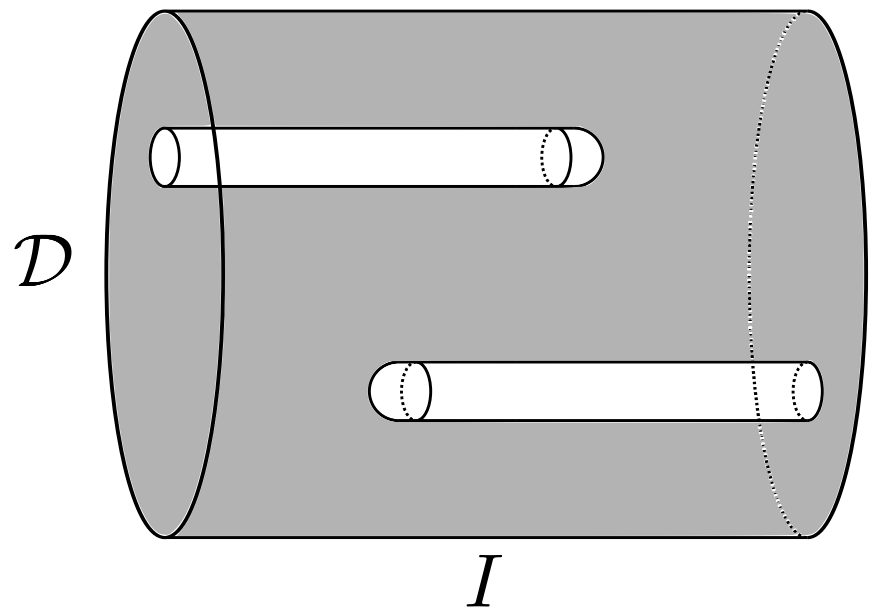

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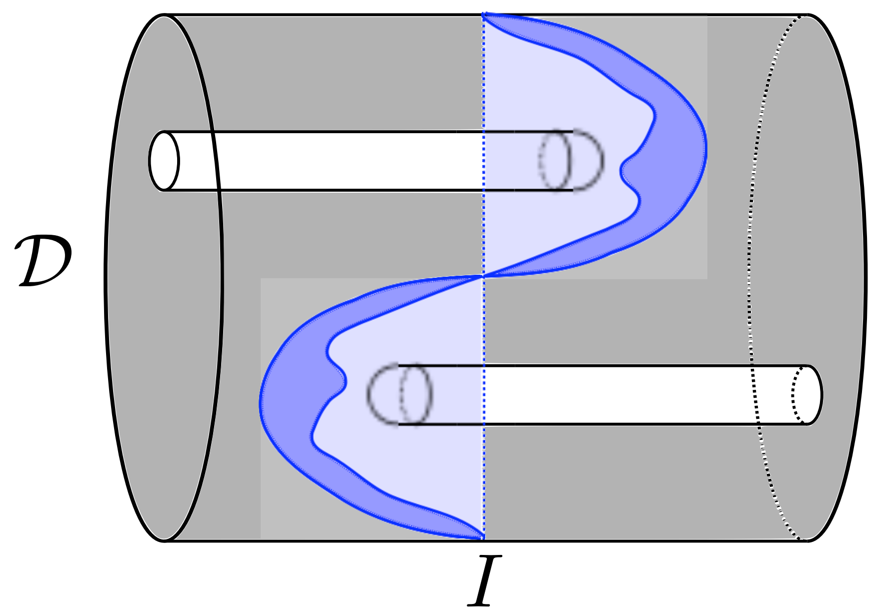

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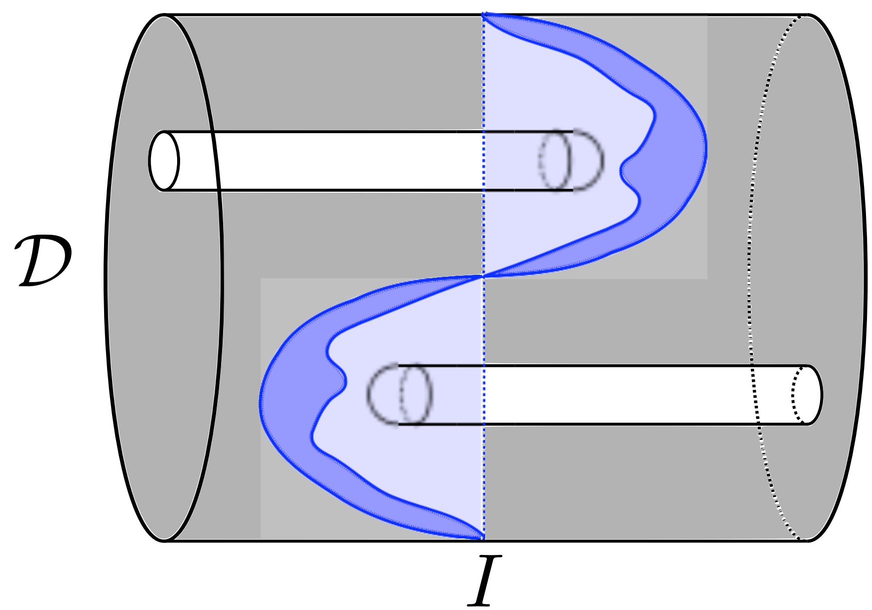

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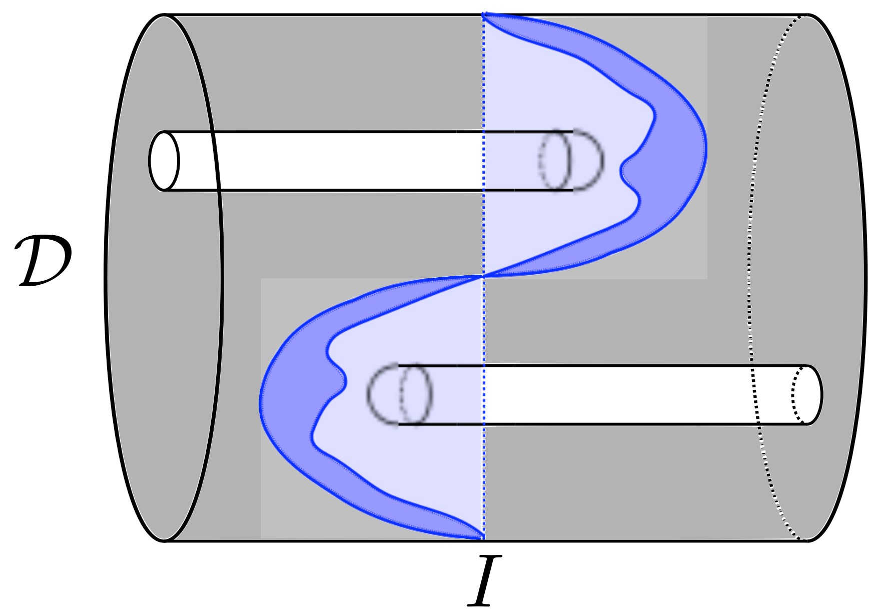

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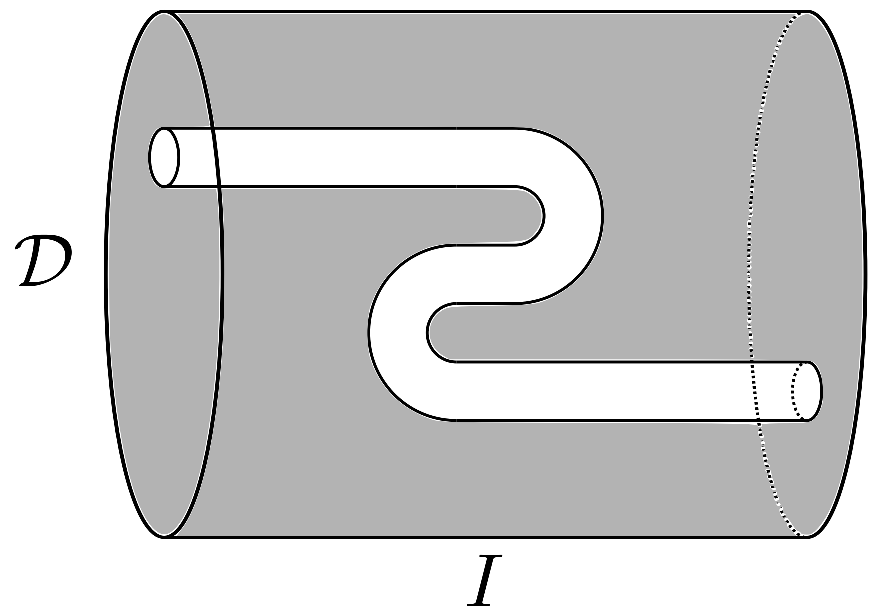

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Zigzag persistence

- Let quiver Q be $\bullet \leftrightarrow \bullet \leftrightarrow \ldots \leftrightarrow \bullet \leftrightarrow \bullet$
- The category of zigzag modules is $(k\text{-Vect})^Q$.

G. Carlsson and V. de Silva, *Zigzag Persistence*, Found. Comput. Math. 10 (2010), 367-405.

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- Zigzag modules are classified by barcodes.

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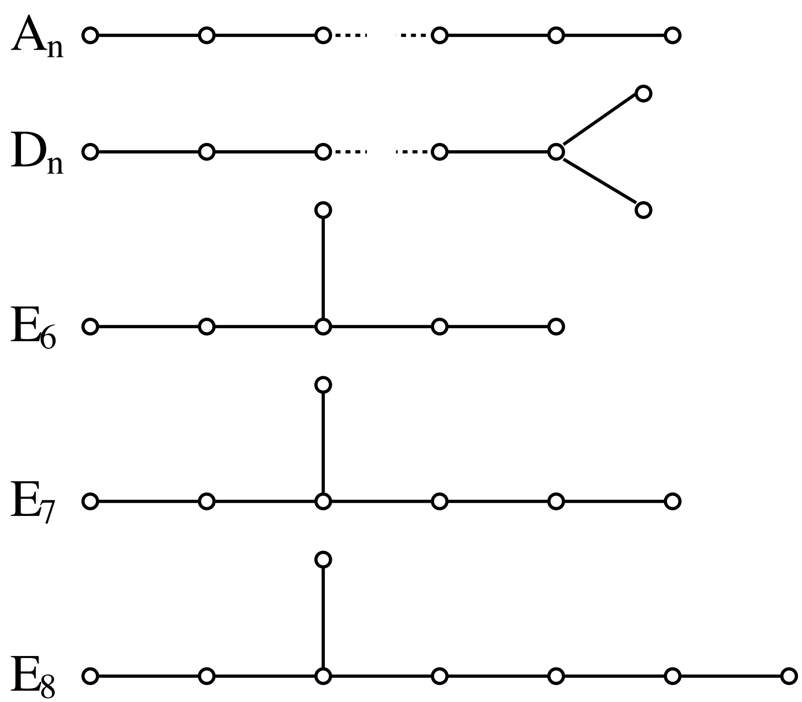
- <u>Theorem (Gabriel)</u>
  Quiver Q has a finite number of indecomposables $\Leftrightarrow$ it's a union of certain Dynkin diagrams.



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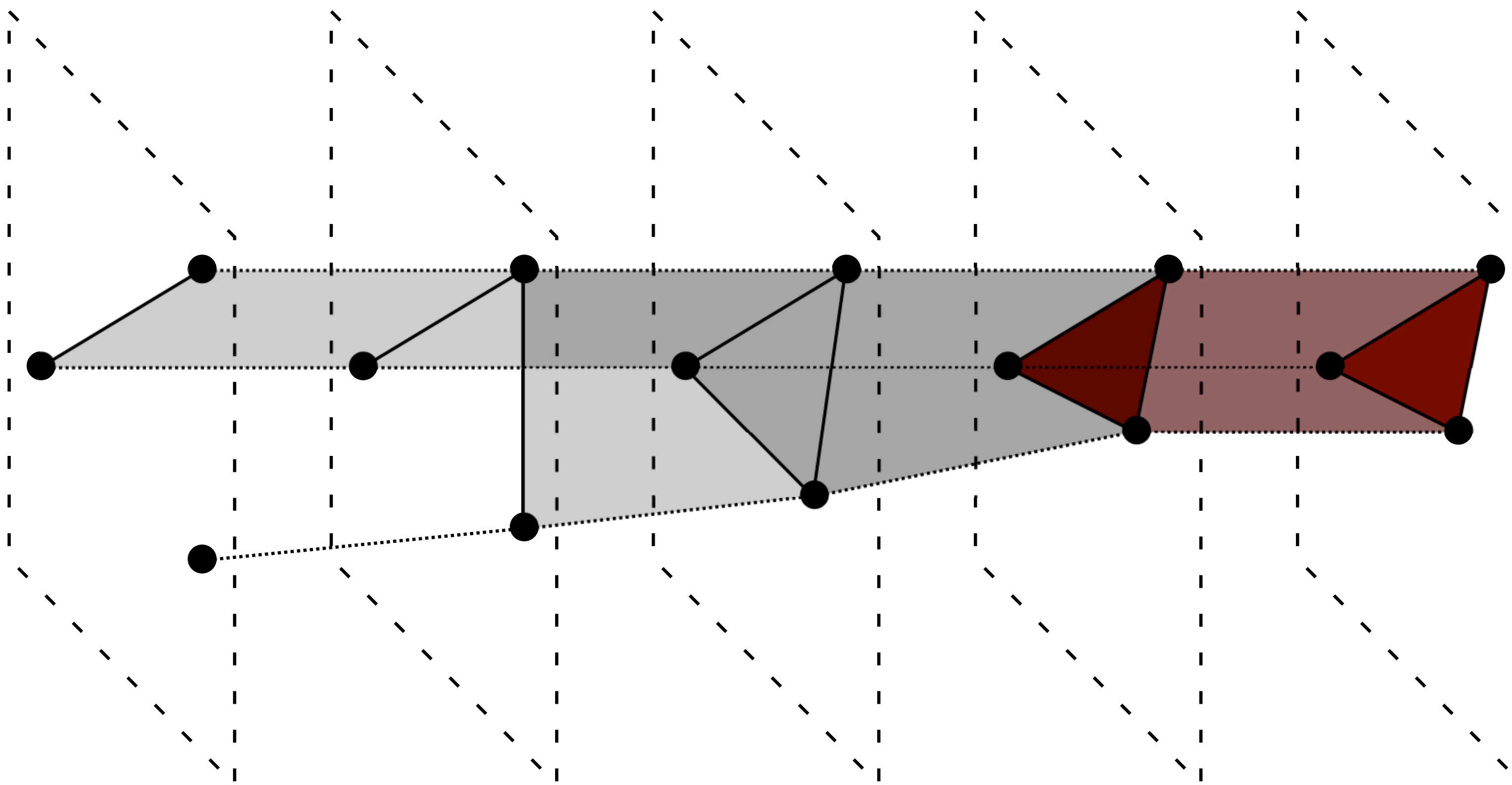

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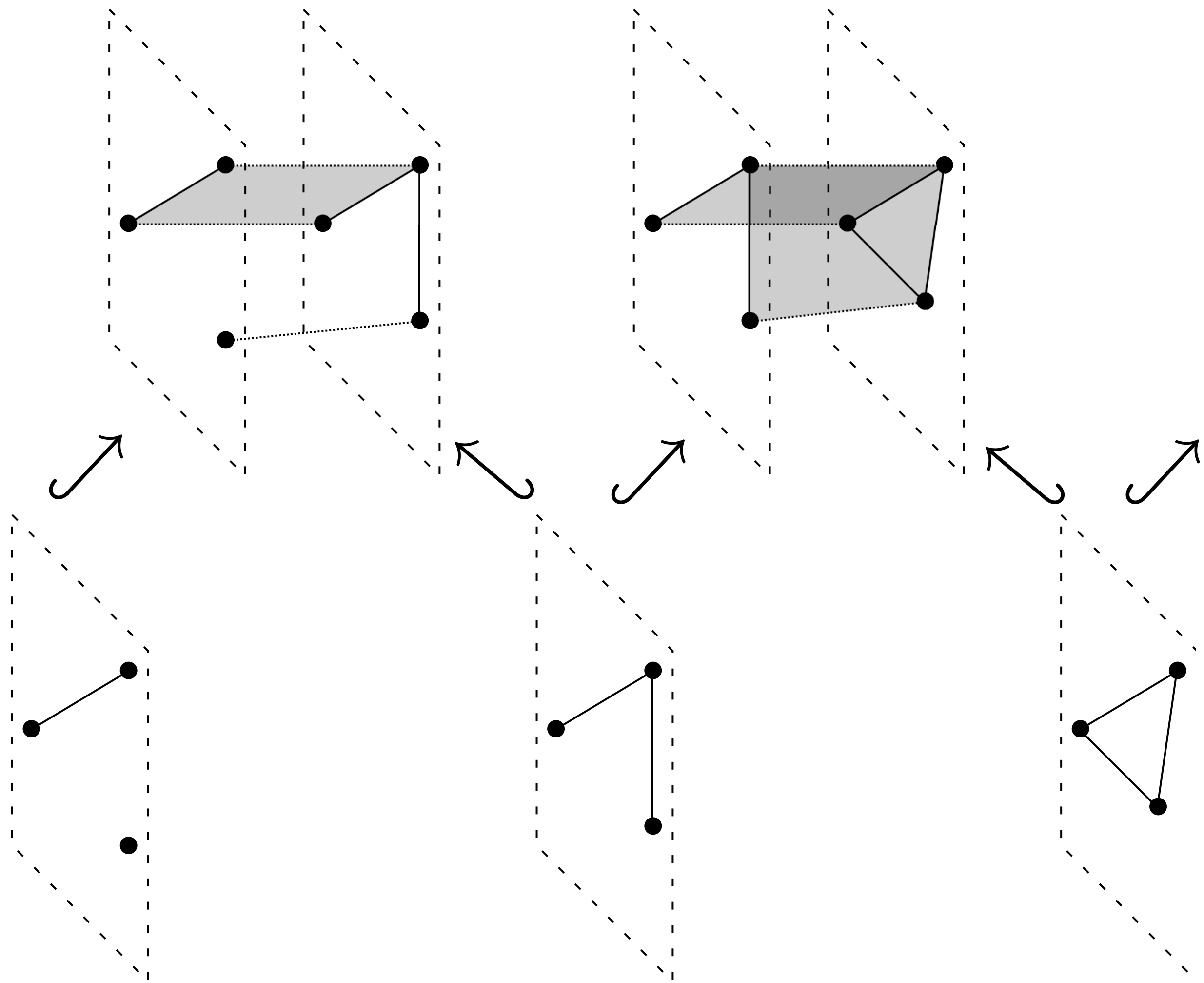

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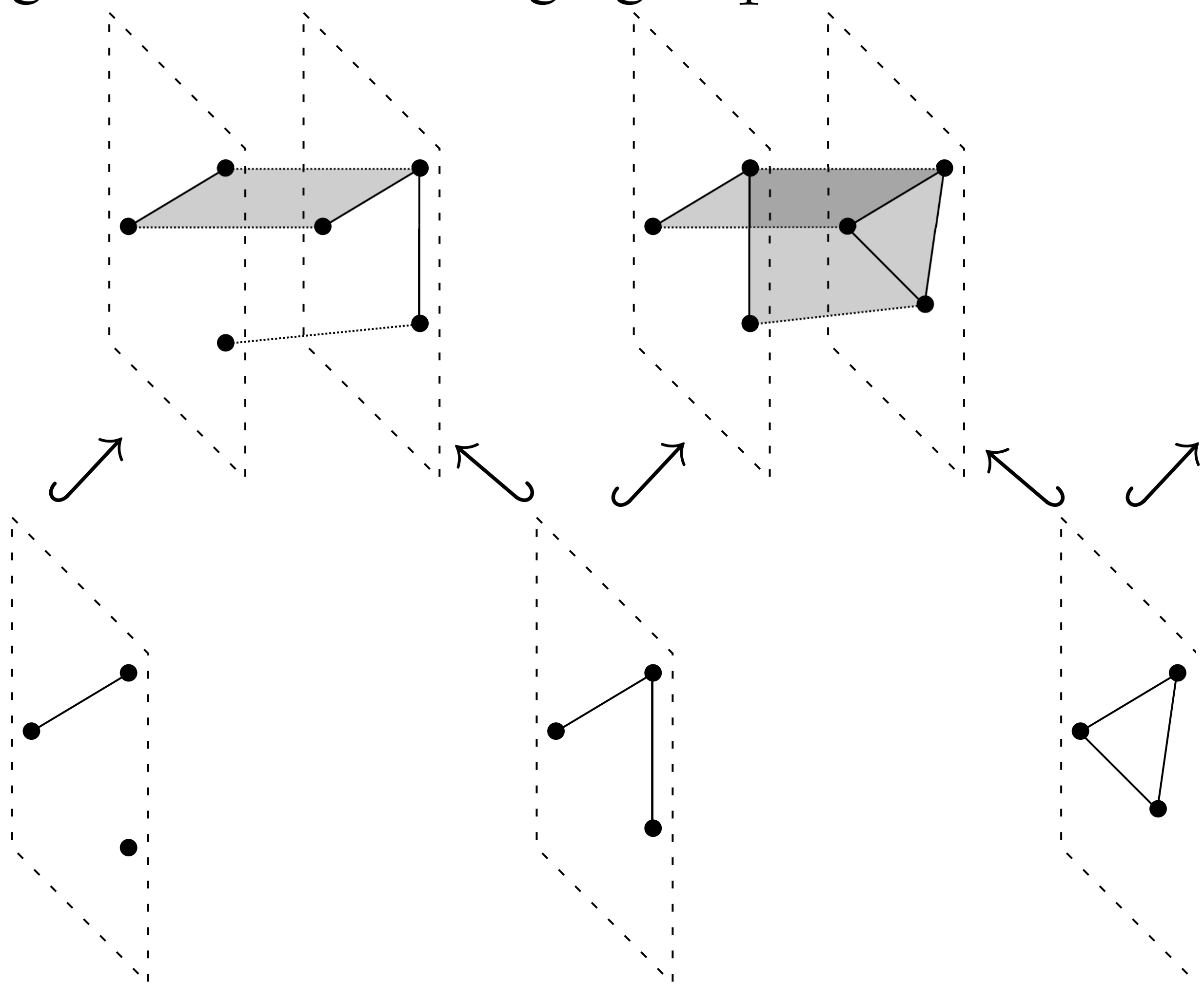

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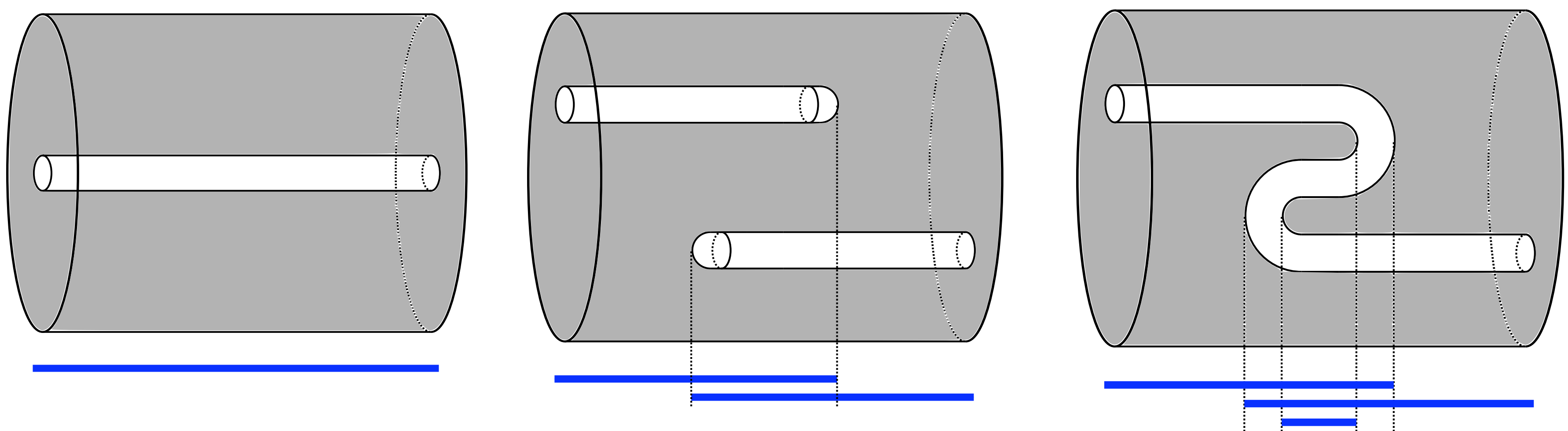

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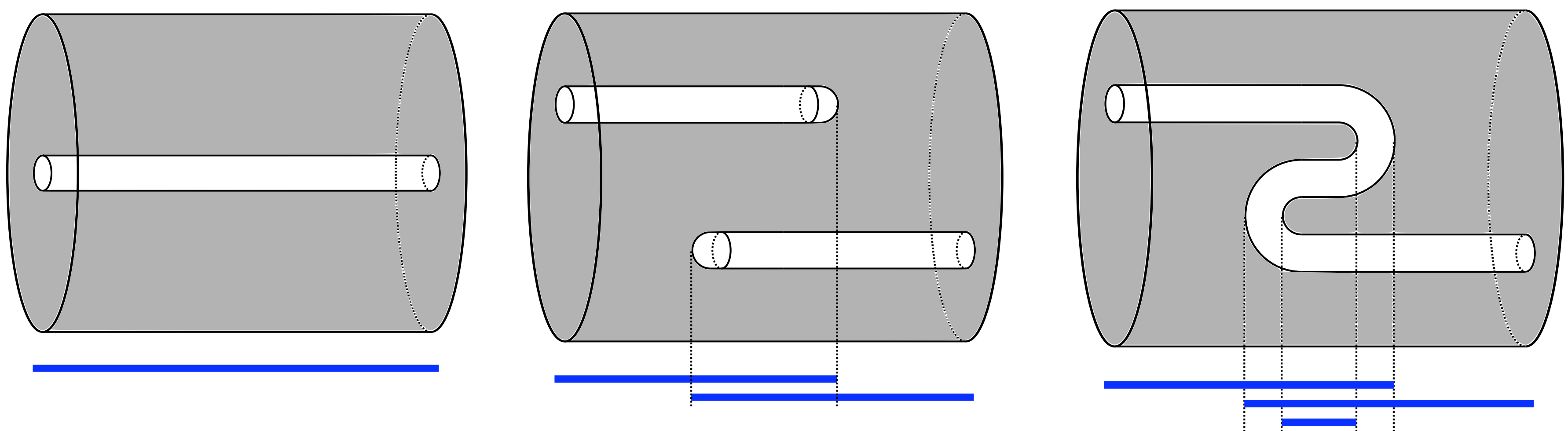

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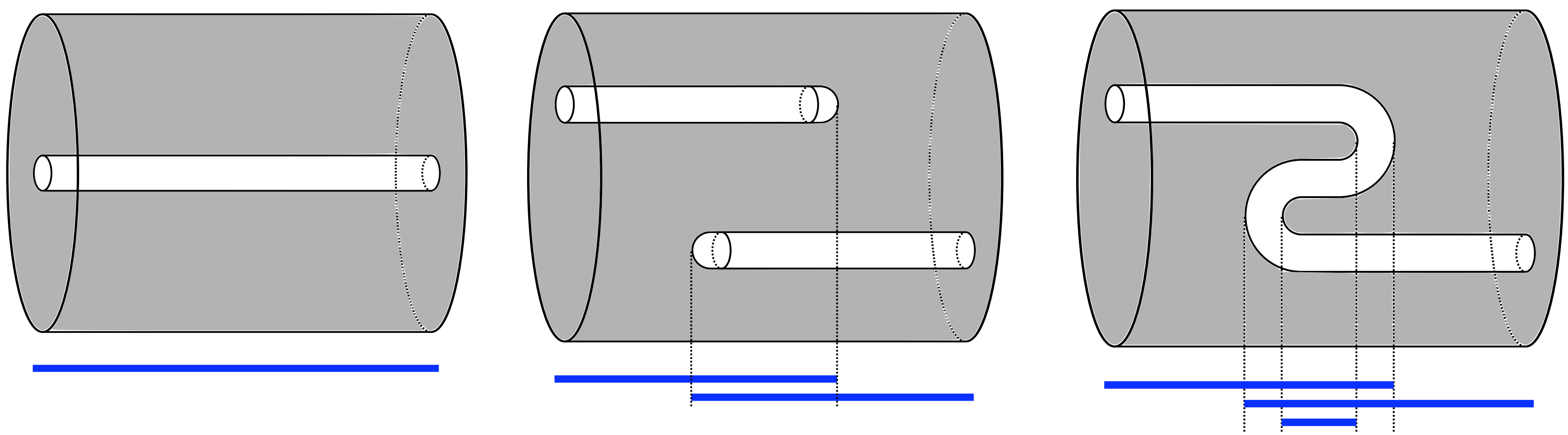

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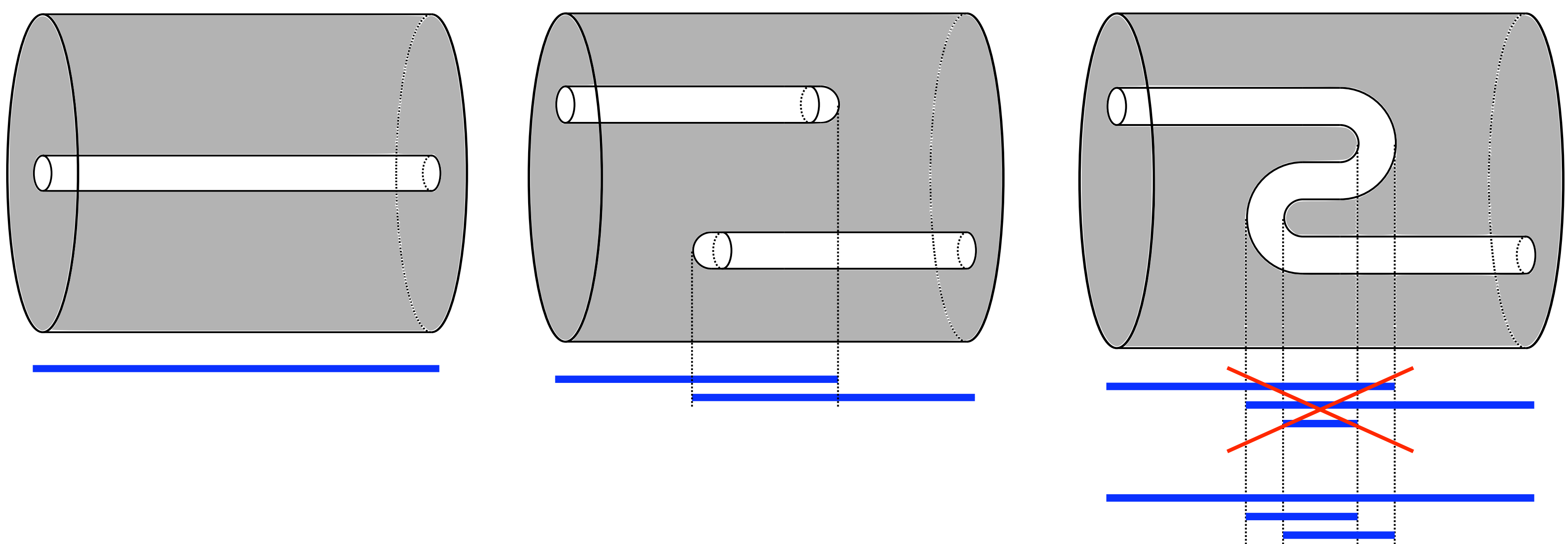

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- $\Rightarrow$ is true, but $\Leftarrow$ is false.

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Dependence on embedding $X \hookrightarrow \mathcal{D} \times I$

- No approach with input $\mathcal{SC}$ can determine (sharply) whether an evasion path exists or not.
- Two networks with the same $\mathcal{SC}$. Top contains evasion path while bottom does not.

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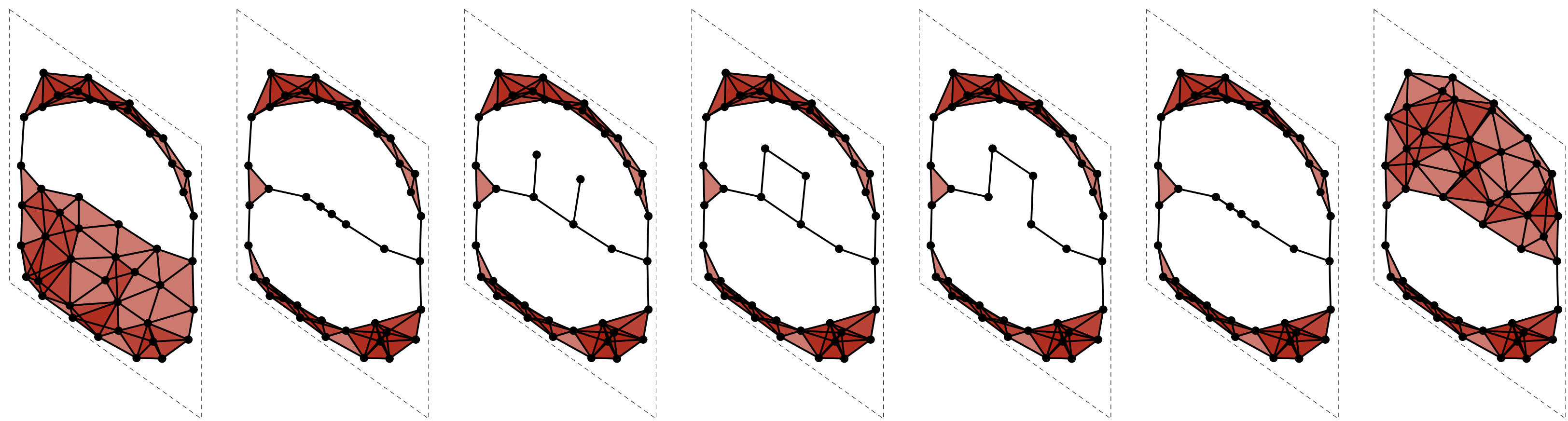

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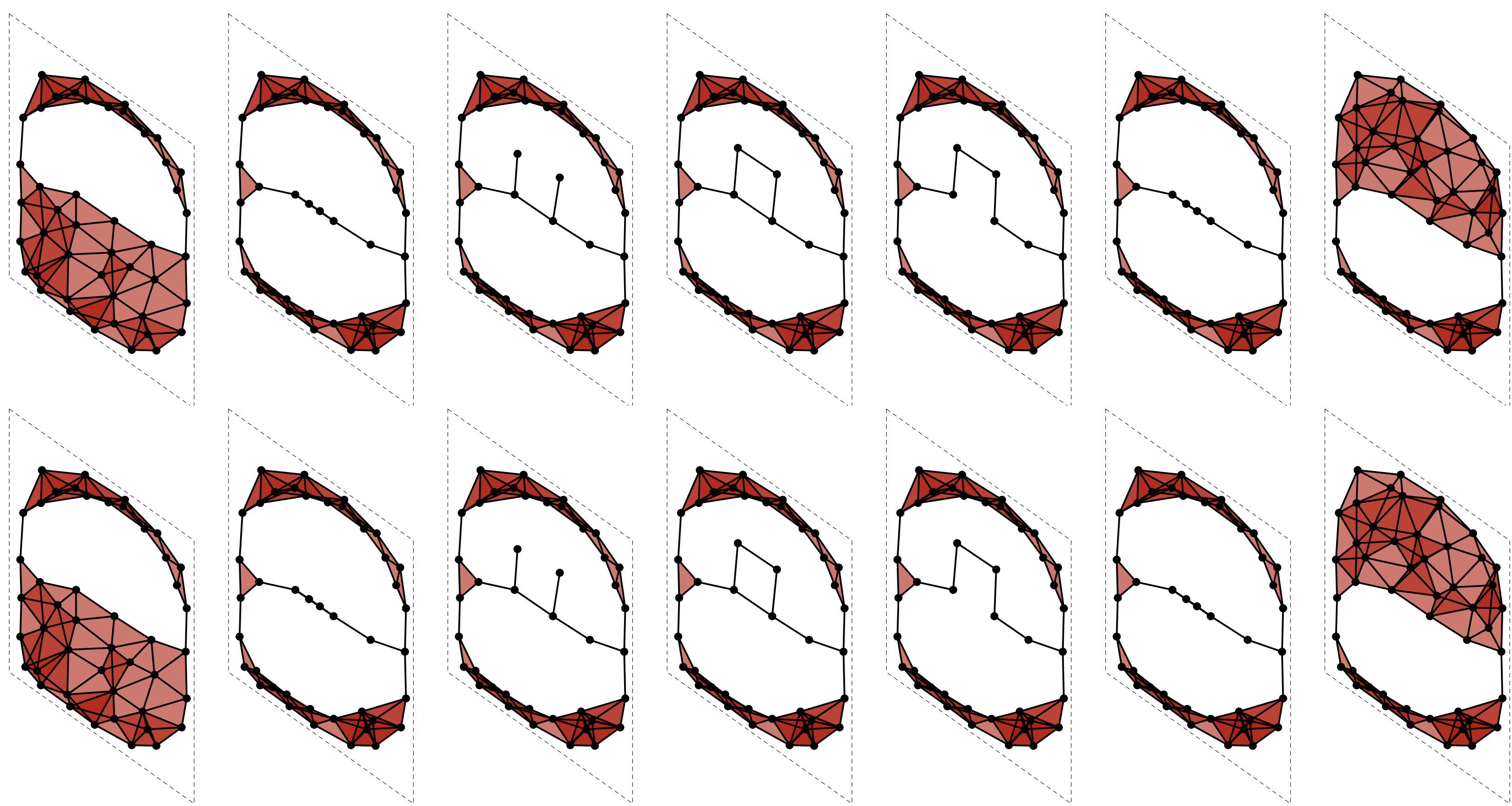

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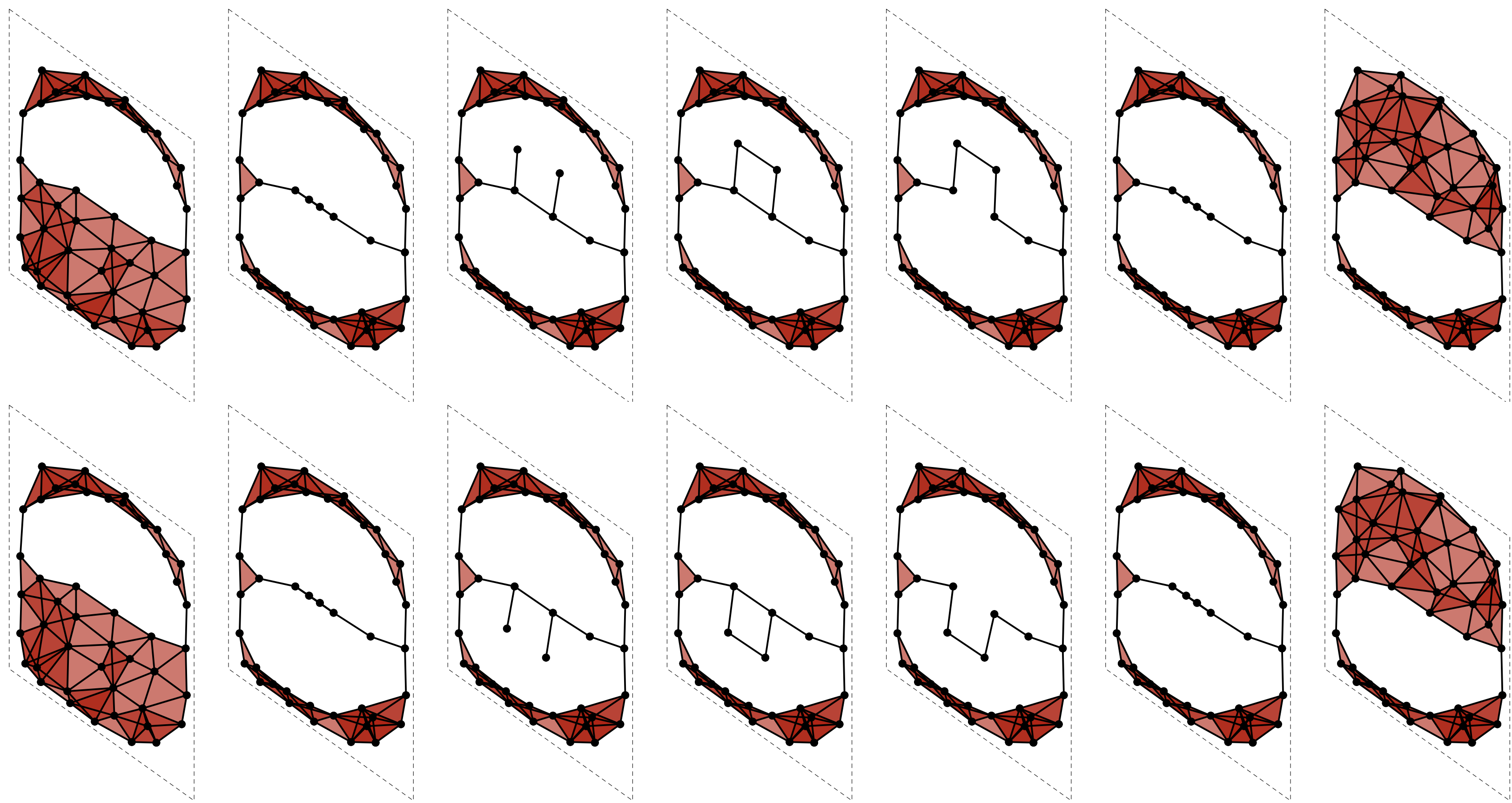

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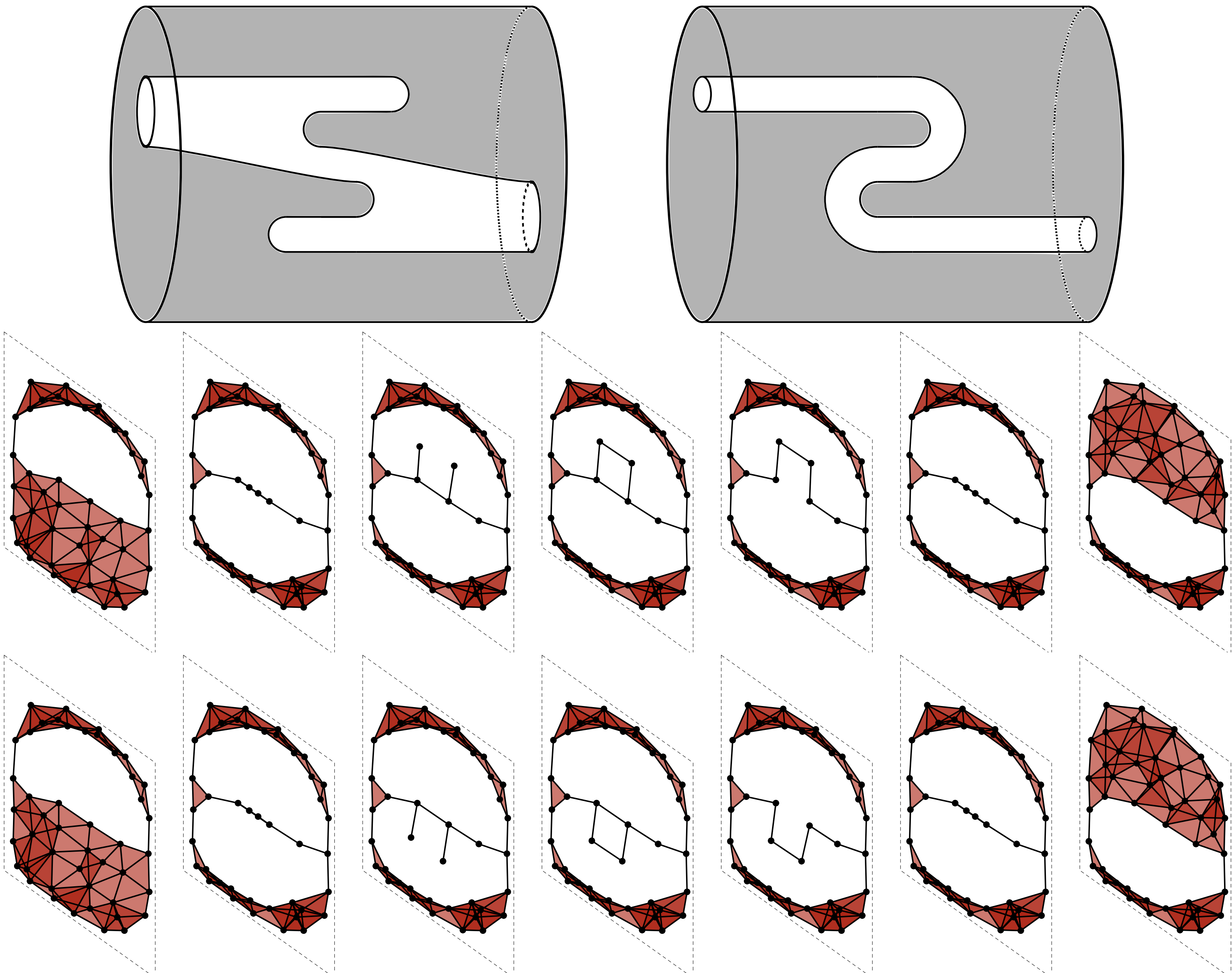

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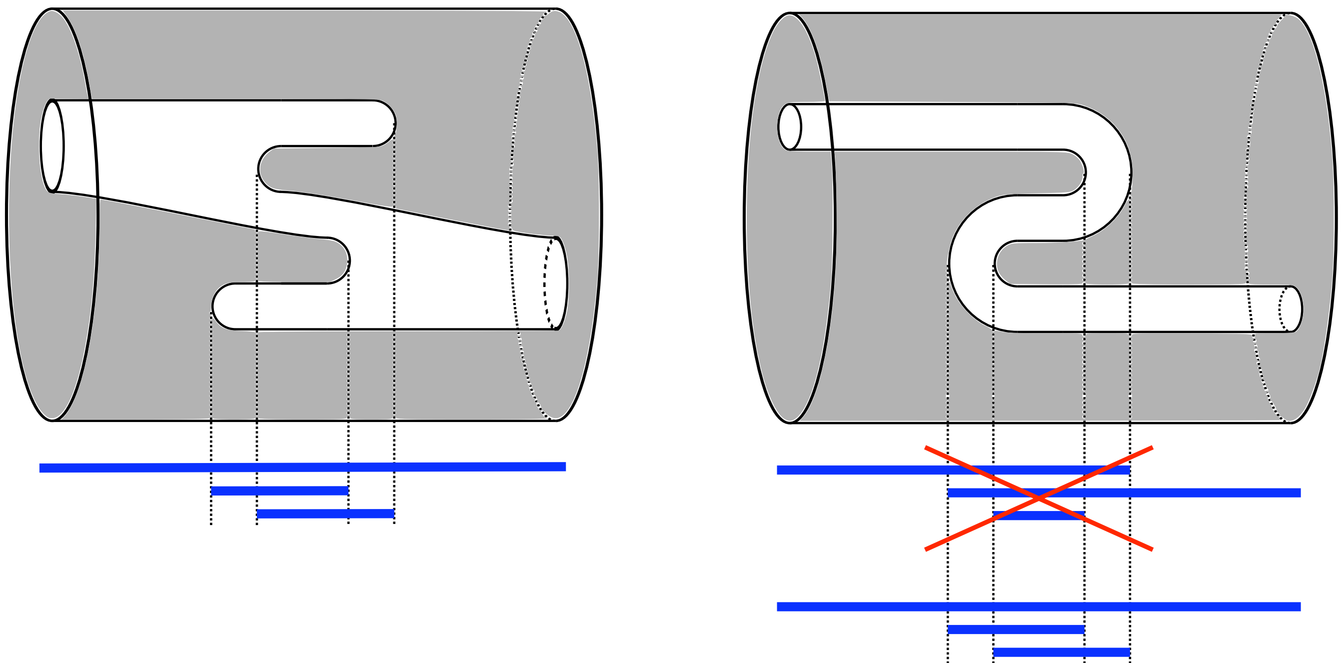
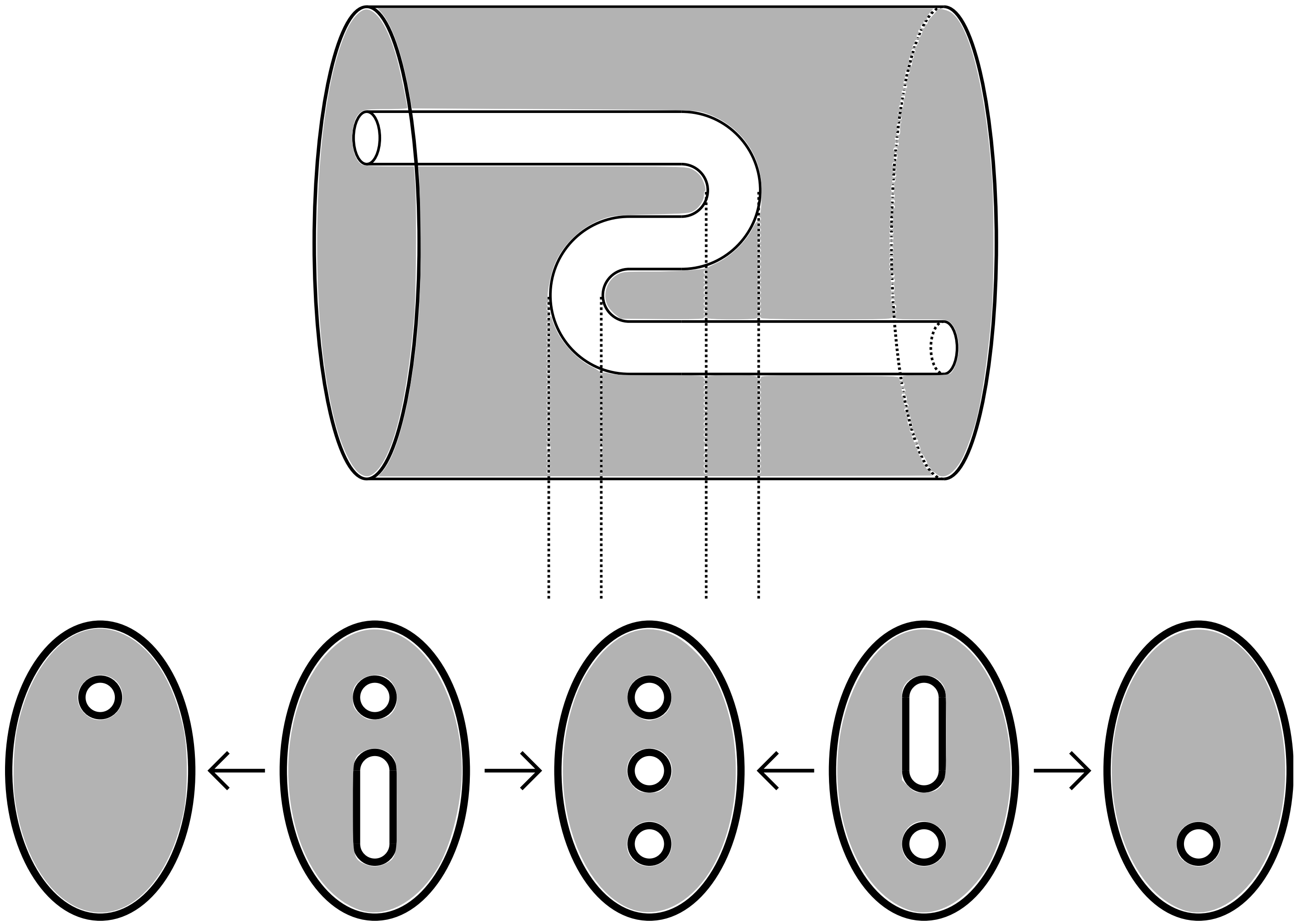


- These two networks X are fibrewise homotopy equivalent but their complements X^c are not.

Zigzag persistence



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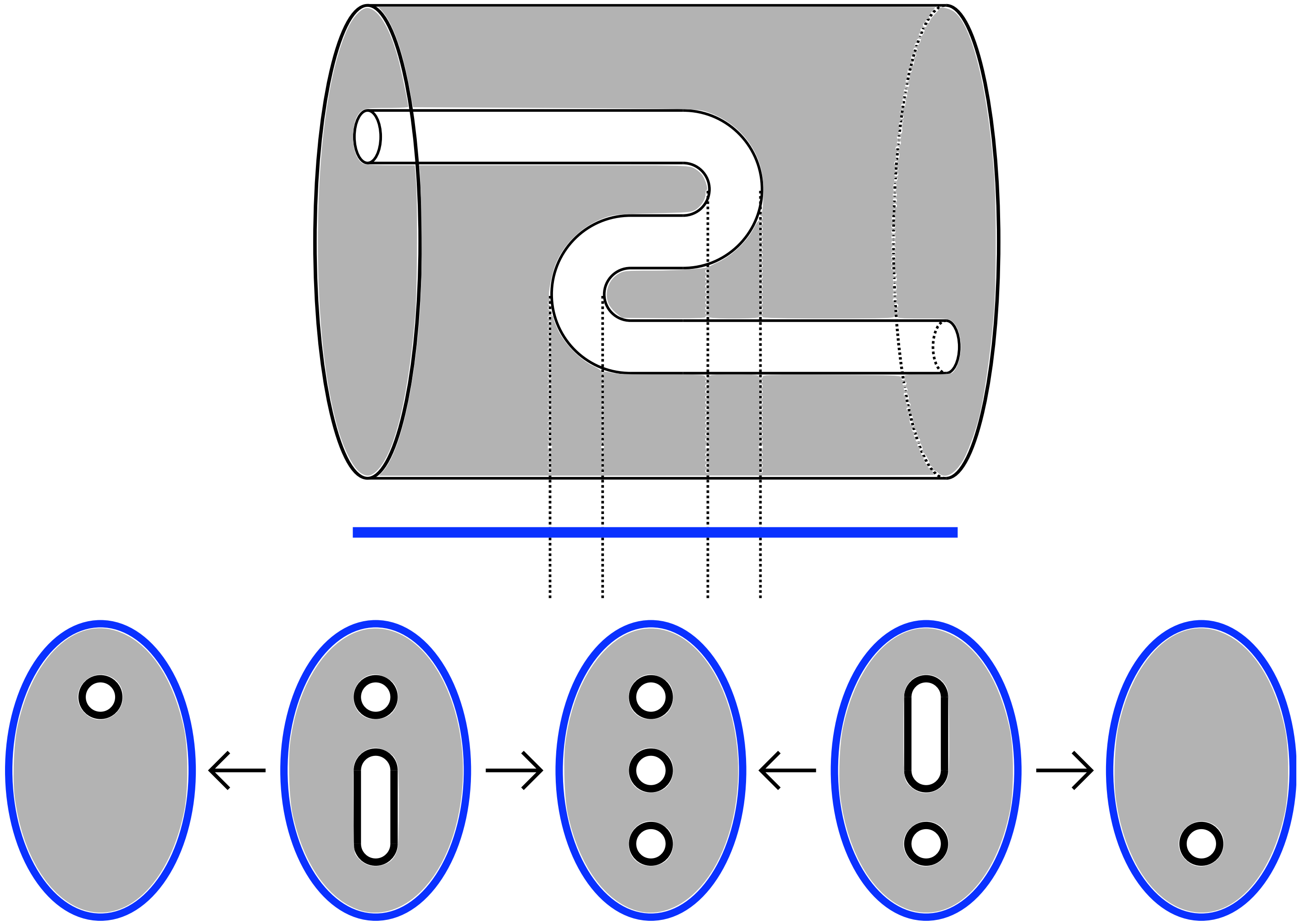

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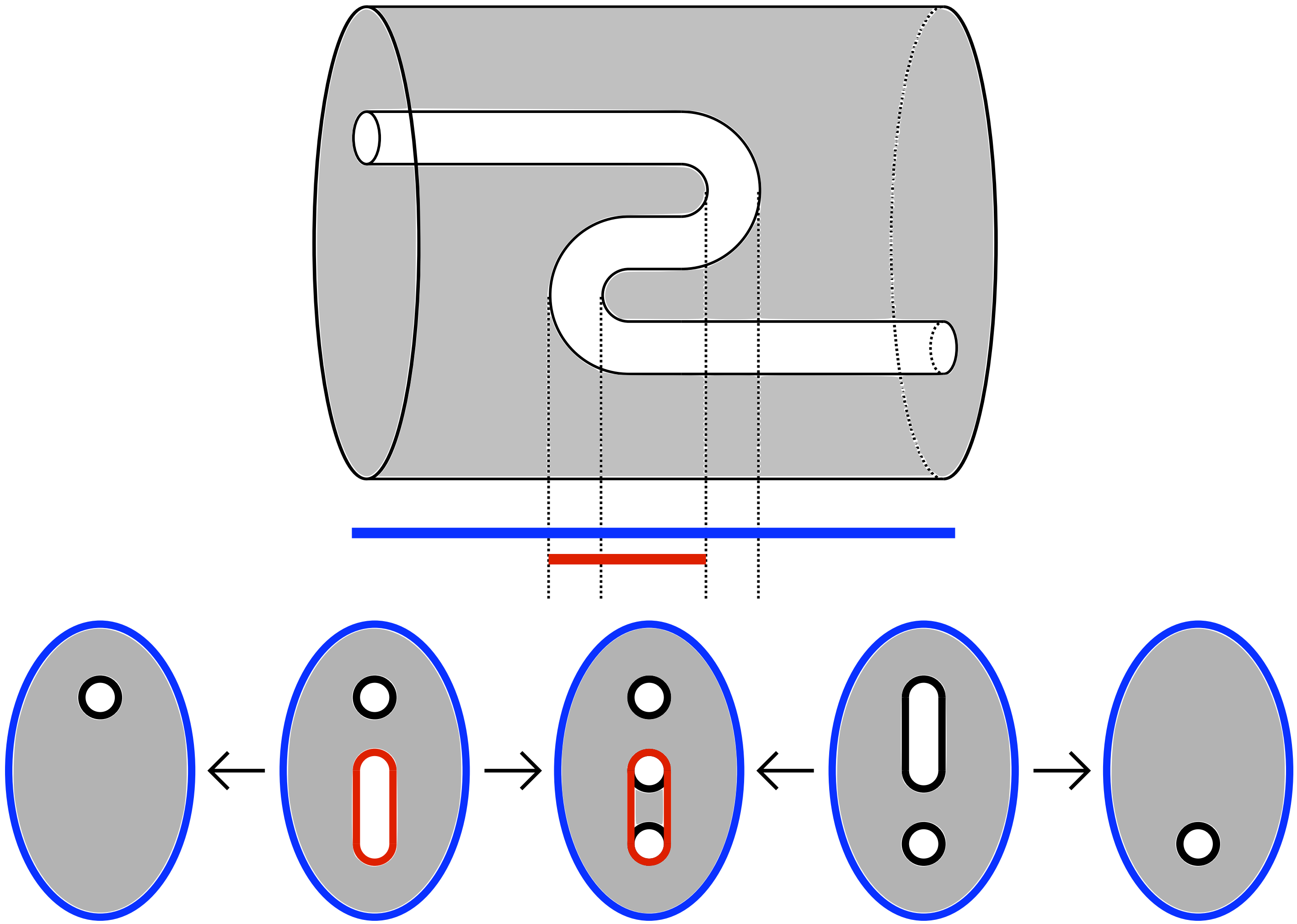

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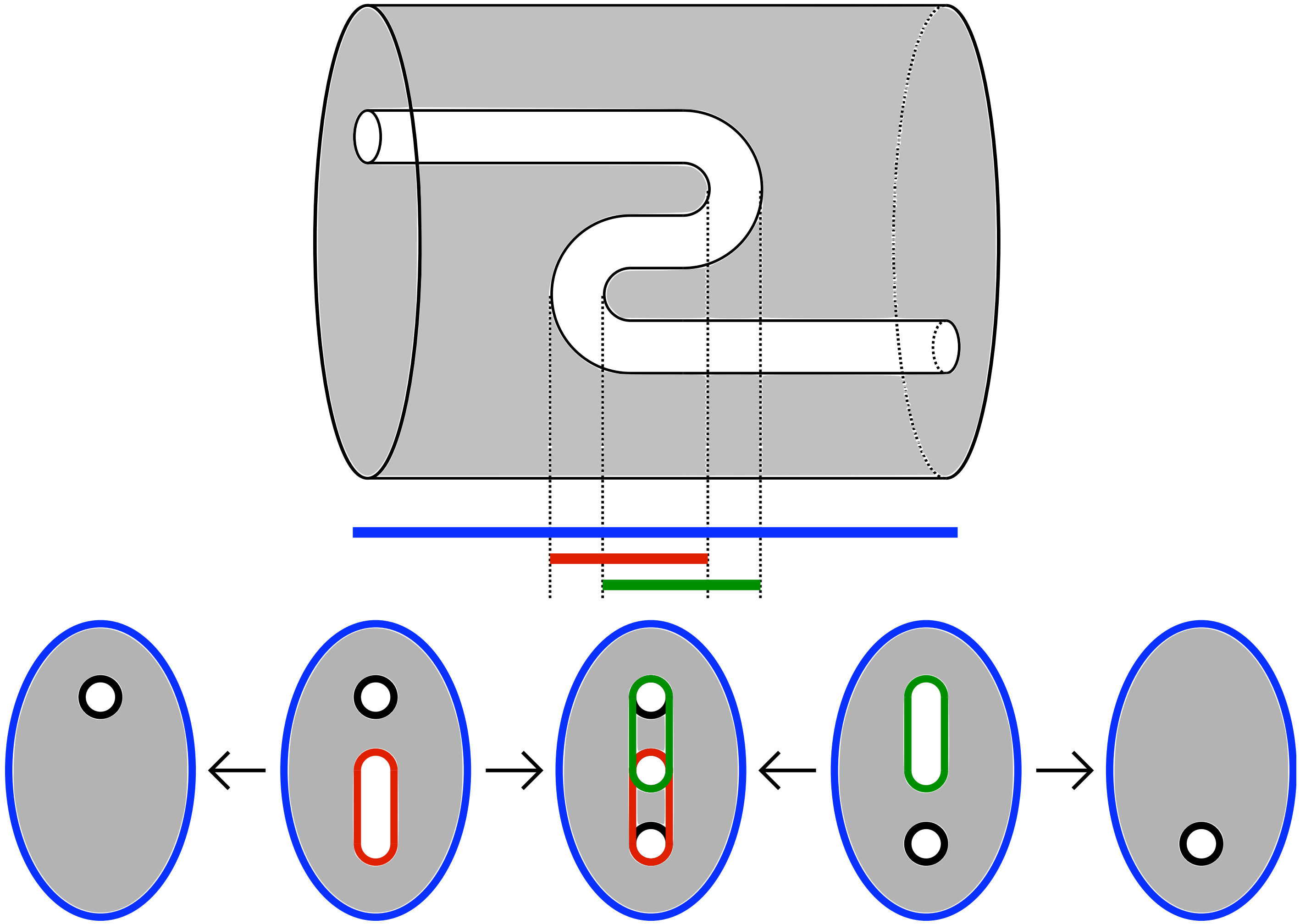

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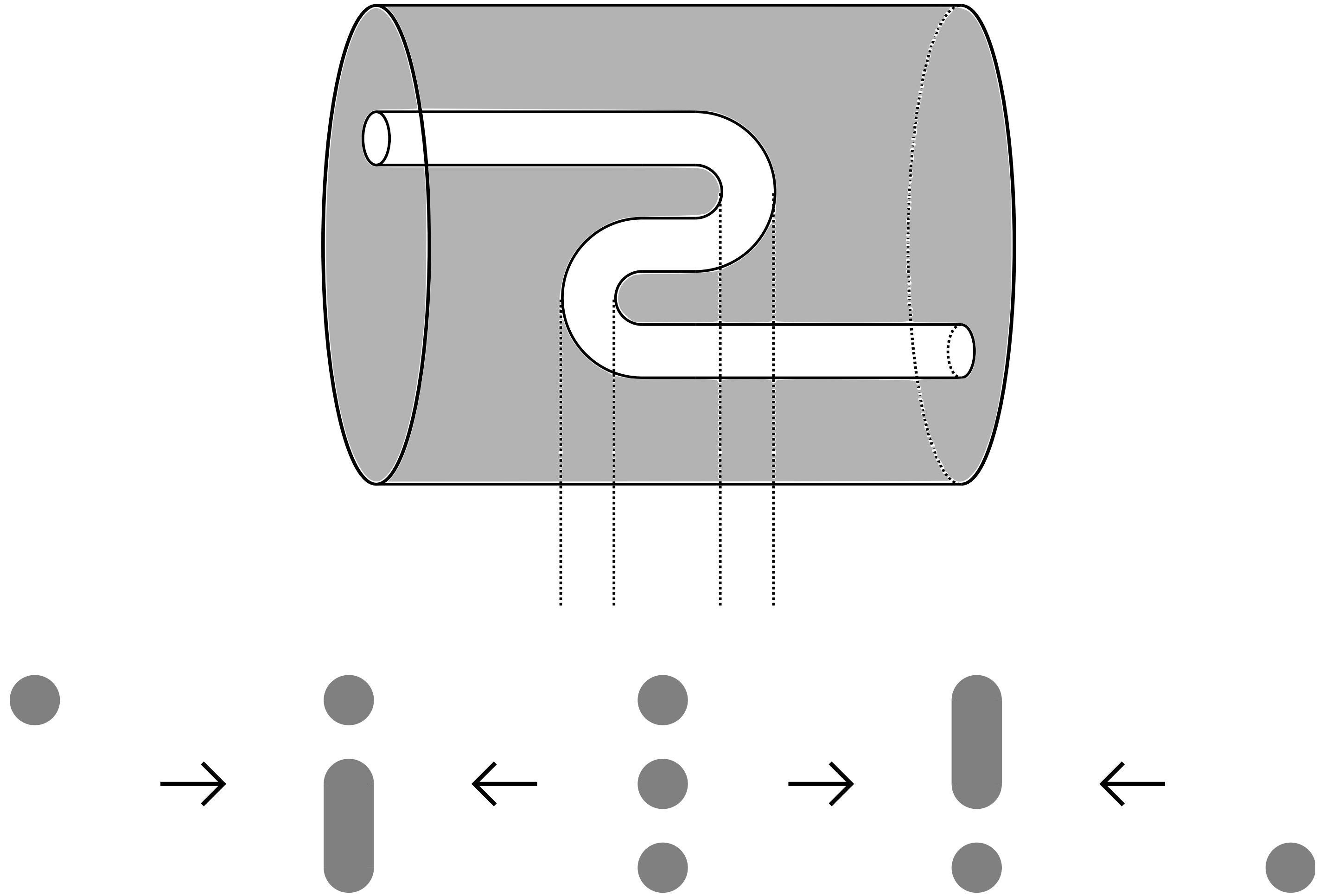

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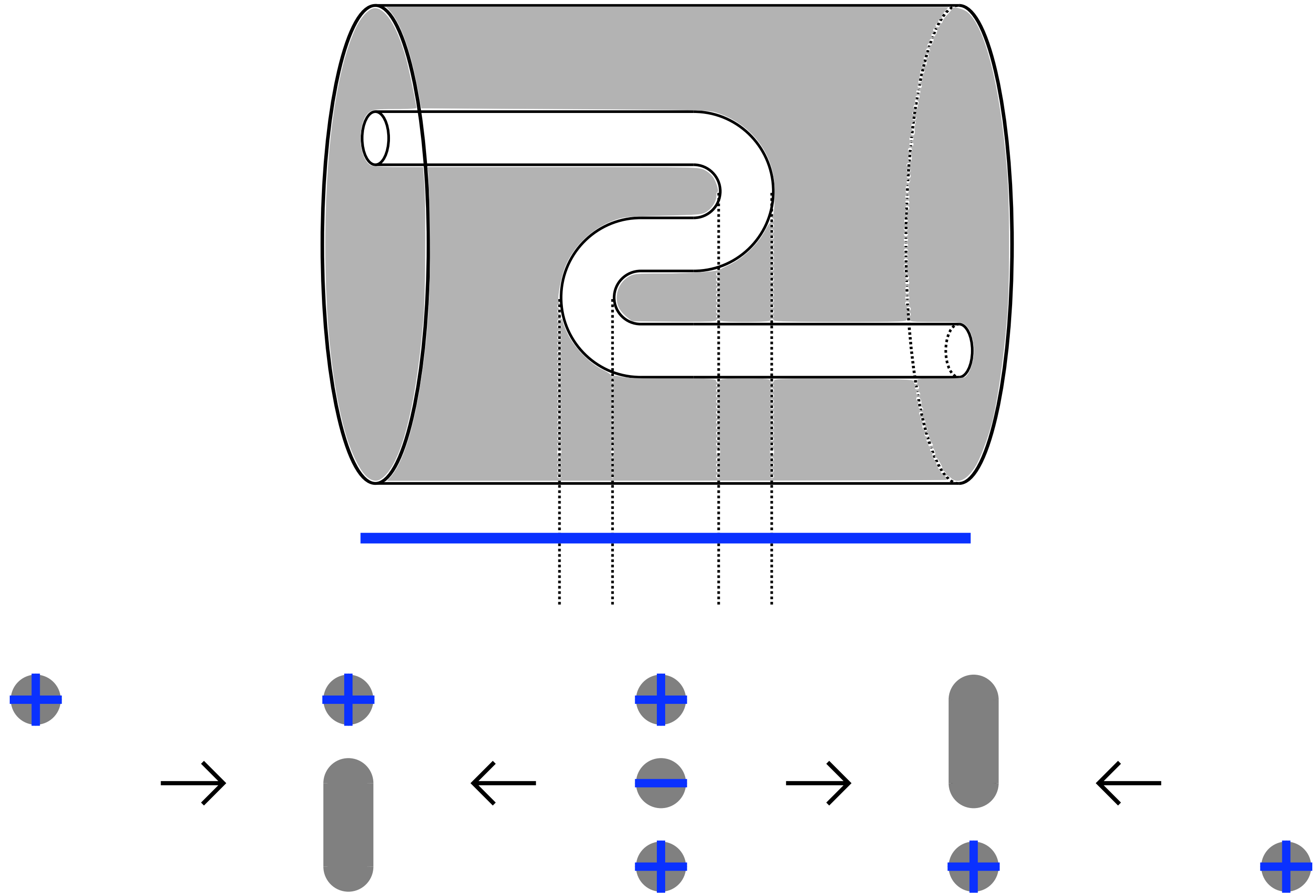

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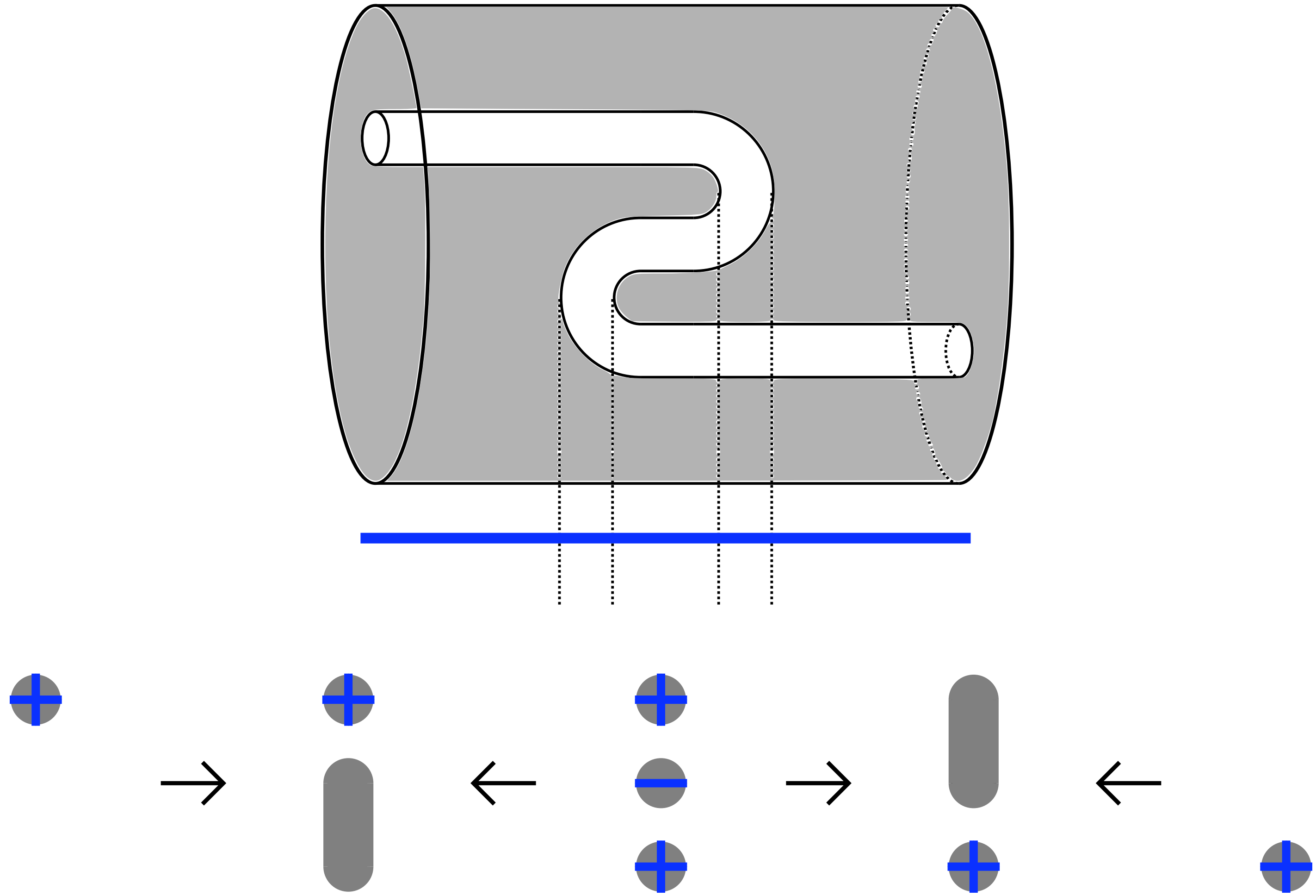

- Zigzag H^0 of X^c tracks linear combinations of components.
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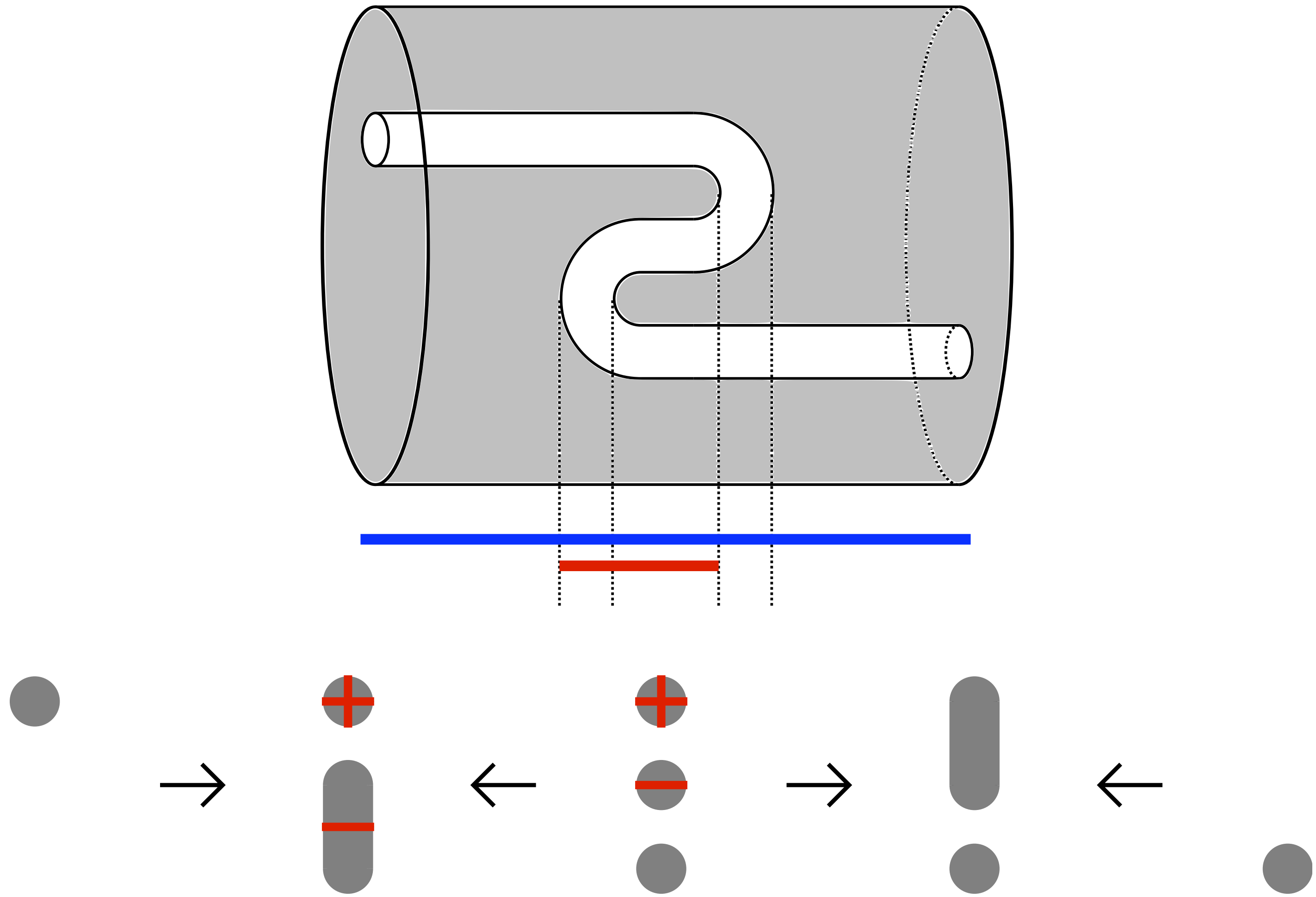

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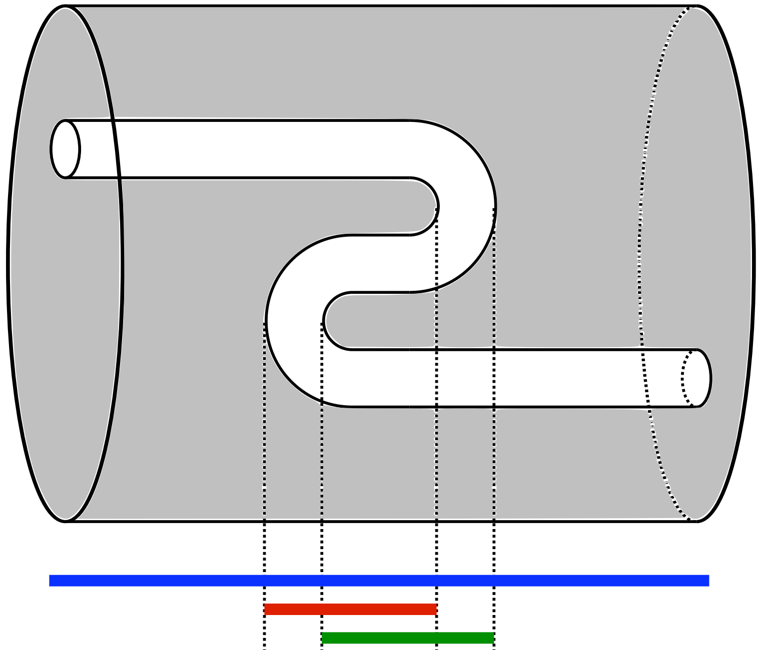

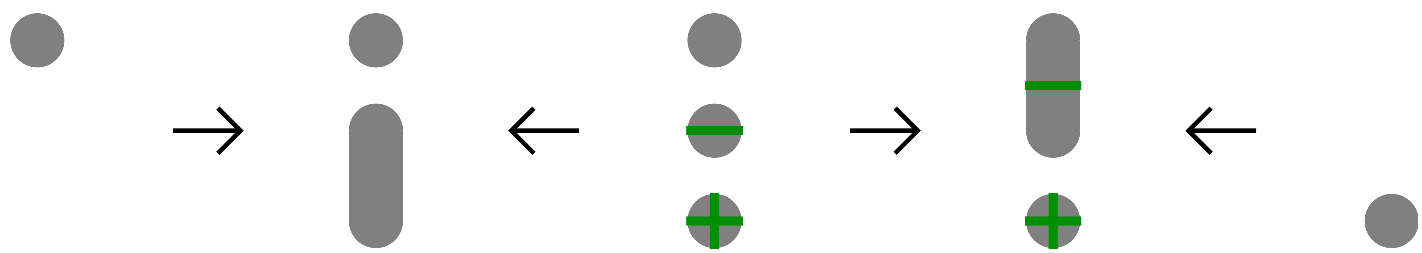

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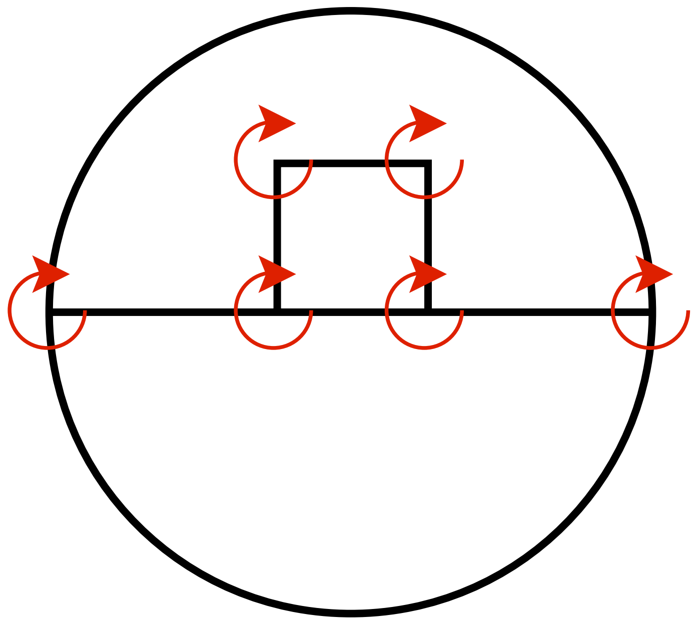
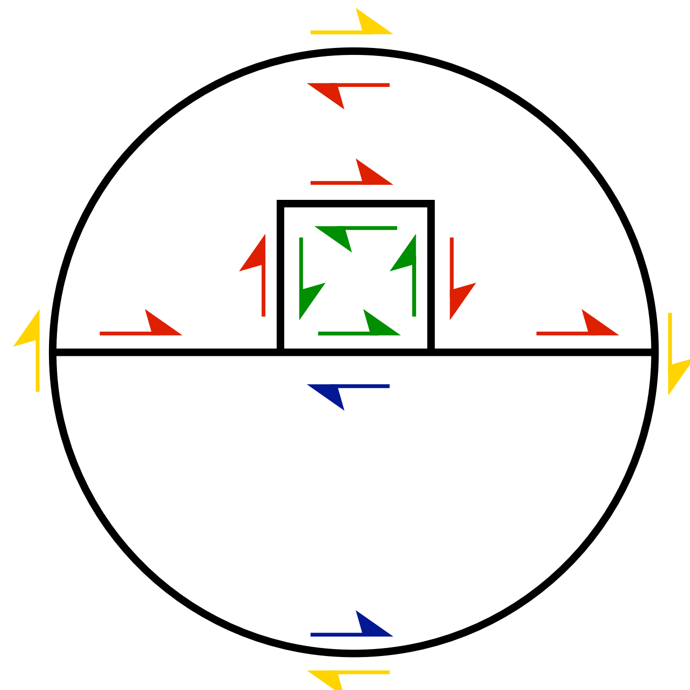
Fat graphs

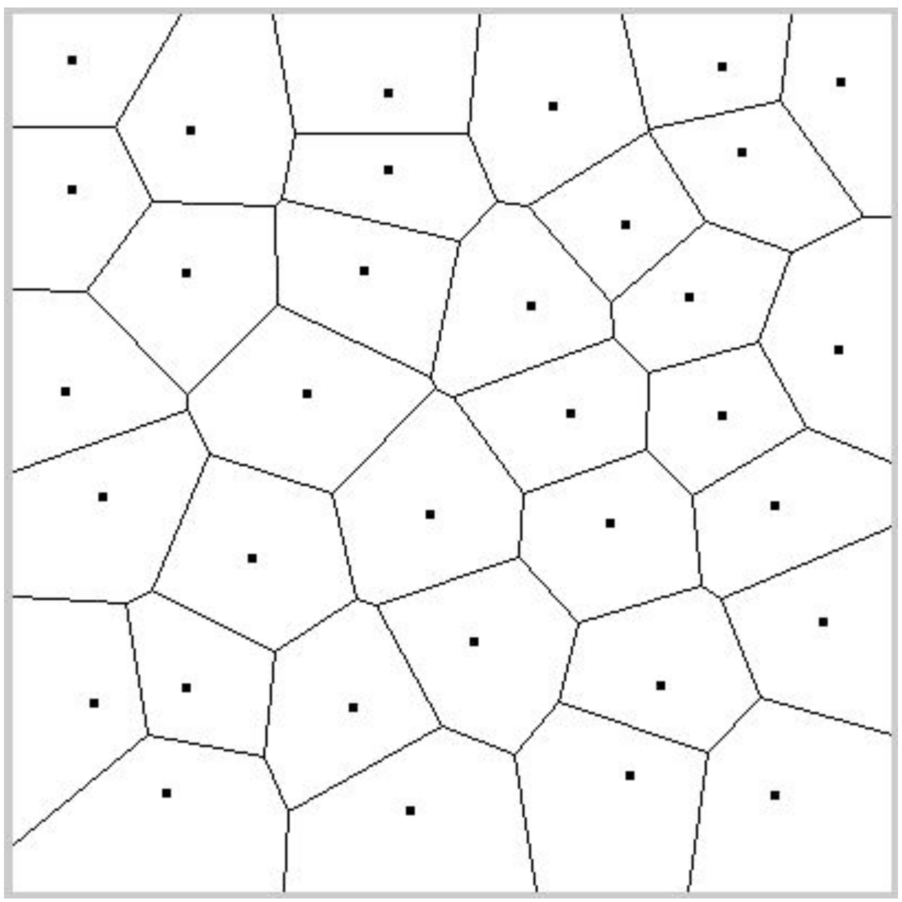
- What minimal sensing capabilities might we add?
- A fat graph structure specifies the cyclic ordering of edges adjacent to each vertex.
- Equivalent to a set of boundary cycles.
- Determines at most one embedding in S^2 up to isotopy.

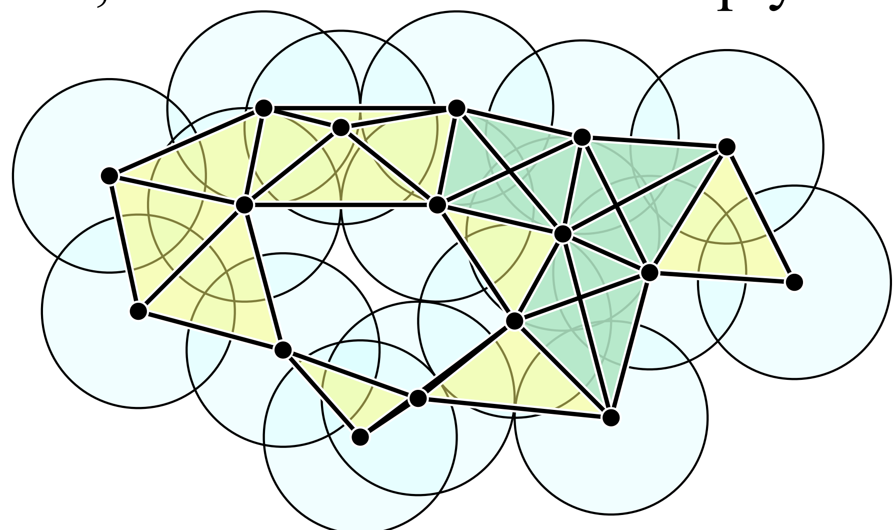


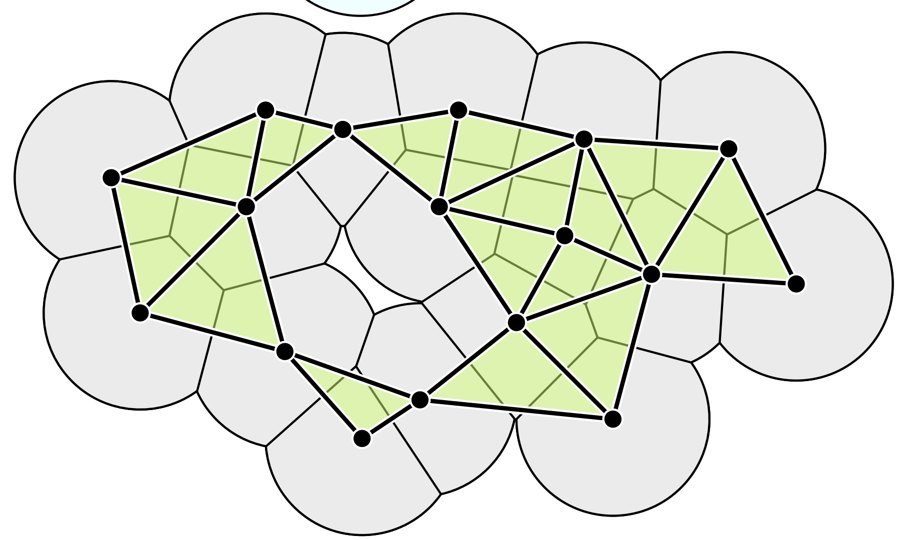


Fat graphs

- Suppose sensor network is connected at each time.
  If given the alpha complex (less coordinate-free than Čech) and fat graph structure at each time, one can determine sharply if an evasion path exists.







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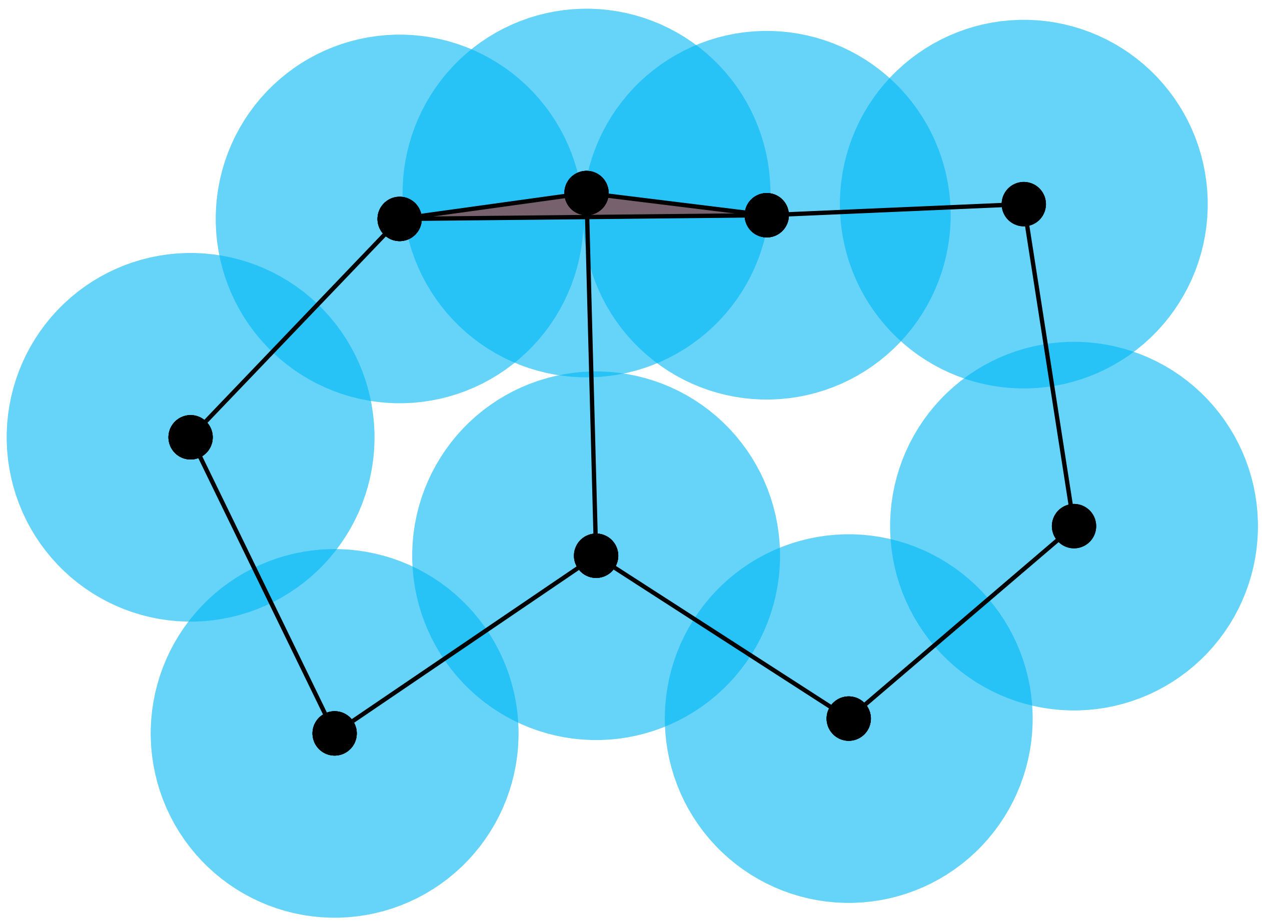

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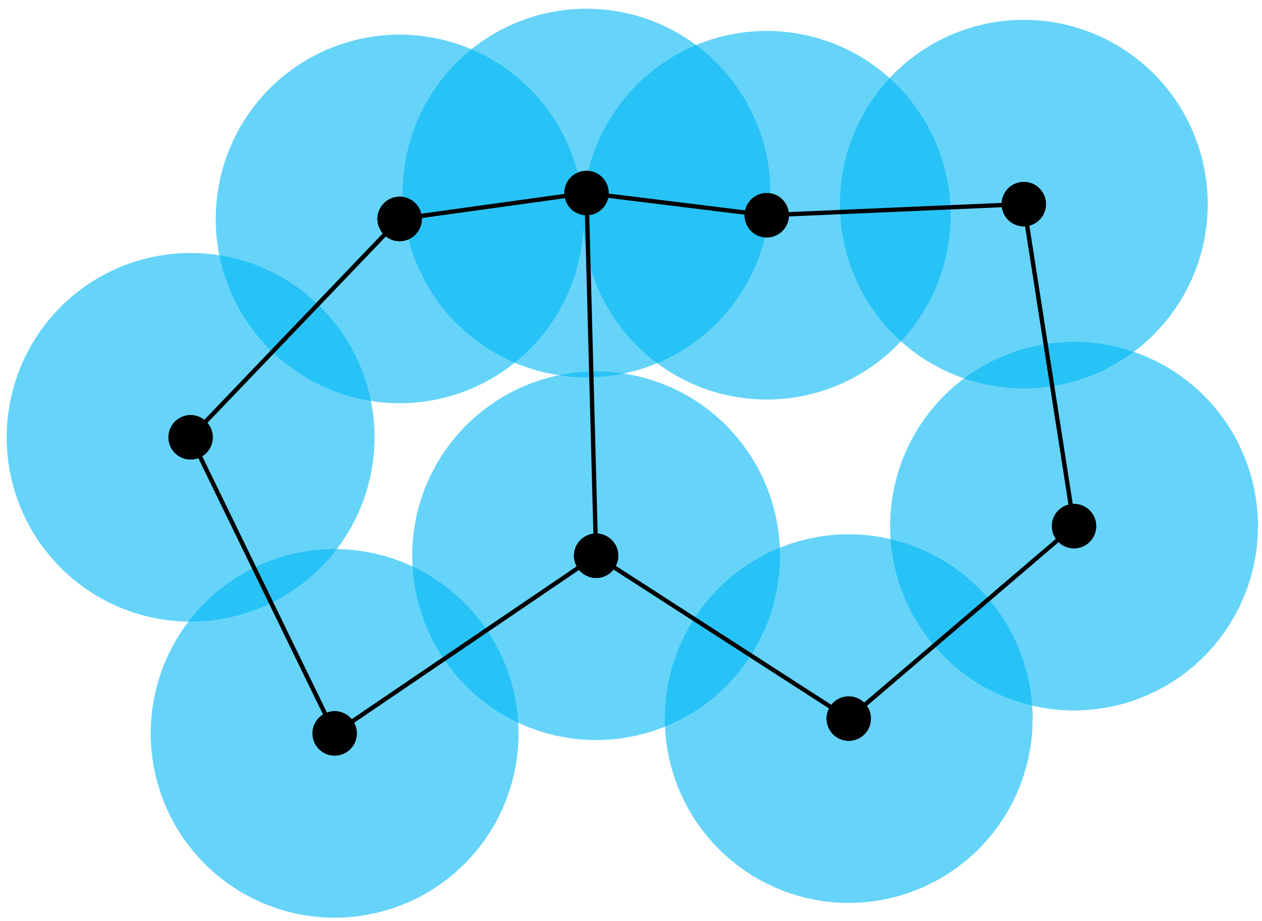

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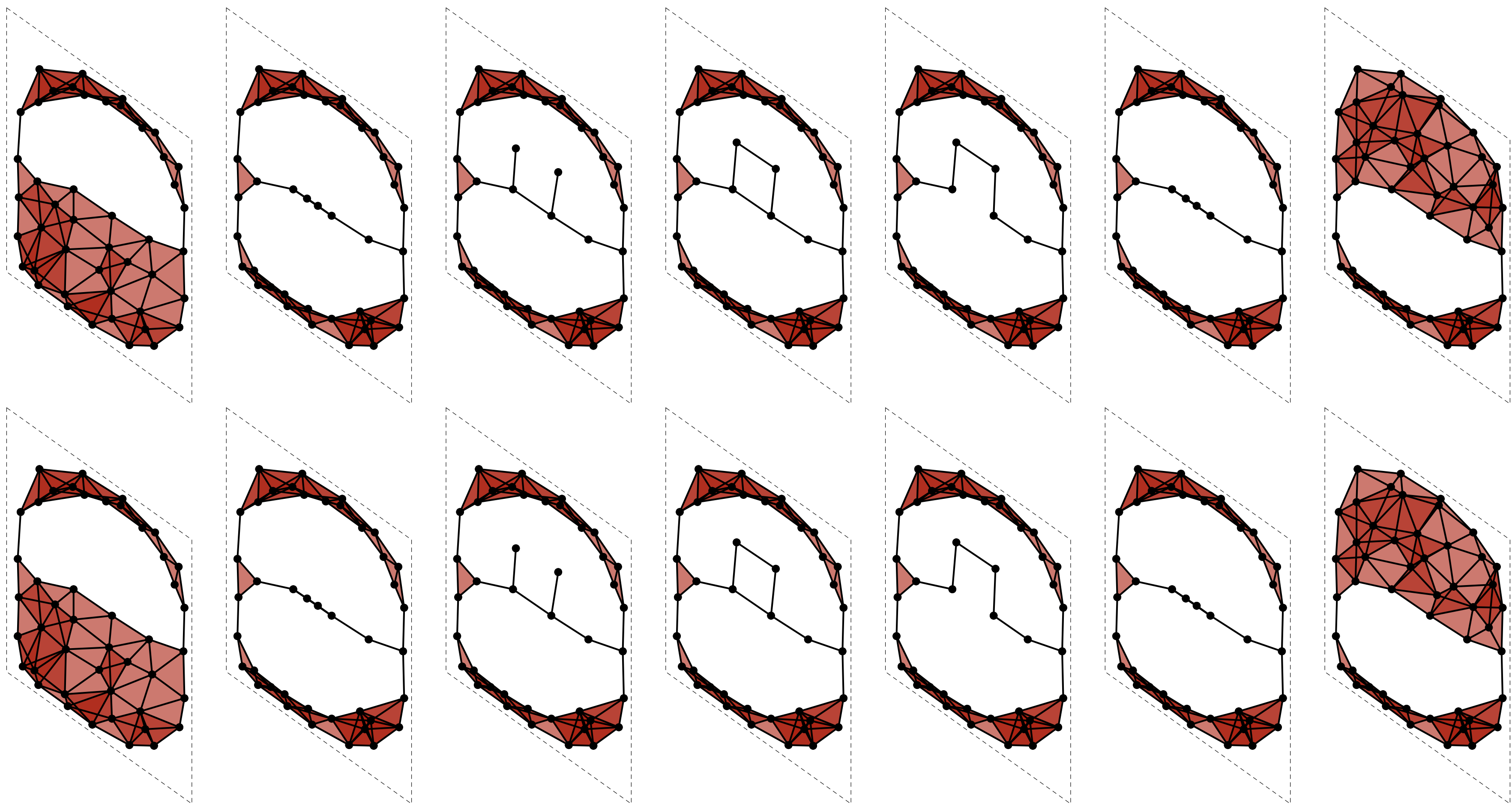

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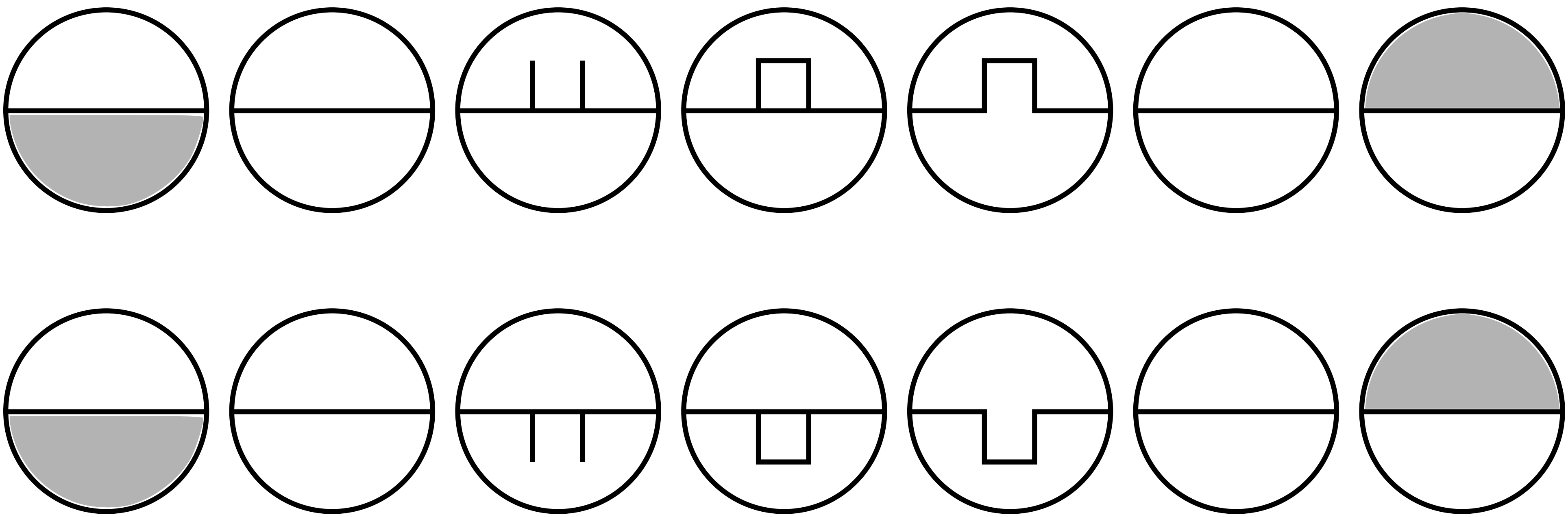

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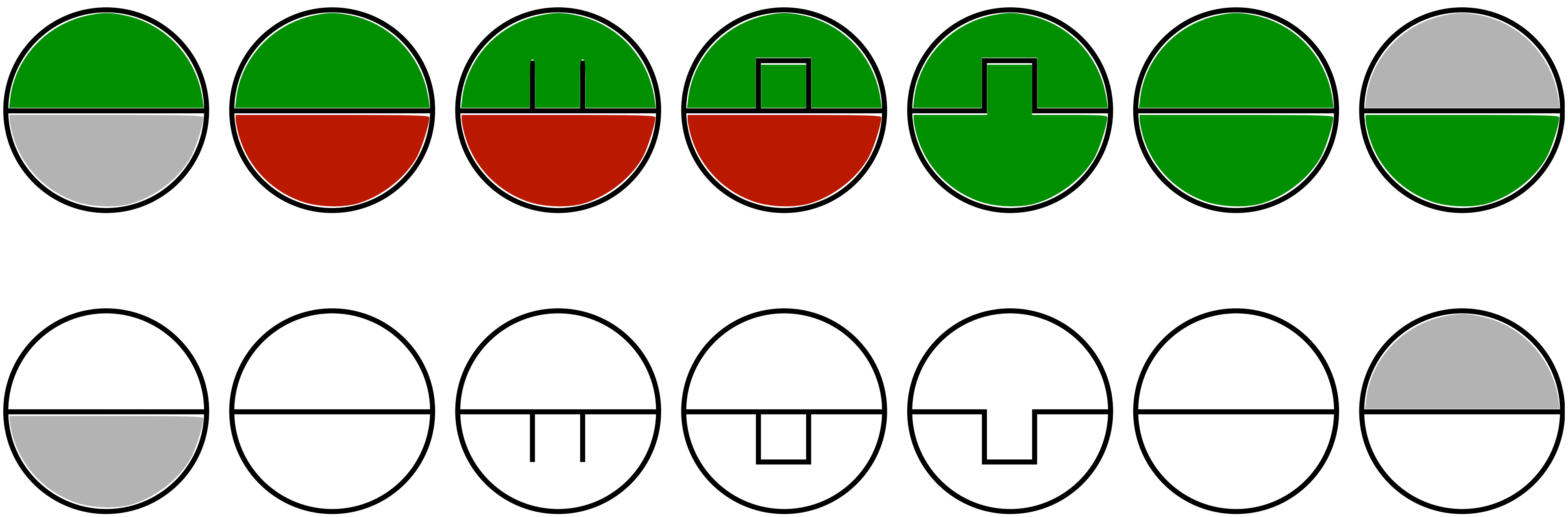

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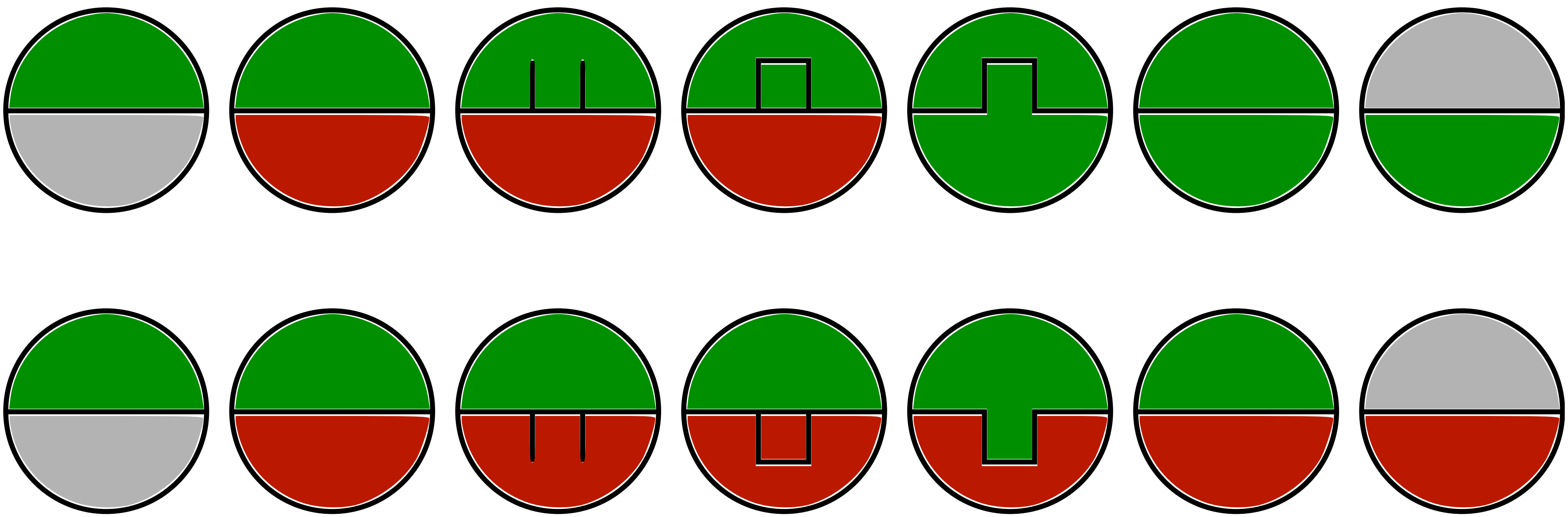

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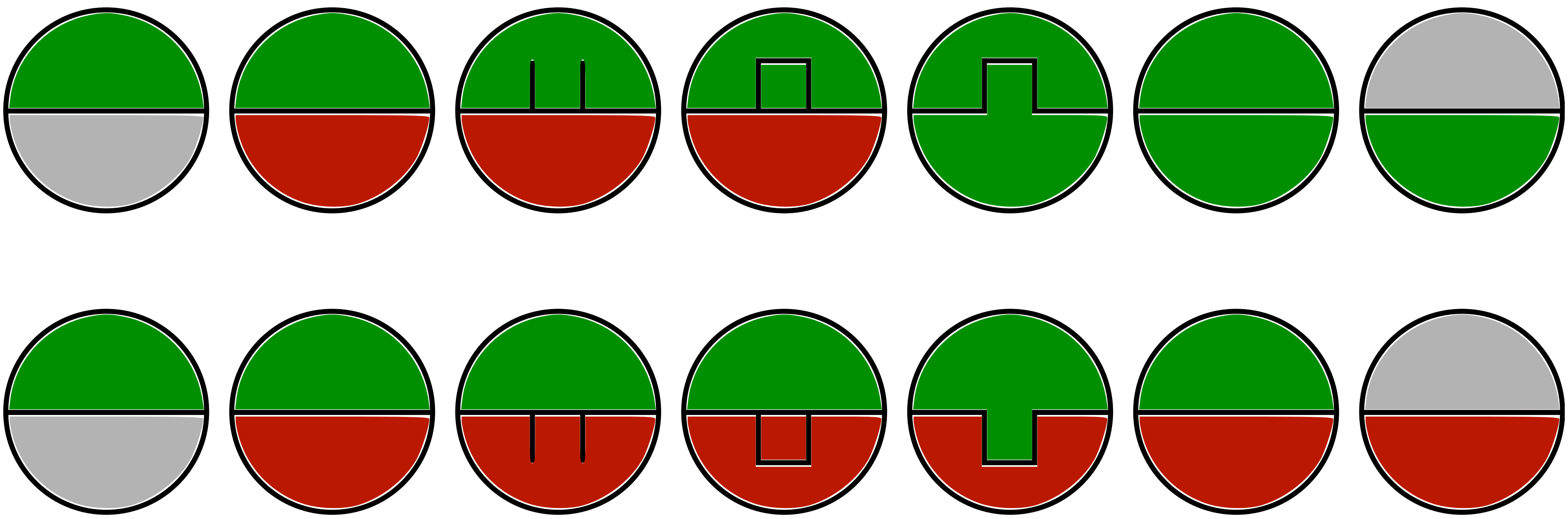

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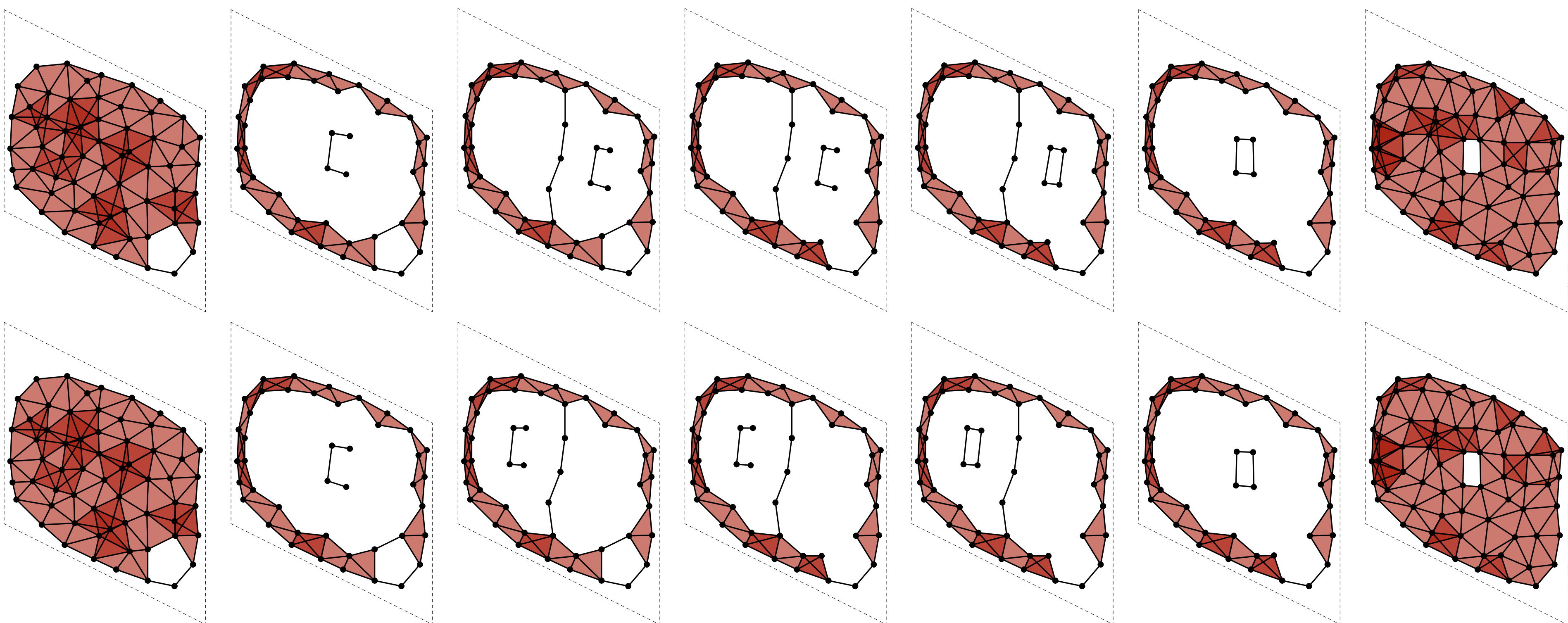

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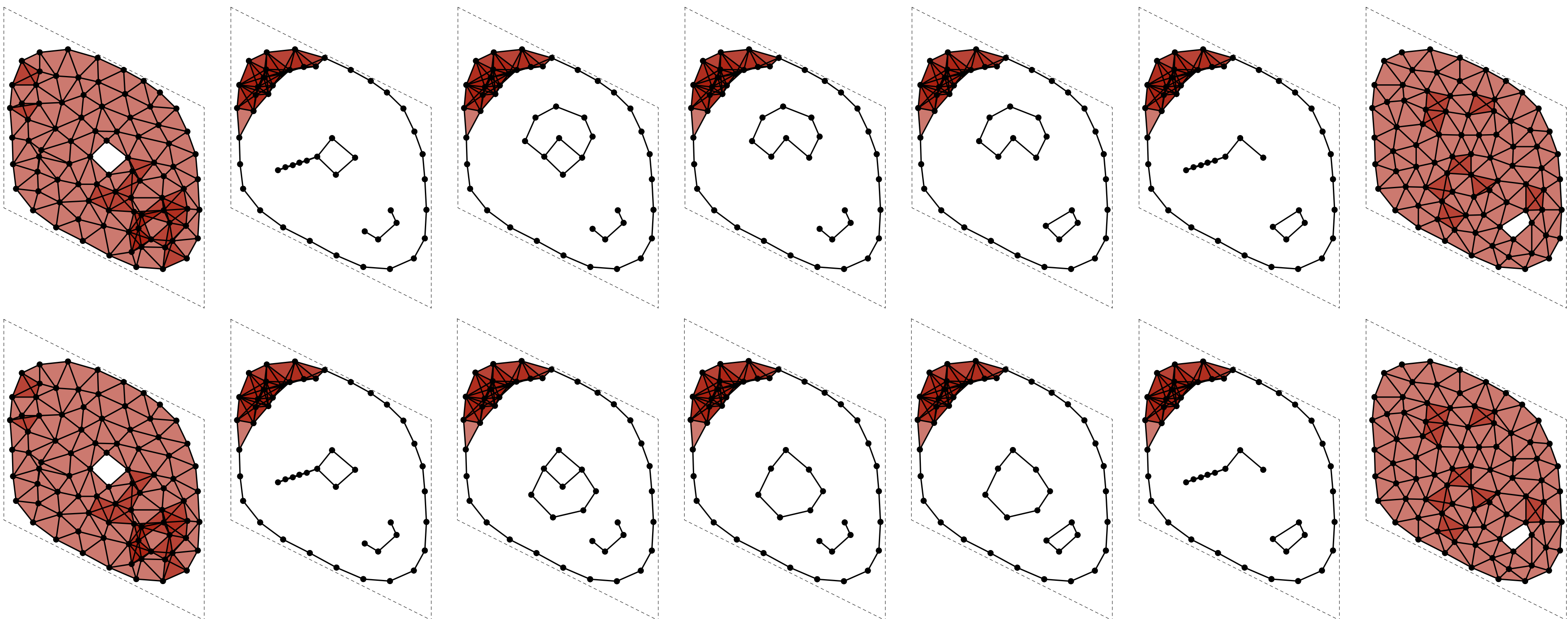

Homotopy theory?

- $\mathbb{R}^n \times I$?
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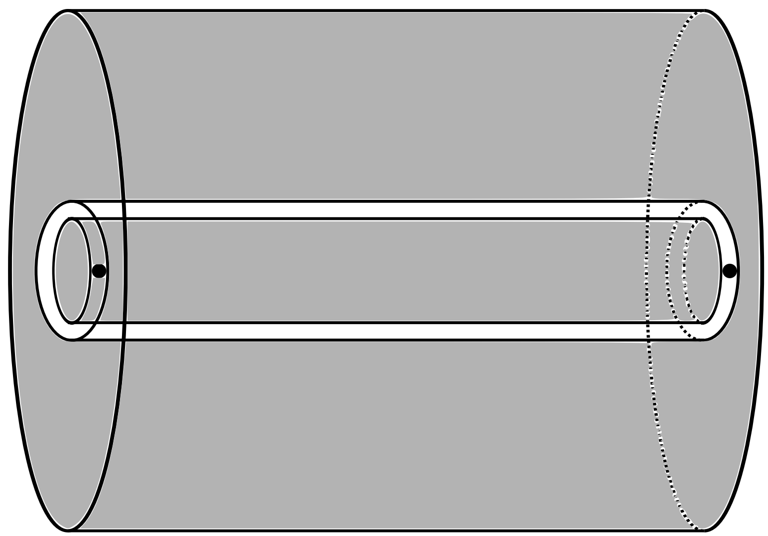

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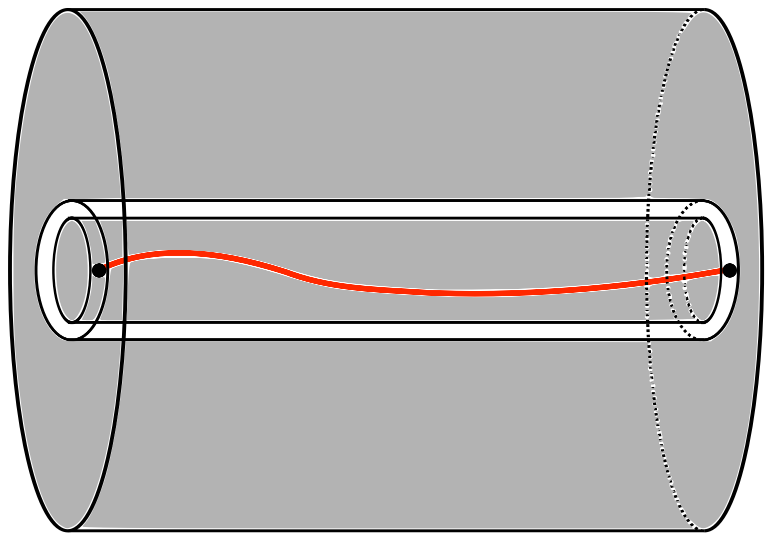

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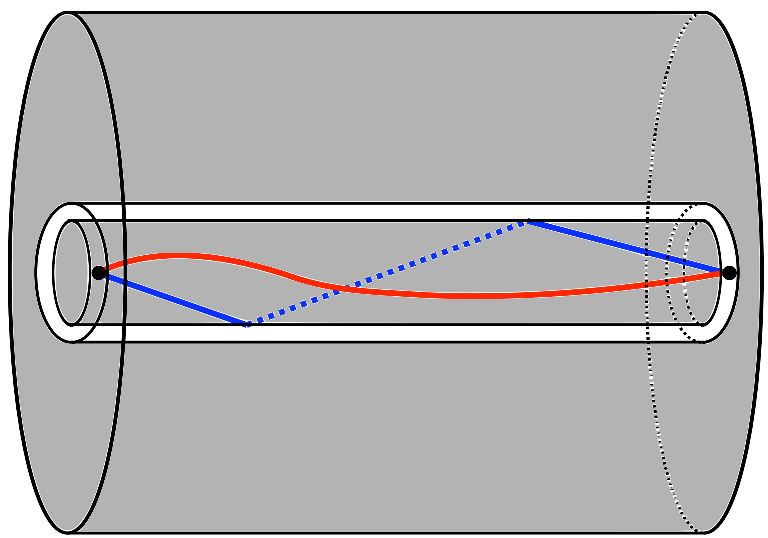

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- Stable and unstable Adams spectral sequence for fibrewise spaces or diagrams of spaces?

$$E_2^{s,t} \cong \mathrm{Ext}^{s,t}_{\mathcal{A}(p)}(H^*(Z), H^*(Y)) \Rightarrow \{Y, Z\}_p$$

Thank you