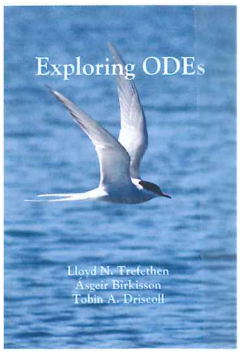
Exploring ODEs

Lloyd N. Trefethen, Ásgeir Birkisson, and Tobin A. Driscoll

SIAM 2018, 335 PAGES
PRICE (HARDBACK) £66.95 ISBN 978-1-61197-515-4

In writing this book, the authors have conceived a unique tome for students of applied mathematics, physics, and engineering sciences. It is not a textbook on analytical methods for solving various classes of ordinary differential equations (ODEs), nor is it about numerical algorithms. Instead, a more rounded approach is adopted by seamlessly weaving together analysis and computation to unpick a diverse range of phenomena within the general field of ODEs. The worked examples are almost always rooted in physical applications – e.g. the nonlinear pendulum, bending beams, chaos, turbulence, neural signals, chemical reactions – and so students may well have already encountered some of the problems considered here.



The computing aspects of the book focus exclusively on MATLAB in combination with the opensource package Chebfun (available for download at www.chebfun.org along with copious supporting documentation). This choice of software turns out to be educationally profitable for three principal reasons: (i) it eliminates the need for any serious programming (clearly advantageous for those students intimidated by the prospect of writing computer code), (ii) it allows the solutions to often analytically-intractable systems of ODEs to be found quickly and with the minimum of fuss (typically just a few lines of code with results being generated very rapidly), and (iii) those solutions may be visualised within MATLAB's figure environment. Appendix A details the Chebfun ODE algorithms and surveys their strengths and weaknesses, but the main text generally does not dwell on those minutiae.

The broad range of material covered in *Exploring ODEs* is impressive and distributed systematically across 22 chapters. With preliminaries such as the 'FLASHI' classification scheme safely dispatched, early chapters make their way steadily through first-order scalar linear and nonlinear ODEs before moving on to second-order ODEs and damped vibrations. Later chapters tackle linear boundary-value problems (BVPs) and their eigenvalues, variable coefficients and adjoints, and resonance. Thereafter, nonlinearity takes centre stage and it is in such regimes where the power and versatility of Chebfun really come into play. Key topics include phase-plane methods for systems of equations, the fundamental existence theorem (existence, uniqueness, Lipschitz continuity), random functions and random ODEs, chaos, linear systems and linearisation, stable and unstable fixed points, multiple solutions to nonlinear BVPs, bifurcations, and much more besides. The final chapter concludes by touching on partial differential equations, most notably the Korteweg–de Vries equation and its sech^2 solitons. Were all that not enough, Appendix B provides 100 additional examples of solutions to other ODEs as templates for further independent study.

Every chapter follows largely the same prescription: opening with a well-defined problem, re-capping essential analysis, and demonstrating how Chebfun may be deployed simply and effectively. Throughout, the authors place strong emphasis on understanding solution behaviour. There is sufficient mathematical detail in each chapter to introduce the topic at hand, without getting lost in a sea of theorem/proof formality that less confident students might find disengaging. The codes are almost trivial in early chapters, but not surprisingly they increase slightly in complexity as the subject matter is developed. Each chapter also finishes with set exercises that comprise a mixture of mathematical problem-solving and Chebfun-based calculations.

In reviewing their book, I was immediately struck by the care and attention that Trefethen, Birkisson, and Driscoll have devoted not just to the text (which is concise and very readable) but to the presentation (ranging from exactly which results to give in the steps of a derivation, to the preparation of around 400 colour figures). The authors' enthusiasm for their subject, and for the long-running Chebfun project in general, is evident from the very beginning. A further nice touch is that the PDF version of the book along with all the codes from each chapter is freely available online (see people.maths.ox.ac.uk/trefethen/ExplODE, and note that the .m files supplied also include formatting statements for the in-text figures).

Exploring ODEs is, in my view, an elegantly written, well thought-out, and beautifully-presented supplement to any of the standard textbooks covering ODEs. The instructive *analysis + computation* spirit allows the authors to probe a much greater breadth of material than might otherwise be possible for a book of 335 pages, and readers are left in no doubt as to the importance of this rich vein of applied mathematics.

James Christian CMath MIMA
University of Salford