An Applied Mathematician's Apology

Lloyd N. Trefethen

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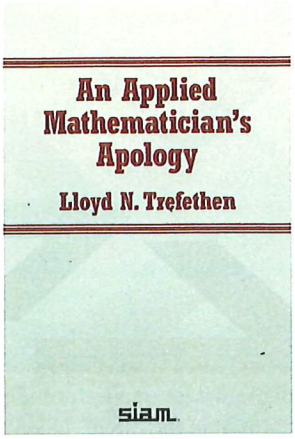
The title deliberately provokes comparison and contrast with G.H. Hardy's *A Mathematician's Apology*. His descriptions of the differences between mathematics with and without the 'applied' are the clearest I have seen. This theme, and accounts of his work as a numerical analyst, occur repeatedly throughout what Trefethen calls his 'meditation'.



His alternative title was *Confessions of a Numerical Analyst*. He contrasts numerical analysis with what he calls 'Fields Medal mathematics'. By any measure – prizes, citations, honorary degrees and a prestigious chair – Trefethen is a successful mathematician. But after listing all 60 Fields medallists, 'the gods of mathematics' (p. 7), he asks (p. 8):

> … how many works by these Fields medalists have I read? … exactly one.

He guesses that the same number of the sixty have read a paper by him.

To read this as sour grapes would be a mistake. Rather, the disconnect intrigues him. There are several entangled aspects. He admits having made discoveries (p. 66):

> … without … even attempting to master the results of the nonnumerical experts in the area.

He cites, initially without comment, Atiyah's remark that applied mathematics feeds off the crumbs dropped from the table of pure mathematics, but appreciates that the matter is more subtle: the crumbs 'travel … in both directions' (p. 67).

Such distinctions, born of a wish to classify, are ultimately uninteresting: Trefethen is a mathematician, full stop. I work on theoretical physics and have also contributed to mathematics, so people occasionally ask: Are you a physicist or a mathematician? I answer, 'yes'.

He is generous in recognising senior colleagues who influenced him and is especially warm in praising the late Gene Golub, who was a master of matrix computations. My single encounter with Golub nicely exemplifies Trefethen's cultural discordances. I told Golub that I was calculating nearly-degenerate eigenvalues of non-Hermitian matrices. He replied 'They're horrible', referring to the instability of common algorithms. He instantly understood my complementary perspective: 'No, they're beautiful' because physics concerns the universal behaviour of eigenvalues of such matrices as parameters vary.

Trefethen recounts beating his fellow student Bill Gates in a speed-typing competition. His mathematically competitive friend Nat Foote went on to earn vastly more money than him (p. 11):

> … although I note the gap, it doesn't disturb me too much.

One of my physicist friends put it differently:

> We professors could make much more in business or finance, but I regard myself as infinitely rich because I can afford to buy any book I want.

He describes his enthusiasm for numerical mathematics as a child, and in his high school and university years, arriving at his (p. 13, Trefethen's italics):

> … special vision of mathematics. We numerical people are the ones who see the show live. … We *make it happen.*

He worries that numerics isn't more widely recognised (p. 13):

> … as an indispensable way to explore mathematics.

Trefethen details how he develops algorithms for continuous, rather than discrete, mathematics, emphasising the power of extending real functions into the complex plane. He regrets that numerical analysts conventionally devoted their efforts to studying the errors in algorithms, rather than using them to deliver insight through numbers and pictures.

An example is Newman's theorem concerning approximations for non-smooth functions such as the modulus $|x|$: quotients of polynomials (rational functions) converge exponentially faster than polynomials. For forty years following this spectacular result, no numerical analysts (including Trefethen) applied it: they merely sharpened it.

Over many years he has been involved with, and contributed to, the software MATLAB. On his laptop, he can calculate all eigenvalues of a 1000×1000 matrix, using MATLAB's eig(A), in half a second. My laptop can do the same with Mathematica, illustrating that software developers learn from each other, so our allegiance (MATLAB? Mathematica? Maple? …) is largely determined by which system one first encounters – almost a matter of religion.

Trefethen states that MATLAB has changed his research life for the past 37 years. Mine has been changed for almost as long by Mathematica. But there is a difference: as a user, I don't know what goes on under the hood; I depend on the creativity of numerical analysts who do – people like Trefethen.

Trefethen writes with clarity, wit and generosity (even when seeming to complain), providing a unique perspective into numerical analysis and its place within mathematics.

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