SIAM Review **Vol. 57, Issue 3 (September 2015)**

Book Reviews

BOOK REVIEWS

The greatest challenge of this job is to find just the right reviewer for each book. Personal contacts get one only so far; I know lots of people in or near my own research area, but not so many in the larger applied mathematics community. So where do I look for reviewers? The Internet, of course! By searching the web I can find lots of names. But this approach has a drawback: the names I am most likely to find are those of very senior people, who are often too busy to write a review. Up-and-coming younger workers, who might have the time and might benefit from the exposure, are much less visible. I would like to find you.

Writing book reviews can be fun and rewarding. You spend some time with the book, get to know it well, then try to write something interesting and informative for the community. This is not a waste of time. You will surely learn something in the process. Moreover, if you write good reviews, they will be noticed.

If you (young *or* old) think that you would like to try your hand at writing reviews, please let me know (or contact one of the other members of the editorial board). Send me an email or introduce yourself at a meeting. Tell me (preferably in not too much detail) what interests you; perhaps I'll be able to find something for you.

In this issue we are pleased to offer you a timely review by Nick Trefethen of a large tome that is scheduled to appear this month, namely, *The Princeton Companion to Applied Mathematics*, edited by Nicholas J. Higham. This ambitious project is in the same spirit and style as *The Princeton Companion to Mathematics*, edited by Timothy Gowers, which appeared seven years ago. I hope you enjoy Nick's review.

In addition we have reviews of books on a wide variety of topics, including numerical linear algebra, risk and portfolio analysis, stochastic chemical kinetics, quantum mechanics, and computational complexity theory.

David S. Watkins
Section Editor
siam.book.review@gmail.com

SIAM Review
Vol. 57, No. 3, pp. 469–480

Book Reviews

Edited by David S. Watkins

Featured Review: The Princeton Companion to Applied Mathematics. *Edited by Nicholas J. Higham.* Princeton University Press, Princeton, NJ, 2015. xx+988 pp., hardcover. ISBN 978-0-6911-5039-0.

What is applied mathematics? How does it relate to pure mathematics, or should we simply say, to mathematics? With the appearance of the *Princeton Companion to Applied Mathematics*, we have two magnificent data points, 1000 pages each, to help us reflect on these questions.

The first thing one feels on looking at this volume is, quite simply, pleasure. Seven years ago *The Princeton Companion to Mathematics* was published to wide acclaim, and it was clear that a similar work on applied mathematics might be a good idea. Now it has appeared. The look and feel are the same, and this is highly satisfying. Figure 1 shows maps of the two volumes, which I will have more to say about in a moment. In each case the editors divided the collection into eight parts featuring pieces of differing lengths and flavors. The formatting and the typesetting are closely matched, and the new *Companion* is a perfect companion to the earlier one.

PCM was masterminded by Timothy Gowers of the University of Cambridge, and *PCAM* has been created by Nick Higham of the University of Manchester. These must be two of the most capable editors on earth. Anyone who knows Gowers and Higham will be aware of their combination of broad mathematical vision with phenomenal attention to detail. Higham, the Richardson Professor of Applied Mathematics at Manchester and a Fellow of the Royal Society, is celebrated not just for his research in numerical analysis but also for his outreach activities, including his *Handbook of Writing for the Mathematical Sciences*, the *MATLAB Guide* (coauthored with brother Des), and his blog. Like Gowers, he is a leader who cares deeply about his field. Princeton's appointment of him as editor was the perfect choice.

Of course, it takes a village. Like *PCM* before it, *PCAM* has a board of associate editors who helped shape the volume and contributed some of the articles: Mark Dennis, Paul Glendinning, Paul Martin, Fadil Santosa, and Jared Tanner. (I am pleased to note that five of the six editors are connected with England.) It has 165 authors, experts in their topics, many of them very eminent. A key person in the back office was Sam Clark of T&T Productions Ltd, who as project manager for both *PCM* and *PCAM* was involved in all the details and deserves much of the credit for making them such a comfortable pair.

So, what can you do with a book so big that its weight is measured in kilograms? With a million words of first-rate applied mathematics?

One possibility is to read it cover to cover. I more or less did that, but I doubt you will. (I did it as much to learn about myself as to learn about applied mathematics, which brings us to our first difference between mathematics and applied mathematics. I could not have read *PCM* cover to cover.)

Publishers are invited to send books for review to Book Reviews Editor, SIAM, 3600 *Market St.,* 6^{th} *Floor, Philadelphia, PA* 19104-2688.

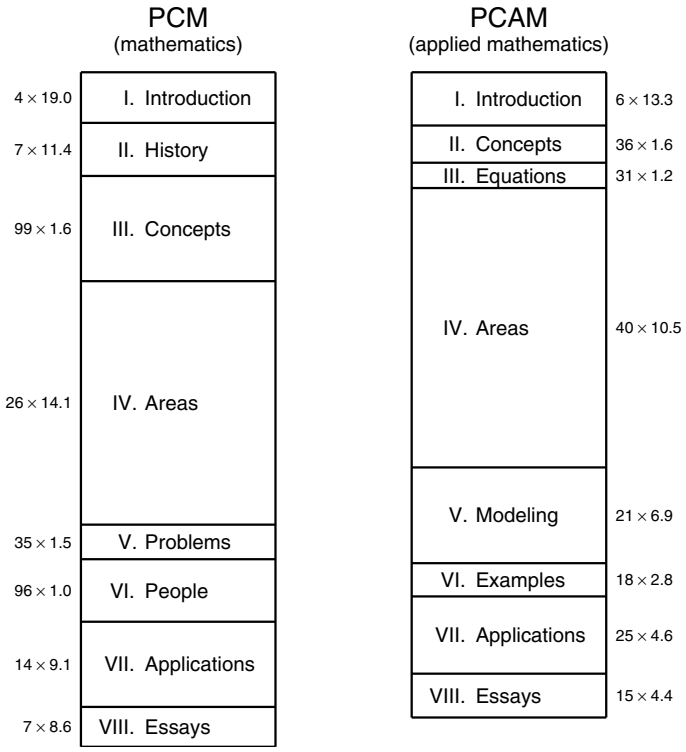

Fig. 1 *The eight parts of the two Princeton* Companions*, with shortened names. Annotations indicate numbers and average lengths of articles. For example,* PCAM *has* 18 *articles in Part VI, on Example Problems, and their average length is* 2.8 *pages.*

Another approach is to dip in and out at whim, as Higham et al. encourage.

> *Despite the careful organization, the editors expect that many readers will flick through the book to find something interesting, start reading, and by following cross-references navigate the book in an unpredictable fashion. This approach is perfectly reasonable.*

You bet it's reasonable! Your eye will be caught by Barbara Keyfitz on conservation laws, by Berry and Howls on divergent series, or by Jane Wang on insect flight. Maybe by Phil Holmes on dynamical systems, or Jack Dongarra on high-performance computing, or Andreas Griewank on automatic differentiation. The treasures go on and on.

I loved some of the accounts of things I hadn't known about. Ken Golden on the mathematics of sea ice, showing us how percolation theory applies to actual percolation—fascinating. Villani and Mouhot's article on kinetic theory—idiosyncratic and thoughtful, including a summary of 50 important papers in the field from 1912 to 2013. Donald Saari's beautifully simple explanation of why physicists believe the universe is full of dark matter. Doug Arnold's flight of a golf ball, a perfect example of how, by focusing on something small, we can see things that are big.

A third approach to this book is to use it for reference on smaller subjects or serious learning of bigger ones. I think the potential here is very great. All of us have areas we've touched upon but not immersed ourselves in properly, and some of these

pieces offer outstanding opportunities for taking that next step. For example, I was grabbed by David Tong's article on classical mechanics, whose clarity is illustrated by its opening lines.

> *Classical mechanics is an ambitious subject. Its purpose is to predict the future and reconstruct the past, to determine the history of every particle in the universe.*

There are dozens of truly deep and expert survey articles in *PCAM,* such as Brian Davies on spectral theory, Stephen Wright on continuous optimization, Hairer and Lubich on the numerical solution of ODEs, David Griffiths on quantum mechanics, and Emily Shuckburgh on the dynamics of the Earth's ocean and atmosphere. Graduate students and established researchers will be profitably reading these articles for many years.

But *eight parts*?? What's going on here?

Those of us of a certain age remember when the *Encyclopedia Britannica* raised eyebrows with its 15th edition in the 1970s. Instead of the traditional flat collection of articles, they brought out 28 volumes divided into the *Propedia,* the *Macropedia*, and the *Micropedia*. Was this controversial organizational principle a success?

If you take a look at the eight parts of *PCAM,* charted in Figure 1, the presence of some of them seems self-explanatory. Of course there is going to be an Introduction to Applied Mathematics, which here consists of six articles on foundational material. (Five are by Higham, with a generally numerical viewpoint, but this lack of diversity in the opening 55 pages is unrepresentative. Overall the book is very balanced, not at all dominated by numerics.) And following *PCM's* successful model, it is satisfying to find a final Part VIII of Final Perspectives. This assortment begins with opinion pieces by Gowers and Higham themselves and moves on to Ian Stewart, David Donoho and Victoria Stodden, David Bailey and Jonathan Borwein, Heather Mendick, David Acheson, Peter Turner, Gil Strang, Rachel Levy, Ya-xiang Yuan, Maria Esteban, Jim Crowley, and Alistair Fitt. One of my favorites is Stewart's zestful essay on "How to Write a General Interest Mathematics Book."

> *I have called mathematics the Cinderella science. It does all the hard work but never gets to go to the ball.*

PCAM's organizational confusion lies in the middle 800 pages, Parts II to VII, which are devoted to Concepts, Equations, Areas, Modeling, Examples, and Applications. Though one can only admire the impulse to classify, I am afraid it is very difficult to keep these six headings straight. Why is Benford's Law an Equation and the traveling salesman problem an Example? Why does modern optics belong to Modeling and control theory to Areas of Applied Mathematics? It all feels rather arbitrary.

Naturally you get curious and wonder, were the middle six parts of the earlier *PCM* equally hard to keep straight? On inspection it turns out that no, they were not, because three of them had specially memorable themes. Part II was on history, a category that *PCAM* does not repeat. Part VI was on mathematicians, 96 one-page mini-biographies from Pythagoras to Bourbaki, which *PCAM* also does not repeat. (Too many of the best applied mathematicians had already been covered as mathematicians, I suspect, though Bourbaki, I hasten to add, was certainly not applied, quite apart from the question of existence.) And Part VII of the original *Companion* was also special, being devoted to applications ("The Influence of Mathematics"). Perplexingly, this is a category that *PCAM does* repeat, giving us effectively Applications of Applied Mathematics. (One muses about GNU's Not Unix and turtles all the way down.)

So the organization of *PCAM* is unconvincing, and Princeton could have maintained the *PCM* look and feel with five parts instead of eight, but to tell the truth, it doesn't matter. No *Britannica* reader worried much about the *Propedia* back in the 1970s, and nobody's going to lose sleep over *PCAM's* structure today. The gold is in the individual pieces, not their organization.

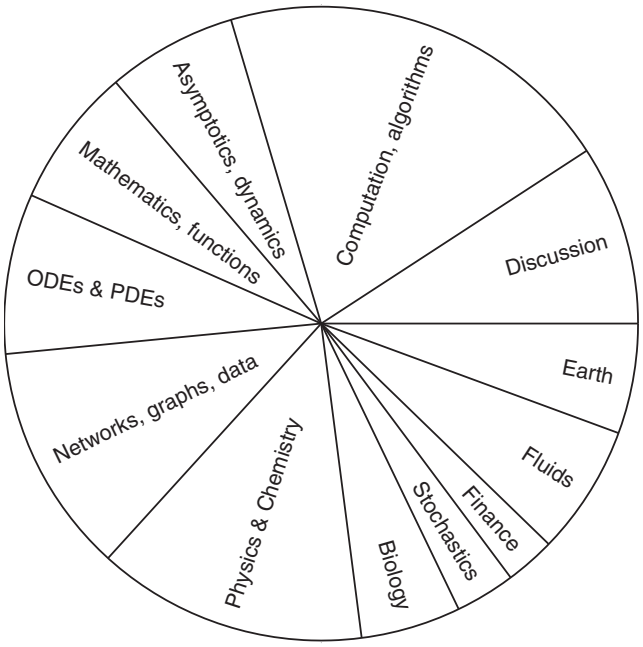

Fig. 2 *Rough breakdown of the pages of* PCAM.

Which brings us back to our opening question. What is applied mathematics? What view of the field does *PCAM* convey? To explore these matters it may be helpful to consider Figure 2, which outlines roughly where *PCAM* spends its pages.

One thing you notice is that there is a lot of physics and fluid mechanics here. These are the oldest, best established parts of applied mathematics, as important as ever, and their strength is powerfully displayed in articles on magnetohydrodynamics, quantum mechanics, optics, gravitation, and many other topics. Newer fields like finance, network theory, and biology are also well represented, although one can't help noting that whereas an article on a physics topic like kinetic theory or solid mechanics, say, will most likely be found in Areas of Applied Mathematics, a biological topic like physiology or biomechanics is more likely to appear in Modeling. It would seem that applied mathematics has terrain it has conquered and terrain it is still exploring. A century from now, will the boundaries have shifted?

It is striking that applied mathematics as displayed in *PCAM* does not define itself by its relationship to (pure?) mathematics, not at all. The book robustly stands on its own, and the proverbial Martian, if he stepped out of his flying saucer and read this volume of mathematics, would not suspect that earthlings knew any other kind.

PCAM shows us that applied mathematics is *vast* and it is *confident*. We see here a discipline engaged in every corner of the human enterprise, from cosmology to the

spread of infectious diseases, from pattern formation to aircraft design, from financial portfolio optimization to the ranking of movie preferences. As Strang writes, "Our subject is extremely large!"

LLOYD N. TREFETHEN
University of Oxford

Numerical Linear Algebra with Applications: Using MATLAB. *By William Ford.* Academic Press, San Diego, CA, 2015. $120.00. xxvi+628 pp., hardcover. ISBN 978-0-12-394435-1.

Numerical linear algebra is a course often offered to upper-level undergraduates or early graduate students from a variety of fields, including mathematics and computer science as well as engineering and physical science disciplines. Compared to the other available texts on this subject, this book is meant to be "an entry point" to encyclopedic treatments like Golub and Van Loan's [2] or Higham's [3] as well as to more advanced texts like Demmel's [1] or Trefethen and Bau's [4]. Ford's approach is more closely aligned with Trefethen and Bau's focus on mathematical foundations over Demmel's consideration of efficiency of algorithms and implementations.

An important distinction of this book is the inclusion of the first six chapters on (nonnumerical) linear algebra. One intended audience comprises engineers and scientists who do not have the mathematical background typically supplied by an undergraduate course in linear algebra; the first section of the book is aimed at bringing those readers up to speed before delving into numerical computations. While I appreciate the convenience of having useful background material in the same book (as opposed to referencing a separate linear algebra text), I would like the author to have done more to integrate numerical ideas into the first six chapters. Forward pointers and other references to (the exciting!) numerical ideas to come later could better motivate the material, instead of presenting the ideas more as a stand-alone primer.

The material is fairly comprehensive, spanning nearly the same set of topics as the advanced textbooks [1, 4], but the presentation is certainly gentler. Aside from the initial linear algebra refresher chapters, the book is organized like most others on the topic. It is roughly divided into the following six sections (each given more or less equal weight): introduction to (nonnumerical) linear algebra, introduction to numerical algorithms, solving linear systems (LU factorization and its variants), solving least squares problems (with QR decompositions and the SVD), computing eigenvalue decompositions, and iterative methods. Much more time and space is spent on presenting concrete examples than on in-depth topics; for example, Cholesky decomposition is presented with multiple examples including MATLAB input and output, but the reader is referred elsewhere for the proof of the backwards stability.

Perhaps the biggest gap is a lack of information about available software. Just as the mathematical foundations of linear algebra are well established, algorithms and techniques for achieving high performance in matrix computations are also mature. Software libraries for linear algebra have been evolving with computer architectures, and it's important for engineers and scientists to be aware of the general-purpose, efficient, and up-to-date software already available so they don't waste time reinventing the wheel (or more likely, failing to do so).

I think the ideal reader is a scientist or engineer who takes a class based on this book and then uses it as a reference later in his or her career. It is an accessible introduction to the fundamental matrix computations, particularly to students lacking some of the mathematical background. While a single course is not sufficient to cover the breadth of material, the reader can later revisit sections in order to brush up on the standard

techniques for solving the problems of interest and find pointers to more details and information about software in the literature.

REFERENCES

[1] J. Demmel, *Applied Numerical Linear Algebra*, SIAM, Philadelphia, 1997.

[2] G. H. Golub and C. F. Van Loan, *Matrix Computations*, Johns Hopkins Studies in the Mathematical Sciences, Johns Hopkins University Press, Baltimore, MD, 2012.

[3] N. J. Higham, *Accuracy and Stability of Numerical Algorithms*, 2nd ed., SIAM, Philadelphia, 2002.

[4] L. N. Trefethen and D. Bau, *Numerical Linear Algebra*, SIAM, Philadelphia, 1997.

Grey Ballard
Sandia National Laboratories

Risk and Portfolio Analysis: Principles and Methods. *By Henrik Hult, Filip Lindskog, Ola Hammarlid, and Carl Johan Rehn.* Springer, New York, 2012. $79.95. xiv+338 pp., hardcover. IBSN 978-1-4614-4102-1.

Investment and risk management problems are fundamental problems for financial institutions. A structured approach to these problems naturally leads one to the field of applied mathematics and statistics in order to translate subjective probability beliefs and attitudes toward risk and reward into actual decisions. These fields of applied mathematics and statistics form a natural basis for quantitatively analyzing the consequences of different investment and risk management decisions. Finance being largely a behavioral science, financial decisions strongly depend on subjective probabilities of the future values of financial instruments and investment choices. As such, financial decisions are often suboptimal, making it even difficult to specify a criterion for a desired trade-off between risk and potential reward in an investment situation. Applied mathematics and statistics can, however, assist in translating a probability distribution and an attitude toward risk and reward into a portfolio choice in a consistent way.

This book presents sound principles and useful methods for making investment and risk management decisions using standard principles, methods, and models. The authors combine useful practical insights with rigorous yet elementary mathematics. The material progresses systematically, and topics such as the pricing and hedging of derivative contracts, investment and hedging principles from portfolio theory, and risk measurement and multivariate models from risk management are covered appropriately.

The chapters have many real-world examples followed by several exercises to help reinforce the text and provide insight. The book is organized into two parts, as follows:

- Part I (principles) is composed of principles of portfolio analysis with a chapter on risk management.
- Part II (methods) covers risk measurement methods and multivariate models.

Chapter 1, on interest rates and financial derivatives, is very brief for a subject so broad. However, the principle of no-arbitrage for valuation of financial derivatives is well presented. Many of the investment and hedging problems that we encounter can be formulated as a minimization of a function over a set determined by the investor's risk and budget constraints and other restrictions on the type of positions that the investor can take. This and related issues are the focus of Chapters 2 to 5, which are on hedging and portfolio optimization.

Having taught risk management in a graduate program and with my experience as a practitioner, I found Chapter 6, on risk measurement principle, to be particularly well presented. The discussion on risk measurement properties is formal, yet accessible, and is followed by real-world examples. This chapter, like all of the chapters, ends with very practical examples that practitioners as well as academics will find very instructive. Chapters 7, 8, and 9 are on empirical models.

Considerations are also made on parametric family of distributions for a random variable and approaches to estimating the parameters. The book also discusses multivariate models for the joint distribution of

several risk factors such as returns or log returns for different assets, zero rate changes for different maturity times, changes in implied volatility, and losses due to defaults on risky loans.

The material of this book is based on university lecture notes; as such the organization and structure of the material presented will well serve advanced undergraduate and graduate students. This book will also be beneficial to practitioners in insurance and finance, as well as to regulators. Prerequisites include undergraduate-level courses in linear algebra, analysis, statistics, and probability.

Blessing Mudavanhu
University of Witwatersrand & African Banking Corporation

Stochastic Chemical Kinetics: Theory and (Mostly) Systems Biology Applications. *By Péter Érdi and Gábor Lente.* Springer, New York, 2014. $109.00. xvi+162 pp., hardcover. ISBN 978-1-4939-0386-3.

Compared to the relatively long history of calculus and differential equations going back to Newton and Leibniz, the modern mathematical theory of stochastic processes has a rather young age which can be traced back to A. N. Kolmogorov's foundational work on probability and stochastic processes in the 1930s [1, 2]. While stochastic-process models were widely used in physics following the works of Einstein, Smolochowski, and Langevin, they were not widely taught in chemistry, and even less so in biochemistry. Yet two fundamental papers already appeared in 1940: First, H. A. Kramers [3], the "K" in the WKB method, developed his theory of Brownian motion in a force field with double-well potential and obtained the celebrated "barrier crossing" rate formula that was named after him—using Laplace's asymptotic method for integrals. (This problem is treated briefly in section 3.2.2.2.) Kramers' theory showed how to understand, and compute, the rate constant for a discrete molecular reaction, as a rare event, based on a potential energy function of a collection of interacting atoms immersed in an aqueous "noisy" medium. This theory has become one of the most important items of condensed matter physics and chemistry [4]. In that same year, M. Delbrück, a quantum physicist turned Nobel Laureate in Physiology or Medicine (1969), published a paper [5] in which a continuous-time Markov jump process was used to represent the stochastic dynamics of a small chemical reaction system, assuming the rate constants for each and every reaction are known. This latter theory was later called the *chemical master equation*, and a review article was published in 1967 in the *Journal of Applied Probability* [6]. Together, these two papers paved the computational route from atomic physics to cellular biochemistry.

In 1970s, D. T. Gillespie published several papers, independent of the earlier work, on the stochastic Markov model of discrete chemical reactions, together with the algorithm that generates the exact sample trajectories [7]. This Delbrück–Gillespie approach to chemical reaction systems in a small volume became a major thrust of computational (sometimes called systems) cellular biochemistry in the late 1990s [8]. As a teacher in mathematical biology, I had been stitching together McQuarrie's review [6], a more recent review by Gillespie [9], and a brief Chapter 11 in my own book [10] into one part for my course readings on "Mathematical Theory of Cellular Dynamics." All this time, I have been wishing for a more coherent textbook.

The book under review, therefore, fills an urgent need and will receive warm welcome. I am glad to report that this little book is nicely done, and I will certainly use it next time when I again teach the subject—the only hesitation I have is its steep price of $109!

In many areas of science and mathematics, we in the U.S. have been guilty of ignoring concurrent or even preceding developments in other parts of the world. In this case, as told in the book, M. A. Leontovich had published a theory of chemical kinetics based on random processes as early as 1935 [11]. Furthermore, following an earlier work of A. Rényi in 1953, there is actually a "Hungarian school," centered around Eötvös Loránd University, which has been continuously researching the sub-

ject! The torch has been passed along to P. Arányi, J. Tóth, and the two authors of this book, Professors Érdi and Lente. They have brought a wealth of insight and a very unique perspective to their writing. The product is a pleasure to read. (There was also an earlier book by Érdi and Tóth published in 1989 [12].)

The book has three chapters plus a closing "Retrospect and Prospect." Chapter 1, "Stochastic Kinetics: Why and How," in a fairly accessible and relaxed fashion in 20 pages, breezes through the essential notions of deterministic chemical kinetics, Brownian motion, Markov jump processes in terms of master equations, and some very interesting historical remarks. I don't know how a biochemist will react, but anyone with a undergraduate education in physical sciences will get the big picture pretty well.

Chapter 2 is where one learns the mathematics. Some familiarity with probability and differential equations are certainly required to really understand the material. The main approach is what is known as the multi-dimensional birth-and-death process. One of the most important intellectual insights one gains in this approach to chemical kinetics is that deterministic, nonlinear dynamic behavior is an emergent property of an inherently stochastic process.

More specifically, the stochastic mathematical theory lays the foundation for the system of nonlinear differential equations (1b) describing chemical kinetics of m species involved in n reactions (1a) based on the law of mass action:

$$\text{(1a)}\qquad \nu_1^j X_1 + \nu_2^j X_2 + \cdots + \nu_j^m X_m \underset{k^{j-}}{\overset{k^{j+}}{\rightleftharpoons}} \mu_1^j X_i + \mu_2^j X_2 + \cdots + \mu_m^j X_m,$$

$$\text{(1b)}\qquad \frac{\mathrm{d}x_i}{\mathrm{d}t} = \sum_{j=1}^{n} \left(\kappa_i^j - \nu_i^j\right) \cdot \left(k^{j+} \prod_{k=1}^{m} x_k^{\nu_k^j} - k^{j-} \prod_{k=1}^{m} x_k^{\mu_k^j}\right)$$

$(j = 1, 2, \ldots, n)$, in which x_i is the concentration of chemical species X_i; ν_i^j and μ_i^j are called stoichiometric coefficients; and k^{j+} and k^{j-} are the rate constants for the jth reaction in the forward and backward directions. Corresponding to $\mathbf{x}(t) \in \mathbb{R}^m$, the stochastic dynamics is a Markov jump process $\mathbf{n}(t) \in \mathbb{Z}^m$, where $n_i = Vx_i$ represents the number of X_i in a volume V. A fundamental difference in studying deterministic dynamical systems like (1b) and such a stochastic process is the possibility of dual perspective: One can either study the stochastic sample trajectories $\mathbf{n}(t)$ or its probability distribution $p(\boldsymbol{\ell}, t) = \Pr\{\mathbf{n}(t) = \boldsymbol{\ell}\}$. The latter satisfies a linear differential equation.

Section 2.3.3 contains a nice discussion on the relationship between the deterministic system in (1b) and the stochastic process. Here T. G. Kurtz's "fundamental theorem of stochastic chemical kinetics" is mentioned. It would be nice to make it clear that the linear differential equation for the probability distribution is in a *function space*, which is usually infinite, while the nonlinear dynamics lives in a much lower dimensional space. In other words, as an analogy, $\dot{x} = f(x)$ in $\mathbb{R}^n$ always has a corresponding linear partial differential equation (Liouville) in phase space $\partial_t u = -\nabla_x\big(f(x)u\big)$.

Section 2.3.4 presents new results on stochastic mapping, developed very recently by one of the authors, which identifies in parameter space regions where a stochastic model is required.

Chapter 3 provides an assorted applications of the stochastic chemical kinetic approach. As clearly indicated in the subtitle of the book, they are mostly in systems cellular biochemistry, ranging from applications to membrane noise analysis and fluorescence fluctuation spectroscopy to olfactory neurobiology. The nonlinear bifurcation problem has added new features in stochastic dynamics; population extinction can occur as a rare event, while a differential equation shows $x = 0$ to be an unstable fixed point. Thinking dynamics with multiple time scales are essential. Enzyme kinetics, cellular signaling processes, stochastic gene expression, and chiral symmetry breaking are extensively covered. There is also a section on parameter estimation in stochastic kinetics models, as well as a very brief discussion of stochastic resonance (SR) in chemical systems. It is a pity the authors did not adopt the viewpoint that SR is a consequence of the breakdown of detailed

balance—that would have provided a fresh and accessible introduction to the very popular but rather technical subject.

The last section of the chapter contains the novel idea of using stochastic kinetics in the theory of computation. One can sense the influence of J. J. Hopfield, who turned a "memory problem" into *designing* a differential equation with a large number of attractors with prescribed locations.

Not to diminish the virtue of the book, but there are several places where improvements or corrections can be made. For example, the exponential part of Kramers' formula given in equation (3.25) followed the statement "By evaluating the integral. . . ." This could be made more clear by actually showing the integration process. On p. 94, the mean time of the Michaelis–Menten single enzyme turnover could benefit from a parallel mean first-passage-time computation, which might be more illuminating. Sometime jargon slips in without a clear definition first: "infinitesimal generator" on p. 15; "CDS" and "CCS" on p. 18—though they are later defined on p. 27.

The book could benefit from an online erratum, which certainly will come handy in the second print: On p. 16, anomalous diffusion need not to be non-Gaussian, as the theory of fractional Gaussian processes testifies. Equation (1.30) should read $\mathrm{d}x = \sqrt{2D}\mathrm{d}W(t)$; and the time scale for extinction, on p. 74, must be proportional to e^{cV}, where $c > 0$. Extraneous symbols and characters appear in various places, which is more likely a reflection of the copyediting: An ${}^{n}p_{ij}^{m}$ in equation (2.8); $\mathrm{Var}[\xi(t)] = 2\lambda t$ on p. 73; reference [45] on p. 141 should not be "der Chen Y"; a "618" in reference [108] on p. 143 and again in reference [20] on p. 157.

In summary, this is the first book in a new, highly exciting, growing area of applied mathematics, with applications to modern cell biology. The level is very accessible to a wide range of readers from the physical sciences to biology and the life sciences. The subject area is very important. The book is lucidly written and has an extensive, scholarly researched bibliography which will certainly be useful to anyone who wants to go deeper into the subject.

REFERENCES

[1] A. N. Kolmogorov, *Über die analytischen Methoden in der Wahrscheinlichkeitsrechnung*, Math. Ann., 104 (1931), pp. 415–458.

[2] A. N. Kolmogorov, *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Springer-Verlag, Berlin, 1933; English transl., *Foundations of the Theory of Probability*, Chelsea, New York, 1950.

[3] H. A. Kramers, *Brownian motion in a field of force and the diffusion model of chemical reactions*, Physica, 7 (1940), pp. 284–304.

[4] P. Hänggi, P. Talkner, and M. Borkovec, *Reaction-rate theory: Fifty years after Kramers*, Rev. Modern Phys., 62 (1990), pp. 251–341.

[5] M. Delbrück, *Statistical fluctuations in autocatalytic reactions*, J. Chem. Phys., 8 (1940), pp. 120–124

[6] D. A. McQuarrie, *Stochastic approach to chemical kinetics*, J. Appl. Probability, 4 (1967), pp. 413–478.

[7] D. T. Gillespie, *Exact stochastic simulation of coupled chemical reactions*, J. Phys. Chem., 81 (1977), pp. 2340–2361.

[8] Y. Cao, H. Li, and L. R. Petzold, *Efficient formulation of the stochastic simulation algorithm for chemically reacting systems*, J. Chem. Phys., 121 (2004), pp. 4059–4067.

[9] D. T. Gillespie, *Stochastic simulation of chemical kinetics*, Annu. Rev. Phys. Chem., 58 (2007), pp. 35–55.

[10] D. A. Beard and H. Qian, *Chemical Biophysics: Quantitative Analysis of Cellular Systems*, Cambridge University Press, Cambridge, UK, 2008.

[11] M. A. Leontovich, *Basic equations of kinetic gas theory from the viewpoint of the theory of random processes*, J. Exp. Theoret. Phys., 5 (1935), pp. 211–231.

[12] P. Érdi and J. Tóth, *Mathematical Models of Chemical Reactions. Theory and Applications of Deterministic and Stochastic Models*, Manchester University Press, Manchester, UK, 1989.

Hong Qian
University of Washington

Quantum Theory for Mathematicians. *By Brian Hall.* Springer, New York, 2013. $89.95. xvi+554 pp., hardcover. ISBN 978-1-4614-7115-8.

This book is an introduction to quantum mechanics intended for mathematicians and mathematics students who do not have a particularly strong background in physics. I suspect there are many in this category. I myself was told as an undergraduate that I would not need any physics for what I was going to be doing. I've been playing catch up ever since. This book, which consists of 23 chapters and an appendix (in about 550 pages), will surely prove useful for this audience and others as well.

The book is, of course, written in mathematical language and aims for a level of precision that one would expect in a graduate textbook. In order to make the book accessible to as large an audience as possible, the author has striven to keep the prerequisites to a minimum. The reader is assumed only to have had at least a first course in real analysis, including some Hilbert space theory. The spectral theory of self-adjoint operators is not assumed; it is covered in the text.

But this is not just a mathematics book. The physical consequences of the mathematical results are discussed at every opportunity. The first chapter discusses briefly the physical experiments that forced scientists to look for a new theory. Chapter 2 is an introduction to classical mechanics in the simplest possible terms, starting with Newton's second law for a single particle in $\mathbb{R}^1$. It then moves on to multiple particles in $\mathbb{R}^n$, then to Hamiltonian mechanics, Poisson brackets, and conservation laws.

The study of quantum theory proper begins with Chapter 3. Wave functions and their probabilistic interpretation are introduced, followed by position and momentum operators. A list of axioms is drawn up, including the Schrödinger equation of motion, motivated by the famous $E = \hbar\omega$. The first example is a particle in a box. Chapter 4 discusses the free Schrödinger equation, solution by Fourier transform, and propagation of wave packets. Chapter 5 analyzes a particle in a square well and tunneling.

This is as far as the author cares to go without the spectral theorem. Chapters 6 through 10 constitute a 100-page chunk of the book in which the spectral theorem for unbounded self-adjoint operators on Hilbert space is formulated and proved. The presentation is quite detailed; the author advises the reader to read as much or as little as (s)he needs. The discussion begins with the finite-dimensional result: every Hermitian matrix is unitarily similar to a diagonal matrix. The difficulties associated with the infinite-dimensional case are discussed, as are the goals of spectral theory. A chapter is devoted to the formulation of the spectral theorem for bounded Hermitian operators on a Hilbert space, and the following chapter provides the proof. Then unbounded operators are discussed, and the spectral theorem for unbounded self-adjoining operators is formulated and proved.

With the spectral theorem in hand, the next chapters cover the quantum harmonic oscillator in one dimension, the Heisenberg uncertainty principle, quantization schemes, the Stone–von Neumann theorem, and the WKB approximation. The Stone–von Neumann theorem says, roughly speaking, that if you have two (or $2n$) operators that interact like position and momentum operators, then they *are* position and momentum operators.

Chapter 16 is a brief presentation of the representation theory of Lie groups and Lie algebras. This is preparation for the discussion of angular momentum and spin in Chapter 17. In fact the quantum mechanical theory of angular momentum is the same as the representation theory of the Lie algebra $su(2) \simeq so(3)$. Spherical harmonics are generated, and these are used in Chapter 18 to solve the Schrödinger equation for the hydrogen atom.

The last few chapters of the book are a bit sketchy, and the mathematical prerequisites are higher. The objective is to give the reader a glimpse of each subject. Chapter 19 discusses multiple particle systems and subsystems and introduces the density matrix. Chapter 20 presents the path integral formulation of quantum mechanics. Chapter 21 introduces (classical) Hamiltonian mechanics on manifolds in preparation

for the final two chapters, which are on geometric quantization on Euclidean space and on manifolds, respectively.

A well-qualified graduate student can learn a lot from this book. I found it to be clear and well organized, and I personally enjoyed reading it very much. I already knew a fair bit of this stuff beforehand, but I was surprised at how many details were filled in for me. One warning: the book contains a fair number of typographical errors, so be on your guard.

DAVID S. WATKINS
Washington State University

Boolean Function Complexity: Advances and Frontiers. *By Stasys Jukna.* Springer, New York, 2012. $84.95. xvi+617 pp., hardcover. ISBN 978-3-642-24507-7.

The two main branches of theoretical computer science are algorithms and computational complexity theory. Both of these branches serve the same purpose of understanding the true limits of efficient computation. While algorithm researchers are devising faster ways to solve various computational tasks such as linear programming, matrix multiplication, and sorting, complexity theorists are working in the opposite direction and focusing on *lower bounds*, i.e., establishing that the same computational tasks inherently require a certain nontrivial amount of computational resources such as time, memory, and number of processing units. Apart from the noble challenge of uncovering important scientific truths, the work of complexity theorists has a strong motivation in modern cryptography that is largely based on computational assumptions like hardness of integer factorization, hardness of finding short vectors in lattices, etc. Perhaps surprisingly it turns out that, unlike progress in algorithm, design, progress in computational complexity has been extremely slow. Existing lower bounds are either very weak quantitatively or apply only to very restricted model of computation that do not come close to capturing the power of real computing devices.

Akin to combinatorics and number theory, most problems in computational complexity theory are easy to state and can often be explained even to an eager high school student. Most existing results require a clever insight rather than fancy mathematical tools. The tools used tend to come from algebra, combinatorics, probability theory, or analysis. Complexity theory originated in the 1950s and, despite its relatively young age, has already gained considerable prominence among other mathematical disciplines. One way to recognize its significance is to note that the central question of computational complexity theory, namely, the $\mathbb{P}$ vs. $\mathbb{NP}$ question, is now often considered one of the few most important open problems in mathematics together with the Riemann hypothesis.

Boolean Function Complexity reviews that state of the art in complexity theory focusing on concrete lower bounds. The publication is timely. The only recent book that has extensive coverage of lower bounds is [1, Part 2]. However, there the coverage is far more limited. By contrast, while the current book also does not cover some areas of lower bound work, e.g., models of quantum or algebraic computation, it does cover all major results and models dealing with classical circuit models and does so in a truly encyclopedic manner. To demonstrate the breadth of coverage of material let me mention that the book includes a result of Razborov from late 1980s on application of rigid matrices to separating classes in the communication complexity hierarchy (a result that was circulated as a manuscript and never published [3]). The book also covers some very recent developments such as the breakthrough by Williams [5] separating $\mathbb{NEXP}$ from $\mathbb{ACC}$.

The book is arranged into six parts. The first part is introductory. The second part deals with communication complexity, which is an information theoretic measure of complexity of functions that is useful to prove circuit lower bounds. The presentation here extends the treatment in [2] and covers all the main highlights, e.g., the log-rank conjecture, the number-in-hand and number-on-forehead models, and their applications. The third and fourth parts constitute the core of the book. Here the main lower bound gems such as lower bounds

for the size of monotone circuits, constant depth circuits, and formulas are presented. The treatment is very careful and detailed. For instance, the whole of Chapter 10 is dedicated to a discussion of why proving lower bounds against monotone circuits is so much easier than against general circuits, and various quantitative aspects of this. These parts also cover some less-studied models of computations such as span programs.

Part five of the book covers the model of computation by branching programs. This model captures in a natural way the deterministic space (memory), whereas nondeterministic branching programs do the same for the nondeterministic model of computation. The chapter covers all the main classes of branching programs such as decision trees, read-once programs, and oblivious branching programs. Both lower bounds and upper bounds (Barrington's theorem) are presented. Part six deals with proof complexity. The main goal of this area is to show that some unsatisfiable formulae require long proofs in a formal proof system, as the opposite of this would imply that $\mathbb{NP} = \text{co}-\mathbb{NP}$. The chapter summarizes the state of the art in resolution and cutting plane proof systems including the size vs. width trade-offs for resolution proofs and applications of cutting plane proofs to integrality gaps of integer linear programs.

The last chapter of the book discusses the difficulties in proving lower bounds and focuses on the natural proofs barrier of Razborov and Rudich [4]. The chapter also contains an interesting discussion regarding the possibility that all of the class $\mathbb{P}$ might have linear size circuits. While currently this is considered quite unlikely, apparently in the early days of complexity theory an opposite viewpoint was expressed by the great mathematician Andrey Kolmogorov.

All material in the book is accessible to graduate or even undergraduate students of computer science, mathematics or electrical engineering. The book is self-contained. All necessary mathematical machinery is summarized in the appendix. I gladly recommend this book to beginning students, who will find this book a good starting point in exploring the field of complexity theory, as well as to mature researchers who would like to bring themselves up-to-date on some aspect of the theory. Finally, let me mention that the book contains a large number of open research problems, including some really enticing ones!

REFERENCES

[1] S. Arora and B. Barak, *Computational Complexity: A Modern Approach*, Cambridge University Press, Cambridge, UK, 2009.

[2] E. Kushilevitz and N. Nisan, *Communication Complexity*, Cambridge University Press, Cambridge, UK, 1997.

[3] A. Razborov, *On Rigid Matrices*, manuscript, 1989 (in Russian).

[4] A. Razborov and S. Rudich, *Natural proofs*, J. Comput. System Sci., 55 (1997), pp. 24–35.

[5] R. Williams, *Nonuniform ACC circuit lower bounds*, J. ACM, 61 (2014), article 2.

Sergey Yekhanin
Microsoft Research