### Notes of a Numerical Analyst

## Silly digits

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Web searches just now have told me:

The current world population is 8,182,491,355.

In 2021, Nature had an impact factor of 69.504.

The average body mass index (BMI) in the United States is 29.23.

In 2023, its GDP per capita was \$81,695.20.

What should we make of statements with such preposterous precision?

It's interesting how the flavours of nonsense vary. The world population, say, is a reasonably welldefined number at a given instant; what's absurd is the hint that we might know it to 10 digits of accuracy. (Indeed it may be only 1, for Nick Eberstadt of the American Enterprise Institute tells me "I am not certain we can be confident about that second digit....") In the other cases the absurdity goes deeper, for even if the calculations were exactly correct in terms of the relevant definitions, the numbers still wouldn't have much meaning. An impact factor, for example, is computed from a sample of n papers spread over two years, and n might be in the hundreds if you're lucky. For 5 digits to be meaningful, it would have to be closer to  $10^{10}$ .

# $3.14159{\scriptstyle 265358979323846264338477998}$

Figure 1. All the digits of  $\pi$  as displayed by Don Knuth

Spurious digits don't appear only in online statistics possibly garbled by Al. Flesh and blood students and colleagues record them regularly in homework assignments and published papers:

The computation times grow at the rate  $n^{2.1104}$ .

*E* = 1.7386e-8 [one entry in a table of errors depending on various parameters].

I tell my students to keep their brains in gear when they write down a number. In my course this autumn I tried to convey the message lightly: To look foolish, State results with too many digits.

Some associated emails around this time led to unexpected developments. Tim Cole of UCL told me that Gauss had apparently said the same thing, if more ponderously:

> Lack of mathematical culture is revealed nowhere so conspicuously as in meaningless precision in numerical computations.

Seeing this quote gave me a boost, for it seemed to confirm that in the good old days, even the very top mathematicians cared about numbers. I asked for help in tracking down Gauss's original German from Folkmar Bornemann of TU Munich, who proceeded to unearth a tangled history [1]. According to Bornemann, it seems likely that Gauss never said this. Maybe Wilhelm Weber or Franz Neumann said it, early in the 19th century. In the handbook of chemical analytics by Küster and Thiel, it was printed with the name of Gotthilf Hagen for generations, then re-attributed to Gauss starting with the 41st edition in 1935. The attribution to Gauss spread further with Morgenstern's book *On the Accuracy of Economic Observations* (2nd ed., 1963).

Happily, we know some numbers with unarguable precision, such as  $\pi$  (Figure 1). And the speed of light is 299,792,458 m/s *exactly*! — for the meter has been defined that way since 1983.

### FURTHER READING

[1] F. Bornemann, Tracing a 'quote' of Gauss the numerical calculator, unpublished note, September 2024.

[2] T.J. Cole, Too many digits: the presentation of numerical data, *Arch. Dis. Child.*, 100 (2015) 608–709.



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