Axiomatic Set Theory: Problem sheet 1

1. Which of the ZF axioms (A1)–(A2), (A7) and (A8) hold in the structure $\langle \mathbb{Q}, < \rangle$? Also, find an instance of (A5) that is true in $\langle \mathbb{Q}, < \rangle$ and one that is false.

2. Write the following as formulas of LST:
(a) $x = \langle y, z \rangle$;
(b) $x = y \times z$;
(c) $x = y \cup \{y\}$;
(d) "x is a successor set";
(e) $x = \omega$.

3. Assuming ZF*, show that there exists a *transitive* set M such that
(a) $\varnothing \in M$, and
(b) if $x \in M$ and $y \in M$, then $\{x, y\} \in M$, and
(c) every element of M contains at most two elements.
Show further that if σ is an axiom of ZF*+AC other than (A8), (A4) or (A7), then $\langle M, \in \rangle \models \sigma$. (It follows that if ZF* is consistent then so is (ZF*+AC$\setminus$\{(A8), (A4), (A7)\}.)

4. Assuming ZF show that if a is a non-empty transitive set then $\varnothing \in a$.

5. Deduce (A3) (pairing) from the other axioms of ZF*.