TASHKENT MEETS JAYNUUT

INTERACTING WITH BORIS OVER (AT LEAST) 50 YEARS

FOR OUR COMMUNITY IN LOGIC, HE MERITS THE DESCRIPTION "A MIND ALWAYS IN MOTION" (USED BY OWEN CHAMBERLAIN OF HIS COLLABORATOR EMILIO SEGRÉ, FELLOW NOBELIST FOR THE DISCOVERY OF THE ANTIPROTON).

IN WISHING BORIS
HAPPY BIRTHDAY
I WISH HIM AND
HIS FAMILY PEACE AND GOOD
HEALTH, AND I WISH
OUR COMMUNITY MANY
MORE ~~YEARS~~ OF
VISIONARY IDEAS FROM
THE UNFLAGGING MIND
OF OUR DEAR FRIEND.

"PLAN" OF TALK.

BORIS BORN IN TASHKENT
(1949)

I BORN IN TAYNUILT (1941),

CHANCES OF A NATIVE OF
HIS CITY, AND ONE OF MY
VILLAGE, EVER MEETING,
WERE NOT HIGH.
BUT NOW WE ARE OLD
FRIENDS, SHARING FAROUT
IDEAS. I WILL GIVE A
PERSONAL ACCOUNT, COMPLETELY
NON-TECHNICAL, OF SOME
OF OUR INTERACTIONS.

I STARTED MY Ph.D AT STANFORD IN 1964. I HAD STUDIED MATHS AND ASTRONOMY AT GLASGOW (1958–60), MATHS AND PHILOSOPHY AT ST ANDREWS (1960–62), AND PHILOSOPHY AT CAMBRIDGE (1962–4). I WAS WELL-EDUCATED, LARGELY SELF-TAUGHT IN MATHEMATICS OVER A WIDE RANGE, AND ALREADY INTERESTED IN DEFINABILITY ISSUES IN NUMBER THEORY AND GEOMETRY. AT CAMBRIDGE I DID P-ADICS WITH CASSELS, AND GROUP THEORY WITH PHILIP HALL. I KNEW COHEN'S WORK QUITE WELL.

AT STANFORD I WAS A STUDENT OF DANA SCOTT, A GENEROUS, DYNAMIC SUPERVISOR, SUPERB EXPOSITOR, WITH A WIDE RANGE. HE ASSIGNED ME PROBLEMS OF TARSKI ABOUT PAIRS OF REAL CLOSED FIELDS, ON WHICH I MADE RAPID PROGRESS (I HAD READ TARSKI ON MY OWN, LONG BEFORE, AND SCOTT HAD CASUALLY GIVEN ME THE THREE AX-KOCHEN PAPERS, WHICH HAPPENED TO BE RELEVANT) KUNEN AND I GAVE SEMINARS ON THOSE, AND LATER I STUDIED, TO MY ENORMOUS BENEFIT, PAUL COHEN'S WORK

THEN (AGAIN, IT WAS CRUCIAL FOR ME) I READ MORLEY'S GREAT PAPER, AND EHRENFEUCHT-MOSTOWSKI.

I RETURNED TO THE U.K IN 1967, AND FROM 1968 TO 2003 I WAS IN ABERDEEN, IN A QUIET, BEAUTIFUL PLACE, WHERE I WORKED HARD ON MANY THINGS, IN PARTICULAR ON SOME THINGS THAT WOULD PUT ME AND BORIS (AND OLEG) IN INTELLECTUAL CONTACT BEFORE LONG. AN ACTUAL MEETING DID NOT HAPPEN TILL 1983, IN WARSAW.

IN 1968 (PUBLISHED LATER) I PROVED THAT INFINITE $\aleph_1$-CATEGORICAL FIELDS ARE ALGEBRAICALLY CLOSED. MORLEY'S PAPER HAD SET ME THINKING OF THIS, AND IT GREATLY PLEASED ME TO USE GALOIS THEORY. THE PROOF WORKED FOR ω-STABILITY, AND WAS PRECEDED BY SIMILAR CLASSIFICATIONS FOR ABELIAN GROUPS.

I DID NOT HAVE ANY PHILOSOPHY OF "PERFECT" MODEL THEORY, AS BORIS WAS TO HAVE . . .

I HAD HOWEVER A SENSE THAT ω-STABILITY WOULD TYPICALLY BE CONNECTED TO KNOWN OR CONJECTURED STRUCTURE THEOREMS IN ALGEBRA. I DID MUCH WORK IN THIS DIRECTION FROM 1968, TILL I WENT TO YALE IN 1973 AT THE INVITATION OF ABRAHAM ROBINSON, WHO DIED IN 1974 AGED 56. I OFTEN WISH THAT BORIS AND ABBY HAD MET — EACH WAS WORTHY OF THE TRIBUTE SEGRÉ GOT

ROBINSON AND STABILITY

IN FALL OF 73 ABBY ASKED ME "ANGUS, CAN YOU EXPLAIN TO ME THE SIGNIFICANCE OF STABILITY THEORY?" THOUGH I KNEW THE GENERAL NOTION (FIRST DUE TO ROWBOTTOM) AND HAD READ MANY BEAUTIFL PAPERS, THE RARITY OF EXAMPLES MADE ME DODGE THE REQUEST, BUT I PROMISED TO WRITE TO POIZAT TO GET A GOOD EXPLANATION. POIZAT WROTE A NICE LETTER AND I READ IT TO ABBY. I WOULD LOVE TO KNOW IF BRUNO KEPT A COPY.

IN 1976 I FIRST MET GREG. I GREATLY ENJOYED HIS COMPANY, AND HIS BRILLIANCE. MY ADMIRATION ENDURES. BY THEN WE GOT MUCH MORE INFORMATION FROM RUSSIA, AND MODEL THEORY OF ω-STABLE GROUPS BEGAN TO THRIVE, WITH BORIS (INDECOMPOSABILITY), GREG, BRUNO, SASHA AND OTHERS CONTRIBUTING THEIR EXCEPTIONAL TALENTS TO WONDERFUL CONJECTURES ABOUT GROUPS OF FINITE MORLEY RANK.

SUBJECT IS VERY DEEP, INSPIRED BY IDEAS FROM ALL OVER GROUP THEORY, WITH BRILLIANT YOUNGER PEOPLE, <u>BUT</u> THE CONJECTURES RESIST.

WITH BAUR AND CHERLIN WE MADE REAL PROGRESS ON $\aleph_0$-CATEGORICAL, ω-STABLE GROUPS AND RINGS (GREG HAS VARIOUS BRILLIANT ACCOUNTS OF THE DEEPER MATHEMATICS THAT EMERGED WITH THE INVOLVEMENT OF BOROVIK, ZILBER, AND CONTINUES TO THIS DAY.

I WORKED HARD ON ALGEBRAICALLY CLOSED GROUPS (AND LATER MUCH HARDER ON DIVISION RINGS). STABILITY NOT INVOLVED

STILL, IT WAS MODEL THEORY (<u>AND</u> RECURSION THEORY) AND ATTRACTED ZILBER, BELEGRADEK AND HIGMAN. THE WORK ON DIVISION RINGS REVEALED COMBINATORIAL RESULTS WHICH HELPED FREESE SOLVE THE LONG-OPEN PROBLEM OF UNDECIDABILITY OF THE WORD PROBLEM FOR <u>FREE</u> MODULAR LATTICES

!!

BORIS WAS MUCH CONCERNED IN THE 1970'S WITH $\aleph_1$-CATEGORICITY IN GENERAL, AND WITH WITH THE ISSUE OF FINITE AXIOMATIZABILITY. IN ADDITION HE BEGAN DEEP ANALYSIS OF STRONGLY MINIMAL SETS (AND VARIANT NOTIONS), INSPIRING THE FAMOUS PAPER BY CHERLIN, HARRINGTON AND LACHLAN (PUBLISHED IN 1985) ENTITLED "$\aleph_0$-CATEGORICAL, $\aleph_0$-STABLE STRUCTURES".

IN ADDITION HE STRESSED THE IMPORTANCE, FOR HIS ANALYSIS, OF CLASSICAL GEOMETRIES OVER FINITE FIELDS

CHL SET OUT TO REPAIR VARIOUS ERRORS IN HIS PUBLICATIONS FROM 1977 TO 2004. THE RESULT IS A BEAUTIFUL, INSPIRING PAPER. ZILBER HIMSELF, INFINITELY RESOURCEFUL, FOUND SEVERAL EXOTIC REPAIRS, USING A VARIETY OF METHODS (FOR EXAMPLE DIOPHANTINE, AND BCM). THIS WAS THRILLING (I WELL REMEMBER DISCUSSION SESSIONS AT YALE WITH MORLEY PRESENT, THE AUDIENCE LOST IN ADMIRATION.)

AND THEN WE MET HIM!

IN 1983 AT THE WARSAW ICM. GREG, LOU AND I MET BORIS AND OLEG (I HOPE MY MEMORY DOES NOT FAIL ME). NATURAL, AFFECTIONATE, THAT MIND IN MOTION, AND NOTHING OF THIS HAS CHANGED SINCE.

THE TRICHOTOMY CONJECTURE (TOO BEAUTIFUL TO BE TRUE) WAS VERY MUCH ON HIS MIND. UDI FOUND A POWERFUL CONSTRUCTION THAT REFUTED IT, AND THAT TURNED OUT ALL TO THE GOOD.

GOOD?

1) THE METHOD OF UDI WAS VERY FLEXIBLE, AND HAS BEEN USED BY MANY

2) UDI AND BORIS MODIFIED BORIS' BASIC IDEAS (ONE MOTIVATION CAME FROM REMMERT'S FAMOUS THEOREM ON ANALYTIC SETS, KNOWN TO THE EARLY O-MINIMALISTS) TO MAKE A FLEXIBLE NOTION OF ANALYTIC ZARISKI STRUCTURE, NOW PART OF THE BASIC REPERTOIRE OF APPLIED MODEL THEORY (TOO LATE FOR ROBINSON, ALAS)

MEANWHILE, AT YALE AND OXFORD AND URBANA, AND PARIS AND BERLIN, THE FIRST-ORDER THEORY OF $(\mathbb{R}, +, \cdot, -, 0, 1, \exp)$ WAS BEGINNING TO YIELD (IT FINALLY DID IN 1990-1) AFTER MANY YEARS OF EFFORT (SINCE 1976) BY WILKIE, VAN DEN DRIES, MILLER, MARKER, MACINTYRE, RESSAYRE AND OTHERS,

BEFORE THE MAIN RESULTS WERE PROVED, ONE KNEW THAT SCHANUEL'S CONJECTURE WAS RELEVANT, THAT KHOVANSKI'S 1980 PAPER IN DIFFERENTIAL TOPOLOGY WAS LIKELY TO BE INVOLVED AND O-MINIMALITY (FIRST PROVED BY TARSKI FOR THE REAL FIELD) SHOULD BE RELEVANT. THE AXIOMATICS OF O-MINIMALITY GIVING CELL-DECOMPOSITION IN HIGHER DIMENSIONS ARE CLEARLY VISIBLE IN ŁOJASIEWICZ'S WORK, AND WERE CAREFULLY FORMALIZED BY LOU, KNIGHT-PILLAY-STEINHORN.

– IN 1990 WILKIE USED THE ABOVE TO GET MODEL-COMPLETENESS

IN 1991, ALEX AND I PROVED DECIDABILITY, ASSUMING ONLY THE TRUTH OF SCHANUEL' CONJECTURE.

THE PROOF YIELDS O-MINIMALITY, AND THIS HAS LED TO AMAZING INTERACTIONS WITH DIOPHANTINE GEOMETRY (AND NEURAL NETWORKS, ETC).

NO USE OF STABILITY IN THIS PROOF

OF COURSE THERE HAS BEEN SUCCESS IN GIVING LOCAL VERSIONS OF STABILITY NOTIONS, BUT (TO MY KNOWLEDGE) NO DRAMATIC APPLICATIONS.

BUT BORIS HAS DONE SOMETHING FANTASTIC WITH THE COMLEX EXP, A STRUCTURE NEGLECTED THOUGHTLESSLY BECAUSE OF ITS OBVIOUS UNDECIDABILITY (NO ASSUMPTIONS ON SCHANUEL)

BORIS' RESTLESS MIND LED HIM TO CONSTRUCT AN EXPONENTIAL FIELD $\mathbb{B}$ OF CARDINAL $2^{\aleph_0}$, BY A LIMIT CONSTRUCTION INVOLVING EXTENSIONS BASED ON THE KIND INTRODUCED BY UDI (RECALL THE REFUTATION OF TRICHOTOMY CONJECTURE).

$\mathbb{B}$ SATISFIES SCHANUEL, AND EVERY DEFINABLE SUBSET OF $\mathbb{B}$ IS EITHER COUNTABLE OR COCOUNTABLE. NEITHER PROPERTY IS KNOWN FOR $\mathbb{C}$ (OVER THE YEARS MANY PEOPLE HAVE WONDERED IF $\mathbb{R}$ IS DEFINABLE IN $\mathbb{C}$, USING EXP)

BORIS HAS CONJECTURED THAT

$\mathbb{B} \cong \mathbb{C}$!!

MANY VERY ABLE PEOPLE (MANY HERE TODAY) ARE AIMING TO PROVE THIS, AND SOME WEAKER VERSIONS HAVE BEEN PROVED (e.g. BY KIRBY AND GALLINARDI - THE PROOF IS NOT EASY)

$\mathbb{B}$ (AND RELATED STRUCTURES) HAVE A BEAUTIFUL MODEL THEORY (BASED ULTIMATELY ON PROFOUND WORK OF SHELAH)

BUT : IS PICARD'S LITTLE THEOREM TRUE IN $\mathbb{B}$?

YES (D'AQUINO, MACINTYRE, TERZO)
BUT PROOF IS RATHER INDIRECT

IS SHAPIRO'S CONJECTURE TRUE IN $\mathbb{B}$?
(IT IS NOT KNOWN IN $\mathbb{C}$).

YES (SAME AUTHORS)
RATHER DIFFICULT,

COMPUTABILITY ISSUES

LET $\mathbb{A}$ BE ONE OF ZILBER'S COUNTABLE MODELS (NOTION ASSUMED UNDERSTOOD BY THIS AUDIENCE). THEN I PROVED THAT $\mathbb{A}$ IS A COMPUTABLE EXPONENTIAL FIELD.

I DON'T KNOW WHAT THIS MEANS FOR $\mathbb{C}$!

PERSONAL ASPECTS

THE 1991 SEMESTER AT CHICAGO WAS A HAPPY TIME. BORIS AND I SHARED AN OFFICE, AND PREPARED LECTURES, RESPECTIVELY ON ZARISKI STRUCTURES AND ON THE REAL EXP

A VERY CHEERFUL SOCIAL LIFE STARTED AN ENDURING FRIENDSHIP WITH TAMARA AND ALYOSHA

IT HAS BEEN A
PRIVILEGE TO SHARE
TIME WITH SUCH A
CHARMING AND SINCERE
THINKER, WHO HAS
MODESTLY BROUGHT
THRILLING IDEAS TO
OUR SUBJECT.

THANK YOU, BORIS!