This paper considers the topological dynamics of the automorphism group $\mathrm{Aut}(\mathcal{C})$ of the monster model of a first-order theory T. More precisely, it considers the continous action of $\mathrm{Aut}(\mathcal{C})$ on the space of global types $S_{\overline{c}}(\mathcal{C}) := \{\mathrm{tp}(\overline{a}/\mathcal{C}) : \overline{a} \in \mathcal{C}' \succ \mathcal{C},\ \overline{a} \equiv_{\emptyset} \overline{c}\}$, where $\overline{c}$ is a tuple enumerating the whole of $\mathcal{C}$. In particular, it is shown that the Lascar Galois group $\mathrm{Gal}_L(T)$ can be obtained as a certain continuous quotient of the Ellis group of this flow, and the kernel of the natural map $\mathrm{Gal}_L(T) \to \mathrm{Gal}_{KP}(T)$ to the Kim-Pillay Galois group can also be understood in these terms.

This technology is then applied to establish results on the complexity of bounded invariant equivalence relations. The authors show in particular that if E is a bounded invariant equivalence relation on a complete $\emptyset$-type in a countable language, then E is type-definable if and only if it is smooth. Here, E is *smooth* if the equivalence relation E^M it induces on the type-space $S(M)$ is smooth in the descriptive set theory sense for some (equivalently any) countable model M, i.e. it is of the form $f(x) = f(y)$ for some Borel map to Cantor space $f : S(M) \to 2^{\mathbb{N}}$. They show furthermore that if E is additionally Borel, then it has $2^{\aleph_0}$ classes unless it is relatively definable (in which case it has finitely many classes).

Analogous results for languages of arbitrary cardinality are also obtained, and corresponding results are deduced for bounded-index invariant subgroups of type-definable groups.