This paper treats a small but significant error in the model-theoretic literature on unimodularity.

A unimodular strongly minimal set is one in which for any interalgebraic independent tuples $\bar{a}$ and $\bar{b}$, we have equality of multiplicities $mult(\bar{a}/\bar{b}) = mult(\bar{b}/\bar{a})$. Unimodularity was defined and studied in [Hru92], wherein it is shown that unimodularity implies local modularity, generalising Zilber's proof that locally finite (e.g. ω-categorical) strongly minimal sets are locally modular.

In the abstract and introduction of [Hru92] it is indicated that we can take as an equivalent definition of unimodularity: if f_1 and f_2 are definable surjective functions $U \to V$ which are everywhere k_1-to-1 and k_2-to-1 respectively, then $k_1 = k_2$.

The authors of the reviewed paper term this "weak unimodularity", and demonstrate with a simple example that it is in fact strictly weaker than unimodularity. They further show that if "definable" is replaced with "type-definable" in the definition of weak unimodularity, it does become equivalent to unimodularity (one might deduce from the discussion at the start of section 2 of [Hru92] that this is how the definition in that paper was in fact meant to read).

Meanwhile, the error of assuming equivalence of unimodularity and weak unimodularity has propogated through the literature on measurable structures in the sense of Macpherson and Steinhorn [MS08]. The papers [MS08] and [Elw07] each conflate weak unimodularity with unimodularity, and the latter gives an erroneous proof of equivalence.

This paper deals with this confusion. The authors show that the Zilber functions defined in [Hru92] correspond to Macpherson-Steinhorn-measures in the strongly minimal case, hence that for strongly minimal sets measurablity is equivalent to unimodularity. They further give a correct proof of the result in [Elw07] that Macpherson-Steinhorn-measurable stable theories are 1-based - using Buechler's dichotomy to reduce to strongly minimal sets, then applying Hrushovski's theorem that unimodular strongly minimal sets are locally modular.

The paper is clear and precise. It has no prerequisites beyond a familiarity with the basic notions of stability theory.

References

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