Lec 12

$\int x^3 e^{x^2} dx$

$= \int x u e^u dx$ $\qquad u = x^2$, $\frac{du}{dx} = 2x$

$= \frac{1}{2} \int \frac{du}{dx} u e^u dx$

$= \frac{1}{2} \int u e^u du$ (substitution rule)

$= \frac{1}{2} \left(u e^u - \int 1 \cdot e^u du \right)$

$= \frac{1}{2} (u e^u - e^u) + C$

$= \frac{1}{2} (u-1) e^u + c$

$= \frac{1}{2} (x^2 - 1) e^{x^2} + c$

$\int f g' = fg - \int f' g$

$\int e^{x^2} dx = ?$

- $\int \ln(x^2) dx = \int 2 \ln(x) dx$

$= 2 \int \ln x \, dx$

$= 2(x-1)\ln x + c$

$\int \ln x \, dx = \int 1 \cdot \ln x \, dx$

$= x \ln x - \int x \cdot \frac{1}{x} dx$

$= x \ln x - x + C$

$\cancel{= (x-1) \ln x + c}$

$= x(\ln x - 1) + c$

- $\int \ln(\sqrt{x-1}) dx = \frac{1}{2} \int \ln(x-1) dx$

$= \frac{1}{2} \int \ln(u) dx$ $\qquad u = x - 1$

$= \frac{1}{2} \int \ln u \, du$ $\qquad \frac{du}{dx} = 1$

$= \frac{1}{2} u(\ln u - 1) + c$

$= \frac{1}{2} (x-1)(\ln(x-1) - 1) + c$

(4)

$$= \frac{1}{2} \ln\left(x^2 + \sqrt{x^4+36}\right) + c$$

(2)

$$\int x \ln\sqrt{x-1}\, dx = \frac{1}{2}\int x \ln(x-1)\, dx$$

$$= \frac{1}{2}\int (u+1) \ln u \, du \qquad u = x-1$$

$$= \frac{1}{2}\left(\int u \ln u \, du + \int \ln u \, du\right)$$

$$= u(\ln u - 1)$$

$$\int u \ln u \, du = \frac{u^2}{2}\ln u - \int \frac{u^2}{2}\frac{1}{u}\, du$$

$$= \frac{u^2 \ln u}{2} - \frac{u^2}{4} + c$$

$\therefore$ Answer

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left(u+\sqrt{u^2-a^2}\right) + C \qquad \text{— Table pp. 484, 485}$$

<u>check:</u> $$\frac{d}{du}\left(\ln\left(u+\sqrt{u^2-a^2}\right)\right)$$

$$= \left(1+\frac{2u}{2\sqrt{u^2-a^2}}\right)\cdot\frac{1}{u+\sqrt{u^2-a^2}}$$

$$= \frac{1+\frac{u}{\sqrt{u^2-a^2}}}{u+\sqrt{u^2-a^2}}$$

$$= \frac{\sqrt{u^2-a^2}+u}{\left(\sqrt{u^2-a^2}\right)\left(u+\sqrt{u^2-a^2}\right)}$$

$$= \frac{1}{\sqrt{u^2-a^2}}$$

· $\int x\sqrt{x^2-9}\,dx$ $\qquad u = x^2-9$, $\quad \frac{du}{dx} = 2x$

$G'(x) = \sqrt{x^2-9}$

OR by parts: $\int x\sqrt{x^2-9}\,dx = \cancel{x\int\sqrt{x^2-9}\,dx - \int}$

$x\,G(x) - \int G(x)\,dx$

. . .

· $$\int \frac{x}{\sqrt{x^4+36}}\,dx = \frac{1}{2}\int\frac{1}{\sqrt{u^2+36}}\,du \qquad u = x^2, \quad \frac{du}{dx} = 2x$$

$$= \frac{1}{2}\ln\left(u+\sqrt{36+u^2}\right) + C$$