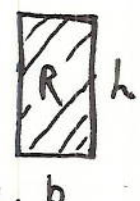
$$\int \ln x\,dx = \int 1\cdot \ln x\,dx$$
$$= x\ln x - \int \frac{x}{x}\,dx$$
$$= x\ln x - x + c$$
$$= x(\ln x - 1) + c$$

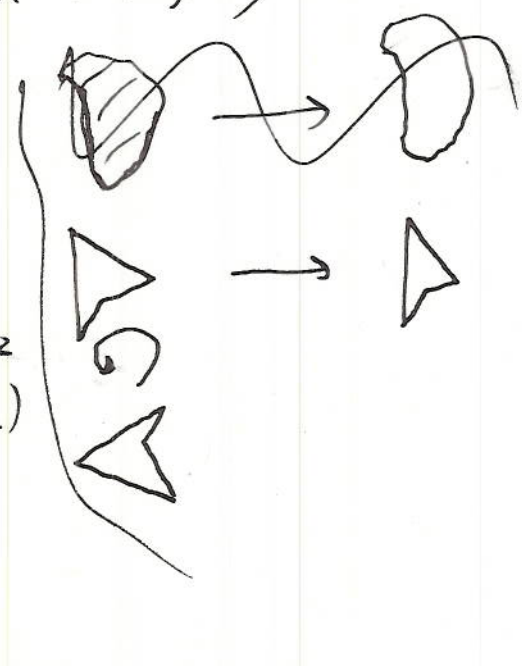
Informal idea of the area of a region in the plane

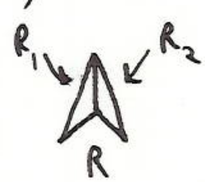


- Area of a rectangle is the product of the length of its base and its height

$$\text{Area}(R) = b\cdot h$$

- If we move or rotate or reflect a region, the area doesn't change

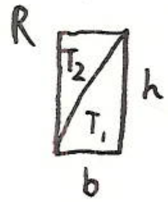




- If we can divide a region R into regions R_1 and R_2

Then $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2)$

Example: Area of a right-angled triangle



T_2 is a rotation of T_1

so $\text{Area}(T_2) = \text{Area}(T_1)$

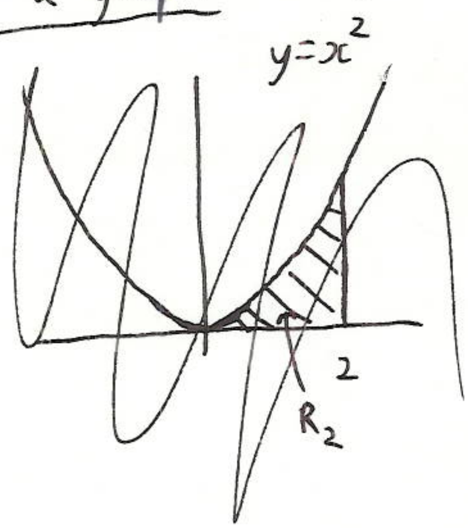
$\text{Area}(R) = bh$

$$bh = \text{Area}(R) = \text{Area}(T_1) + \text{Area}(T_2)$$
$$= 2\,\text{Area}(T_1)$$

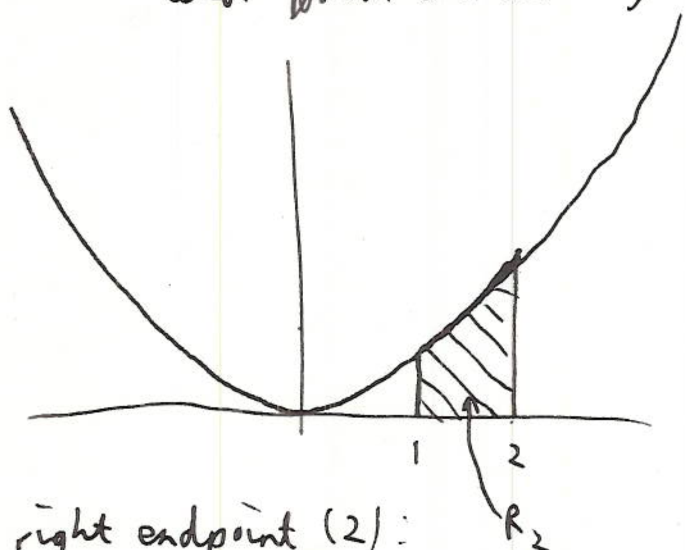
$$\text{Area}(T_1) = \frac{bh}{2}$$

Area under a graph

Example:



Let R_2 be the region bounded by the lines $y=0$, $x=1$, $x=2$ and the curve $y=x^2$



What is $\text{Area}(R_2)$?

Now consider shifting the right endpoint (2):

for any $x \geq 1$, let R_x to be the region:

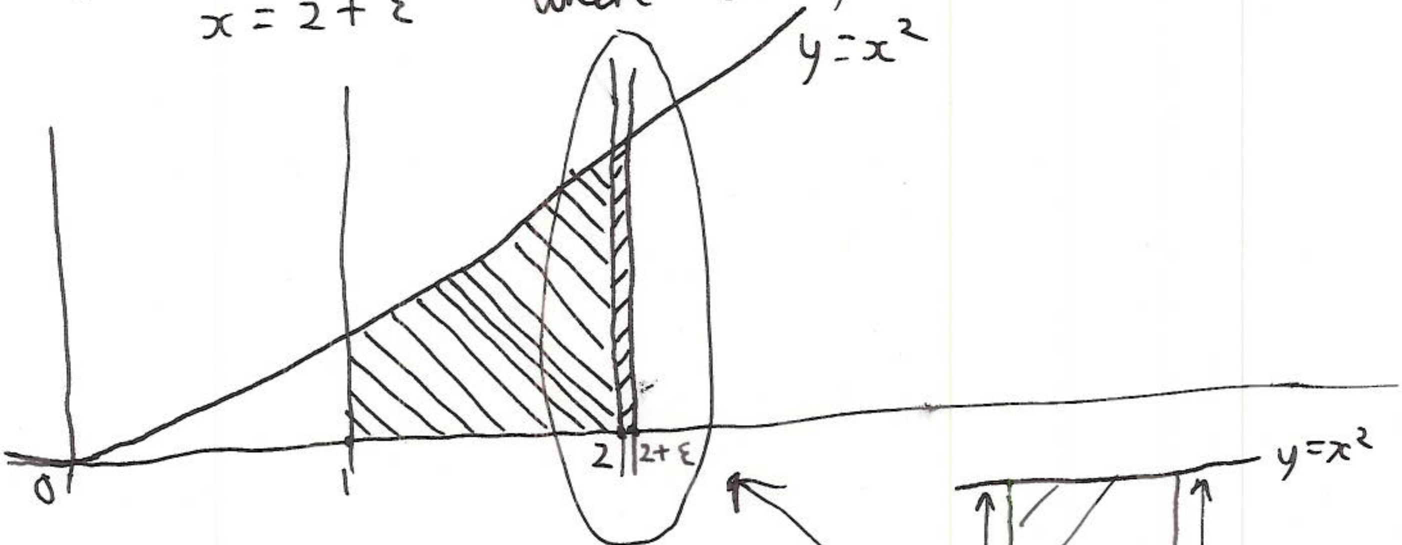
and define

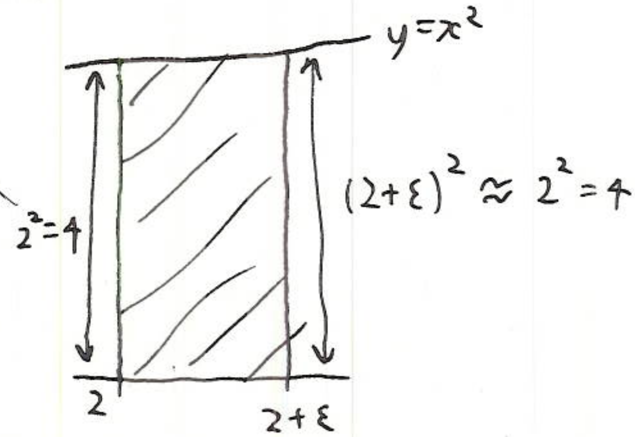
$A(x) := \text{Area}(R_x)$

What is $A(x)$ when x is close to 2?

Say x is very slightly larger than 2

$x = 2 + \varepsilon$ where $\varepsilon > 0$, ε is very small





For small ε, $R_{2+\varepsilon}$ divides into R_2 and a region which is approximately a rectangle with base ε and height $2^2 = 4$

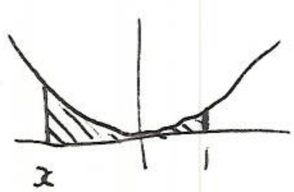
so $A(2+\varepsilon) \approx A(2) + 4\varepsilon$ for small $\varepsilon > 0$
(also for small $\varepsilon < 0$ by similar argument)

so for x near 2, $A(x)$ is approx. linear with slope 4

so $A(x)$ is differentiable at 2 and $A'(2) = 4$

Same argument works in general to show that $A(x)$ is differentiable (on $x \geq 1$) and $A'(x) = x^2$

If we define for $x < 1$

$A(x) := -$[Area of the region 

]

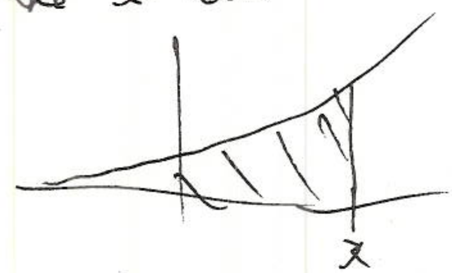
Then A is differentiable everywhere, and $A'(x) = x^2$

Same argument works for $\underline{\text{any}}$ continuous non-negative function in place of x^2 $(f(x) \geq 0$ for all $x)$
and any starting point in place of 1

e.g. $f(x) = e^{2x}$

$A(x) :=$ Area between 0 and x horizontally under the curve $y = e^{2x}$ above the x-axis
(count -vely for $x < 0$)

then $A'(x) = e^{2x}$



(can extend to $\underline{\text{any}}$ cont' f^n)
<u>Punchline</u>: we can find in this way an antiderivative of any continuous function