Remark;

$$f(x)\frac{dy}{dx} + g(x)y + h(x) = 0$$

$$\Leftrightarrow \frac{dy}{dx} + \frac{g(x)}{f(x)}y = -\frac{h(x)}{f(x)} \qquad \text{Linear}$$

Informal Introduction to probability

The probability of a statement is a number between 0 and 1 expressing the chance of it being true

where 0 means "definitely not"

1 means "definitely"

0.5 means "as likely as not"

"fifty-fifty"

e.g. if I toss a fair coin, the probability of getting heads is $\frac{1}{2}$

if I roll a fair die, probability of getting 6 is $\frac{1}{6}$

Generally the probability of a statement is the proportion of the time it will be true, given what we know.

i.e. if we repeatedly "set things up the same way" (in our imagination) (as far as we can tell), and see whether the statement is true, then the probability is the proportion in which it is.

(i.e. if we call these "trials", and they "succeed" if the statement is true,

the probability is

$$\lim_{N\to\infty} \frac{[\text{How many of the first } N \text{ trials are successful}]}{N}$$

Random Variables

A <u>random variable</u> is a variable whose value is random ("probabalistic", "undetermined")

e.g. $X = [\text{number a die turns up}]$

so we can ask
what is the probability that $X = 3$
that $X \geq 4$

we write $P(X=3)$, $P(X \geq 4)$ for these probabilities

so if the die is fair,

$$P(X=3) = 1/6$$

$$\begin{aligned} P(X \geq 4) &= P(X=4 \text{ or } X=5 \text{ or } X=6) \\ &= P(X=4) + P(X=5) + P(X=6) \\ &= 1/6 + 1/6 + 1/6 \\ &= 1/2 \end{aligned}$$

$$P(1 \leq X \leq 6) = 1$$

I spin a coin. Let X be the time (in seconds) it spins for.

What is $P(X=3)$ = Probability that it spins for __exactly__ 3 seconds

Only one reasonable answer;
$P(X=3)=0$!

Why?

Suppose e.g. $P(X=3) \approx 1/10$

Nothing special about 3, so e.g. also $P(X=2.9) \approx 1/10$

but then

$P(X = 2.9 \text{ or } X=2.91 \text{ or } X=2.92 \text{ or} \ldots \text{ or } X=3.09)$

$\approx 1/10 + 1/10 + \ldots + 1/10$

≈ 2 __Nonsense__

X is a __continuous__ random variable – it takes values on the __real__ interval $[0,\infty)$

Similarly

- mm of rainfall tomorrow
- time until your death
- height on its 20^{th} birthday of your firstborn

For a continuous random variable, it is generally the case that for any particular number c,

$$P(X = c) = 0$$

even though if $a < b$

$P(a \leq X \leq b)$ may well be > 0

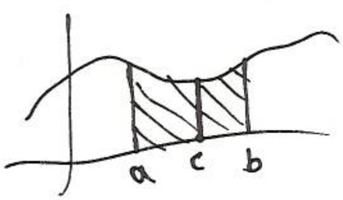
e.g. X = spintime

$P(1 \leq X \leq 60)$ is nearly 1

<u>Just like</u>; if I have a non-negative function $f(x)$,

then the area under the curve between a and b may be > 0

even though at c, the area "between c and c" (i.e. "at c") is 0, since the "thickness" is 0



$$\int_a^b f(x)\,dx > 0$$

$$\int_c^c f(x)\,dx = 0$$

Now even though, for X = spintime,

$$P(X = 5) = 0 = P(X = 5000),$$

there's a sense in which the former is "more likely"

So we associate to X a real-valued <u>probability density function</u>, $p_X(x)$, whose values $p_X(c)$ are non-negative reals (indicating the "relative likelihood" of $X = c$)

such that
$$P(a \leq X \leq b) = \int_a^b p_X(x)\,dx \qquad \text{for any } a \leq b$$

<u>Note</u>: · $P(X = c) = P(c \leq X \leq c) = \int_c^c p_X(x)\,dx = 0$

· We allow $a = -\infty$ or $b = \infty$

e.g. $$P(X \geq 0) = P(0 \leq X < \infty)$$
$$= \int_0^\infty p_X(x)\,dx \qquad \text{(Improper integral}$$
$$= \lim_{N \to \infty} \int_0^N p_X(x)\,dx$$

When does a function make sense as a pdf?

We need two things:

(i) f is non-negative ($f(x) \geq 0$ all x)

(ii) $$\int_{-\infty}^{\infty} f(x)\,dx = P(-\infty < X < \infty) = 1$$

Note we <u>can</u> have $f(x) > 1$ some x