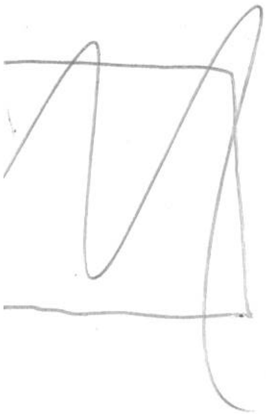
Fact:

The inverse of a function $f$ is unique if it exists,
and it exists iff $f$ is one-to-one



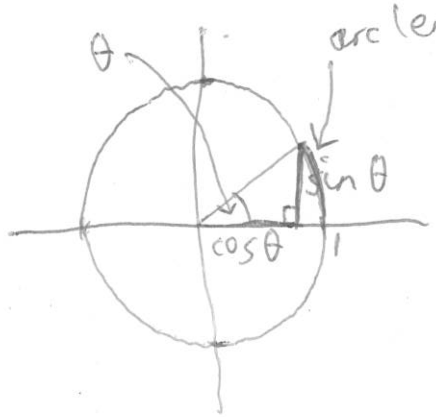
iff : if and only if

means: left hand side is true exactly when the right-hand is true

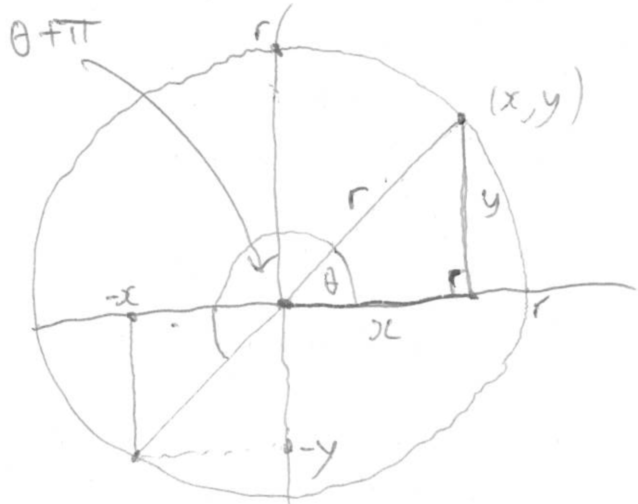
When $f^{-1}$ exists,
the graph of $f^{-1}$ is the reflection in $y=x$ of the graph of $f$.

$$(f^{-1})^{-1} = f$$

# Trig




$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

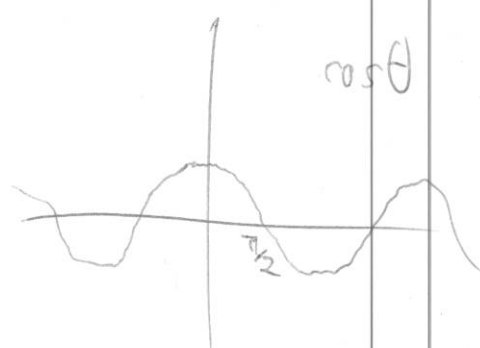


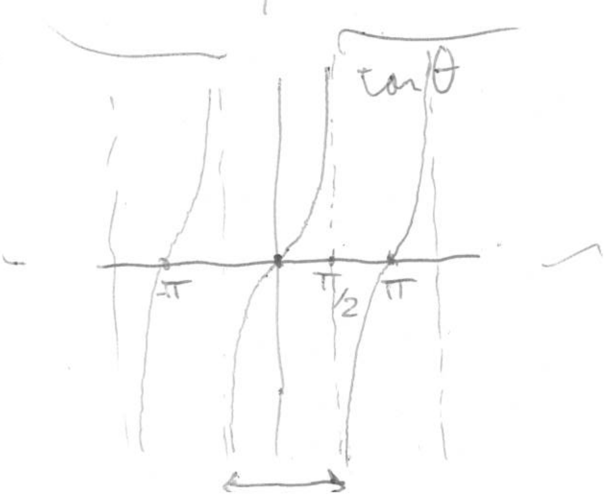

$$r = \sqrt{x^2+y^2}$$

$$x = r\cos\theta \qquad \text{"}\theta = \cos^{-1}(x/r)\text{"}$$

$$y = r\sin\theta$$

$$y/x = \frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \text{"}\theta = \tan^{-1}(y/x)\text{"}$$


not 1-1

but if we restrict to $(-\pi/2, \pi/2)$

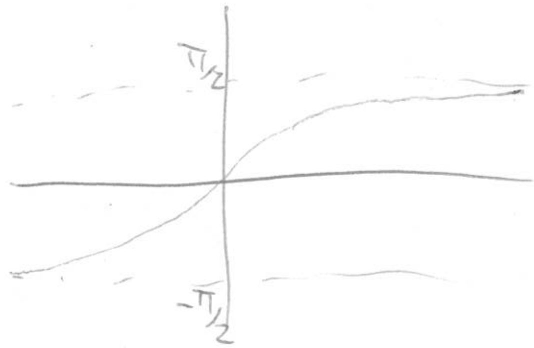
we get a 1-1 function with domain $(-\pi/2, \pi/2)$

range $\mathbb{R}$

its inverse is called arctan

$\text{dom}(\arctan) = \mathbb{R}$

$\text{ran}(\arctan) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$




$\theta = \arctan\left(\frac{y}{x}\right)$

but note $\tan(\theta+\pi) = \frac{-y}{-x} = \frac{y}{x}$