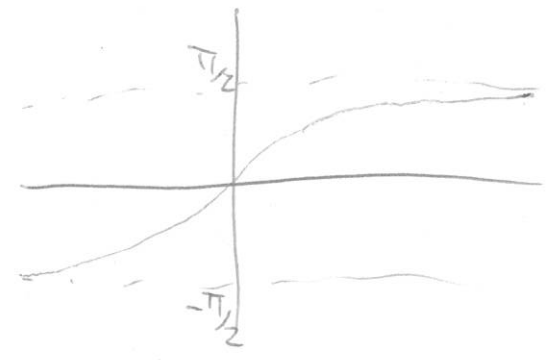


its inverse is called arctan

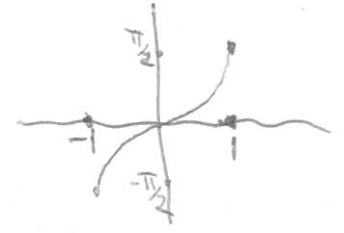
$\text{dom}(\arctan) = \mathbb{R}$   
 $\text{ran}(\arctan) = (-\pi/2, \pi/2)$



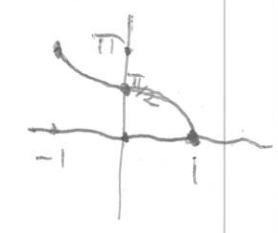
$\theta = \arctan(y/x)$

but note  $\tan(\theta + \pi) = \frac{-y}{-x} = \frac{y}{x}$

arcsin



arccos



Warning:

$\sin^{-1}$  is often used  
 to mean  $\frac{1}{\sin x} \neq \arcsin x$   
 $\text{csc } x = \frac{1}{\sin x}$   
 $\text{cosec } x = \frac{1}{\sin x}$   
 $\text{sec } x = \frac{1}{\cos x}$   
 $\text{cot } x = \frac{1}{\tan x}$

$(11 \text{ (11)})_{1 \rightarrow 2}$

$(x)_{1 \rightarrow 2}$

$1 \rightarrow 2$        $f$

$11 = x + 6 + \ln(x-4)$

$(x-4) \ln(x-4) + 9 + x \rightarrow x$

# Limits

The notation

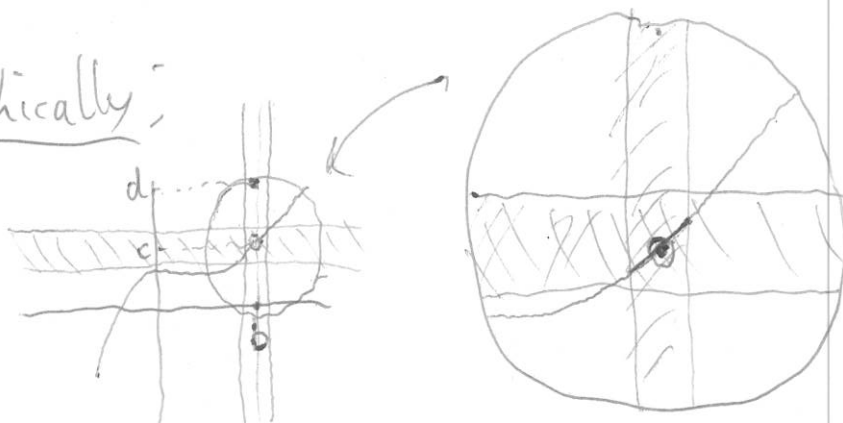
$$\lim_{x \rightarrow b} f(x) = c$$

$b, c$  real numbers

$f$  function

means: defined and  $f(x)$  is arbitrarily close to  $c$  for all  $x$  sufficiently close to  $b$ , but not equal to  $b$ .

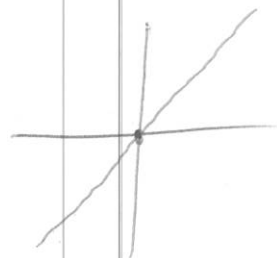
Graphically:



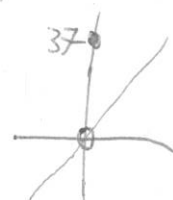
$\lim_{x \rightarrow b} f(x) = c$  This means;

for any horizontal strip around  $c$  there is a vertical strip around  $b$  such that the graph of  $f$  within the vertical strip is entirely contained within the horizontal strip, except maybe above  $b$  itself.

Examples:  $\lim_{x \rightarrow 0} x = 0$



$f(x) = x$  for  $x \neq 0$   
and  $f(0) = 37$



$$\text{dom } f = \mathbb{R}$$

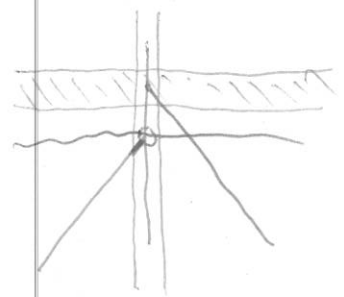
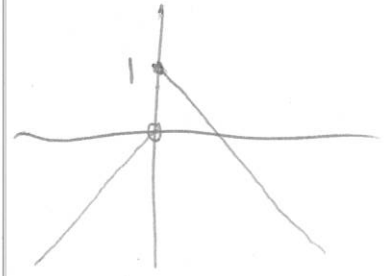
$$\lim_{x \rightarrow 0} f(x) = 0$$

$$f(x) = \begin{cases} x & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$$

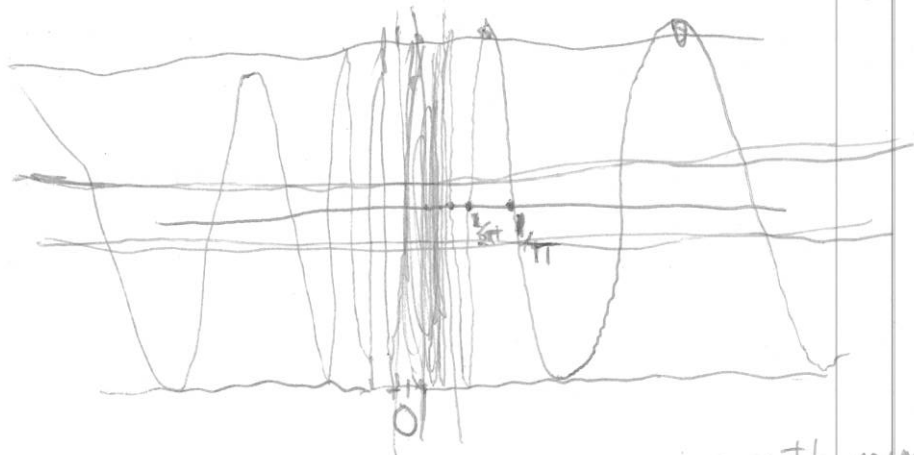
$$\lim_{x \rightarrow 0} f(x) \stackrel{?}{=} 0 \quad \text{NO}$$

$$\lim_{x \rightarrow 0} f(x) \stackrel{?}{=} 1 \quad \text{NO}$$

No limit at 0



$$f(x) = \sin(1/x)$$



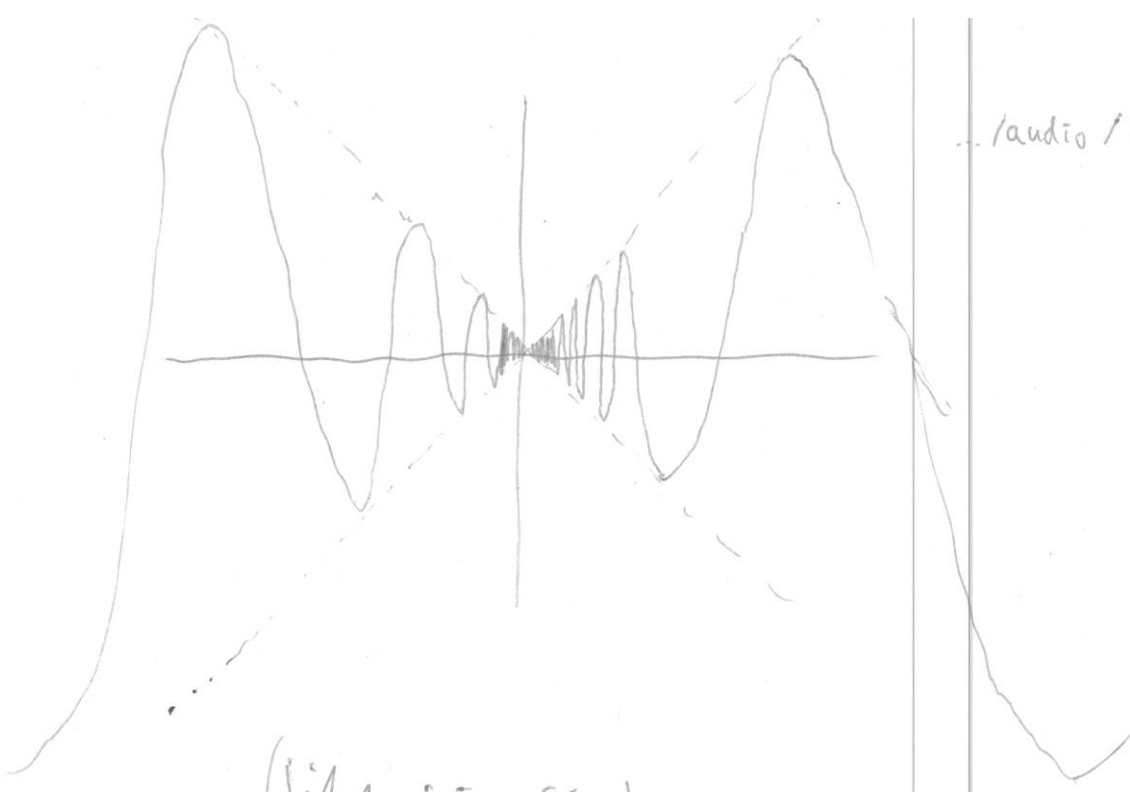
$$\lim_{x \rightarrow 0} f(x) \quad \text{No limit}$$

$$\sin\left(\frac{1}{\left(\frac{1}{k\pi}\right)}\right) = \sin k\pi = 0$$

[www.math.mcmaster.ca/~rmbays/teaching/1za3/audio/sininv.wav](http://www.math.mcmaster.ca/~rmbays/teaching/1za3/audio/sininv.wav)

$$f(x) = x \sin(1/x)$$

audio/x-sininv.wav



$$\lim_{|x| \rightarrow \infty} \frac{\lim_{x \rightarrow 0} f(x)}{f(x)} = 0$$

<http://mbay> www.mcmaster.ca/~mbays

