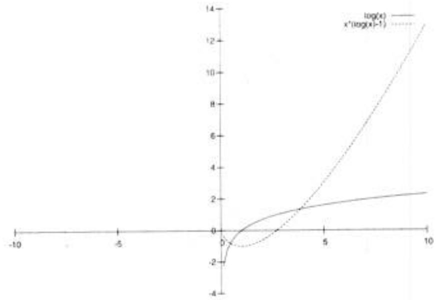
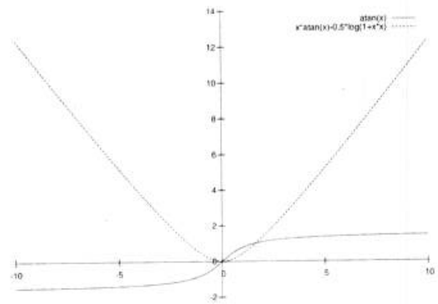
- $\int \ln x dx$

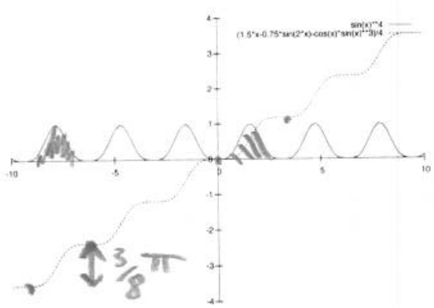


- $\int \arctan x dx$



Trigonometric integrals

- $\int \sin^4(x)\, dx$



- $\int \sin^2(x) \cos(x)\, dx$
- $\int \sin^3(x)\, dx$
- $\int \sin^4(x) \cos^3(x)\, dx$
- $\int \tan(x) \sec^2(x)\, dx$
- $\int \tan^3(x) \sec^3(x)\, dx$

(Recall: $\sin^2(x) + \cos^2(x) = 1, \sin(2x) = 2\sin(x)\cos(x),\ \cos(2x) = \cos^2(x) - \sin^2(x)$)

Integration by parts

$$\frac{d}{\mathrm{dx}} f(x)\, g(x) = f'(x)\, g(x) + f(x)\, g'(x)$$

so

$$\int \left(f'(x)\, g(x) + f(x)\, g'(x)\right) dx = f(x)\, g(x) + C$$

so

$$\int f'(x)\, g(x)\, dx = f(x)\, g(x) - \int f(x)\, g'(x)\, dx.$$

Example:

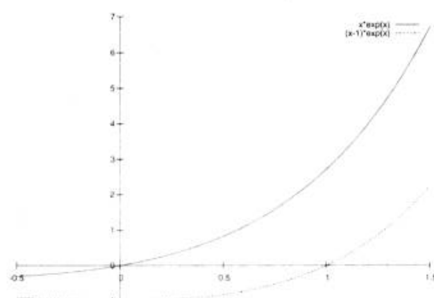
$$\int x \cos(x)\, dx = \int \sin'(x)\, x dx = \sin(x)\, x - \int \sin(x)\, 1 dx = x \sin(x) + \cos(x) + C.$$

Remark: Setting $u = f(x)$ and $v = g(x)$ (and assuming u is a function of v and v is a function of u) we can use the Substitution rule to rewrite this as

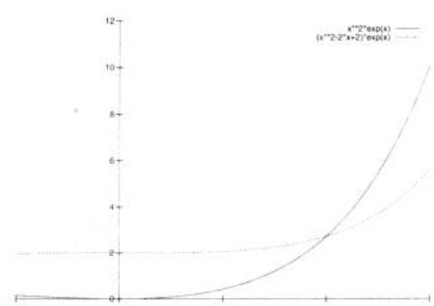
$$\int v\, \mathrm{du} = uv - \int u\, \mathrm{dv}\,.$$

Examples:

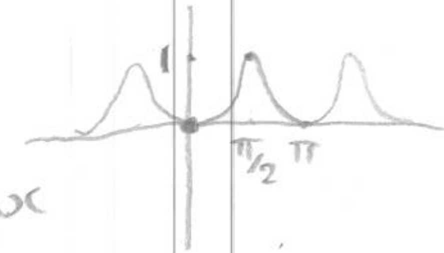
- $\int x e^x dx$



- $\int_0^1 x^2 e^x dx$



$\circ \int \sin^4 x \, dx$



$= \int \sin x \sin^3 x \, dx$

$= -\cos x \sin^3 x + 3\int \cos^2 x \sin^2 x \, dx$

$= -\cos x \sin^3 x + 3\int (1-\sin^2 x) \sin^2 x \, dx$

$f(x) = -\cos x \quad f'(x) = \sin x$
$g(x) = \sin^3 x \quad g'(x) = 3\cos x \sin^2 x$

$= -\cos x \sin^3 x + 3\int \sin^2 x \, dx - 3\int \sin^4 x \, dx$

so $4\int \sin^4 x \, dx = -\cos x \sin^3 x + 3\int \sin^2 x \, dx$

$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos 2x = \cos^2 x - \sin^2 x$
$\qquad\qquad\qquad\qquad\qquad 1 = \cos^2 x + \sin^2 x$

so $\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} \, dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + c$

so $\int \sin^4 x \, dx = \dfrac{-\cos x \sin^3 x + \frac{3}{2}\left(x - \frac{1}{2}\sin 2x\right)}{4} + c$

so e.g. $\int_0^\pi \sin^4 x \, dx = \dfrac{\left(0 + \frac{3}{2}\pi\right)}{4} - 0$

$= \frac{3}{8}\pi$

• $\int \sin^2 x \cos x \, dx$

$= \int u^2 \, du$ $\quad u = \sin x \quad \frac{du}{dx} = \cos x$

$= \frac{u^3}{3} + c$

$= \frac{\sin^3 x}{3} + c$

• $\int \sin^3 x \, dx$

$= \int (1 - \cos^2 x) \sin x \, dx$

$= \int \sin x \, dx - \int \cos^2 x \sin x \, dx$

$= -\cos x - \int \cos^2 x \sin x \, dx$

$= -\cos x + \int u^2 \, du$ $\quad u = \cos x \quad \frac{du}{dx} = -\sin x$

$= -\cos x + \frac{\cos^3 x}{3} + C$

• $\int \sin^4 x \cos^3 x \, dx$

$= \int \sin^4 x (1 - \sin^2 x) \cos x \, dx$

$= \int u^4 (1 - u^2) \, du$ $\quad u = \sin x$

$= \int u^4 \, du - \int u^6 \, du$

$= \frac{u^5}{5} - \frac{u^7}{7} + c$

$= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$

$\cdot \int \tan x \sec^2 x \, dx$

$\tan' x = \sec^2 x$

$\sec' x = \sec x \tan x$

$= \int u \, du \quad u = \tan x$

$\frac{\sin}{\cos^2}$

$= \frac{u^2}{2} + C$

$= \frac{\tan^2 x}{2} + C$

$\cdot \int \tan x \sec x \sec x \, dx$

$= \int u \, du \quad u = \sec x$

$= \frac{\sec^2 x}{2} + c$

equal upto constant by

$\sin^2 + \cos^2 = 1$

$\tan^2 + \cancel{1} = \sec^2$

$\cdot \int \tan^3 x \sec^3 x \, dx$

$= \int \tan x (\sec^2 - 1) \sec^3 x \, dx$

$= \int (\sec^2 x - 1) \sec^2 x \sec x \tan x \, dx$

$= \int (u^2 - 1) u^2 \, du \quad u = \sec x$

$\frac{du}{dx} = \sec x \tan x$

$= \frac{u^5}{5} - \frac{u^3}{3} \, du$

$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3}$