"Moves" available to you when integrating

- Recognise e.g. $\int x^2\,dx \quad \int \sin x\,dx \quad \int e^x\,dx \quad \int \frac{1}{x}\,dx$

  just know an antiderivative

- Manipulate algebraically

  (· multiplying out brackets $\int (x+1)\sin x\,dx = \int (x\sin x + \sin x)\,dx$

  · rules of exp and ln $(a\ln f(x) = \ln f(x)^a)$)

  · trig identities

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$1 = \cos^2 x + \sin^2 x$$

$$\cos^2 x = \frac{\cos 2x + 1}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin 2x = 2\sin x\cos x$$

  · partial fractions)

- Separating linearly

$$\int (a f(x) + b\,g(x))\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

- Substitute (either direction)

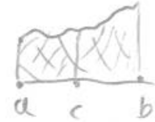
$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du \qquad u = g(x)$$

- Partition (integrate by parts)

$$\int f'(x)\,g(x)\,dx = f(x)\,g(x) - \int f(x)\,g'(x)\,dx$$

Definite Integrals:

- FTC $\displaystyle\int_a^b f(x)\,dx = \left[\int f(x)\,dx\right]_a^b$



- Split up: $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

e.g. $\int_{-\pi}^{\pi} x^2 \sin x \, dx$

$x^2 \sin x$ is odd $\quad f(-x) = -f(x)$

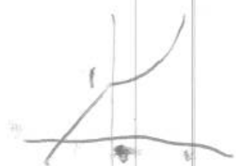
$(-x)^2 \sin(-x) = x^2 \sin(-x) = -x^2 \sin(x)$

so $\int_{-\pi}^{0} x^2 \sin x \, dx = -\int_{0}^{\pi} x^2 \sin x \, dx$

so $\int_{-\pi}^{\pi} x^2 \sin x \, dx = 0$

(even $f(-x) = f(x)$)

$$f(x) = \begin{cases} x+1 & x < 0 \\ 1+x^2 & x \geq 0 \end{cases}$$



$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

$$= \int_{-1}^{0} (x+1)\,dx + \int_{0}^{1} (1+x^2)\,dx$$

Examples

- $\int (x - \sin x)^2 \, dx = \int (x^2 - 2x\sin x + \sin^2 x)\,dx$

$$= \int x^2 \, dx - 2\int x \sin x \, dx + \int \sin^2 x \, dx$$

$\int x^2\,dx = \frac{x^3}{3}$

$\int \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx$

$= \frac{1}{2}\left(\int 1 \, dx - \int \cos 2x \, dx \right)$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2} \right)$

by parts

$\int x \sin x \, dx = -x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x + C$

$\cancel{\frac{d}{dx} \cos^3 x = -3\cos^2 x \sin x}$

- $\int x \sin x^2 \, dx = \frac{1}{2} \int \sin u \, du$ $u = x^2$ $\frac{du}{dx} = 2x$

$= \frac{-\cos u}{2} + c$

$= \frac{-\cos x^2}{2} + c$

$\boxed{\text{even: } f(-x) = f(x)}$

- $\int_{-\pi/2}^{\pi/2} x \cos x^4 \, dx$ $= 0$

$f(x) = x \cos(x^4)$

$f(-x) = (-x) \cos((-x)^4)$

$= (-x) \cos(x^4)$

$= -f(x)$ odd

- $\int \frac{x^4 + 1/5}{x^5 + x} \, dx$ $x^5 + x = x(x^4+1)$

$= \frac{1}{5} \int \frac{5x^4+1}{x^5+x} \, dx$ $u = x^5 + x$ $\frac{du}{dx} = 5x^4 + 1$

$= \frac{1}{5} \int \frac{1}{u} \, du$

$= \frac{1}{5} \ln |x^5 + x| + c = \ln(|x^5+x|)^{1/5} + c$

- $\int \arcsin x \, dx = \int 1 \cdot \arcsin x \, dx = x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} \, dx$

$\int \frac{x}{\sqrt{1-x^2}} \, dx$ $u = 1 - x^2$ $\frac{du}{dx} = 2x$

$= \frac{1}{2} \int \frac{1}{\sqrt{u}} \, du$

$= \sqrt{u} + c$

$= \sqrt{1-x^2} + c$

$\int \arcsin x \, dx = x \arcsin x - \sqrt{1-x^2} + c$