MATH 2A03 Fall 2014 Midterm 1

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Midterm 1
Duration of test: 50 minutes
McMaster University
2014

Name:____

Student No.:____

You should aim to answer all the questions. You are not required to show your working or reasoning except where it is explicitly asked for, but partial credit may be given for correct working with an incorrect answer.
Please be sure to include your name and student number on all sheets of paper that you hand in.

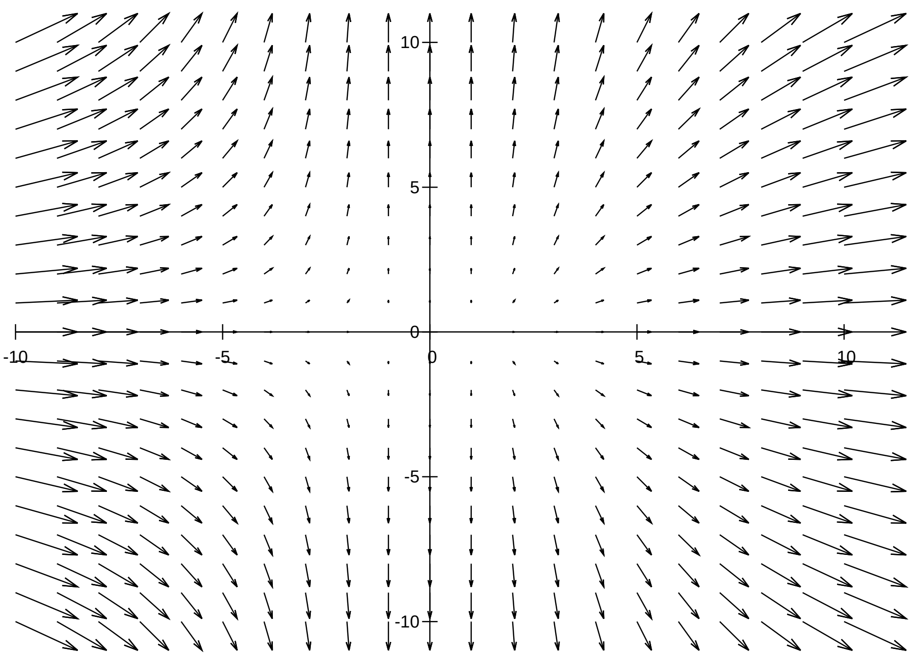
You may consult your textbook and course notes during the test.
The maximal number of marks on this test is 16.

Score

Question	1	2	3	4	5	6	Total
Points	2	2	2	3	2	5	16
Score							

1. [2] Below is the vector field representation of the gradient ∇f of a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$.

   Draw on the diagram a rough sketch of the level curve of f which passes through (0,5). (You may wish to use pencil, so you can correct any mistakes).



2. [2] Let

$$F(x,y) = (ye^x, \ln(x+y))$$

   and

$$f(x,y) = \ln(x) + y\cos(e^x).$$

   (a) Is f continuous at (1,0)? Is F continuous at (0,1)?

   (b) What is $\lim_{(x,y)\to(0,1)} f(F(x,y))$?

3. [2] Let

$$f(x,y) = x^3 + y^3 - 3x - 3y.$$

For what points (x,y) is the tangent plane to the surface $z = f(x,y)$ at $(x,y,f(x,y))$ parallel to the tangent plane to the surface at $(2,1,2)$?

4. [3] The temperature at a point of a metal plate expressed in Cartesian co-ordinates as (x,y) is

$$T_C(x,y) = x^2 + xy - y^2.$$

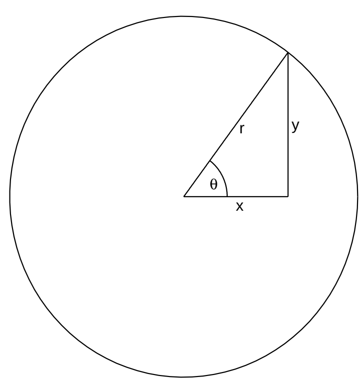
(a) [1] Express the temperature at a point (r,θ) in polar co-ordinates as a function of r and θ.

(b) [2] Use the chain rule to find formulae in polar co-ordinates (r,θ) for the first derivatives with respect to r and with respect to θ of the temperature at a point. Show your working.

(You may wish to check your answer by directly differentiating the function our found in part (a).)

[Recall: the function P converts from polar to Cartesian co-ordinates:

$$P(r,\theta) = (r\cos\theta, r\sin\theta).]$$



5. [2] Find

$$\frac{\partial^6}{\partial x \partial y \partial x \partial x \partial x \partial x}(yx^5 + e^{\cos x}).$$

6. [5] Let

$$f(x,y) = xy - (x+y)^3.$$

   (a) [1] Give the second-order Taylor expansion of f at a point (a,b),

   $$f(a+x, b+y) = \ldots$$

   (b) [2] At what points of $\mathbb{R}^2$ does f have a local minimum? At what points has it a local maximum?

   (c) [1] Calculate $f(-1,-1)$. Does f have a global maximum value on $\mathbb{R}^2$? If so, what is it? Similarly for global minimum. Briefly explain your reasoning.

   (d) [1] WARNING: this part of the question is hard! Don't spend time on it until you've done all you can on the rest of the test.

   Suppose we restrict the domain of f to the half-plane

   $$X := \{(x,y) \mid x+y \geq 0\}.$$

   Does f have a global maximum value on X? If so, what is it? Similarly for global minimum. Briefly explain your reasoning.