MATH 3TP3 Assignment #1

Due: Friday, 21st of September, in class

1. Find a derivation in the MIU-system of the string **MIUI**.

2. Write down **all** the derivations (including the "stupid" ones) in the MIU-system of length (number of lines) at most 3 (Hint: there's only 1 of length 1, there are 3 of length 2, and there are 12 of length 3).

3. Consider the system MIU+, the variant of the MIU-system obtained by adding as a fifth production rule: given **MU**x and **MU**y, produce **MU**xy. Find a derivation of **MU** in this system.

4. This question concerns the original MIU-system, **not** the MIU+-system considered in the previous question.

    (a) Let x be a string of **I**s of length 2^n for some integer $n \geq 0$. Show that **M**x is an MIU-theorem.

    (b) Let x be a string of **I**s whose length l is not a multiple of 3. Show that the string **M**x is an MIU-theorem. *Hint: first show that there is some number n such that 2^n is congruent to l modulo 3 and $2^n \geq l$.*

    (c) Let x be a string over the alphabet $\{\mathbf{I}, \mathbf{U}\}$ such that the number of occurrences of the symbol **I** in x is **not** a multiple of 3. Show that the string **M**x is an MIU-theorem.

    (d) From this and the solution to the **MU**-puzzle given in lectures, you can conclude that there **is** a decision procedure for MIU-theoremhood. Explain briefly how to decide, given an MIU-string S, whether or not S is an MIU-theorem.

BONUS: Consider the function C from natural numbers to natural numbers which takes n to $n/2$ if n is even, and to $3n + 1$ if n is odd. The Collatz conjecture (currently unsolved!) states that for every $n > 0$, if you repeatedly apply this function starting with n, you will eventually get 1 (i.e. $C(n) = 1$ or $C(C(n)) = 1$ or $C(C(C(n))) = 1$ or ...).

Your challenge, should you choose to accept it: find a formal system, of the kind we've been discussing, with an alphabet including **C**, such that the Collatz conjecture is true if and only if every string consisting entirely of **C**s is a theorem of the system.